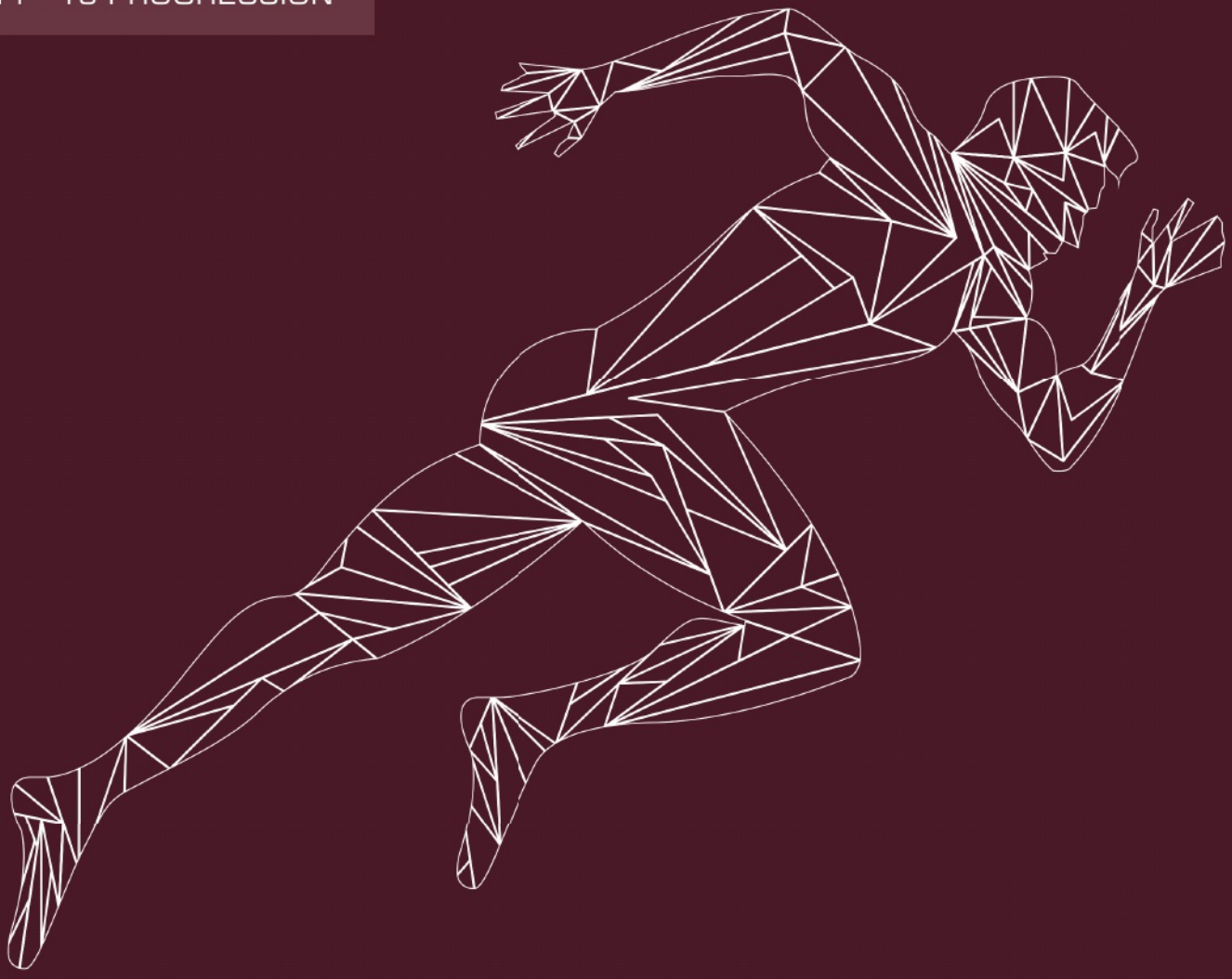
11 - 19 PROGRESSION



Pearson Edexcel A level Mathematics

Statistics and Mechanics

Year 2

Practice Book

Series Editor: Harry Smith
Authors: Laura Connell, Keith Gallick

Pearson

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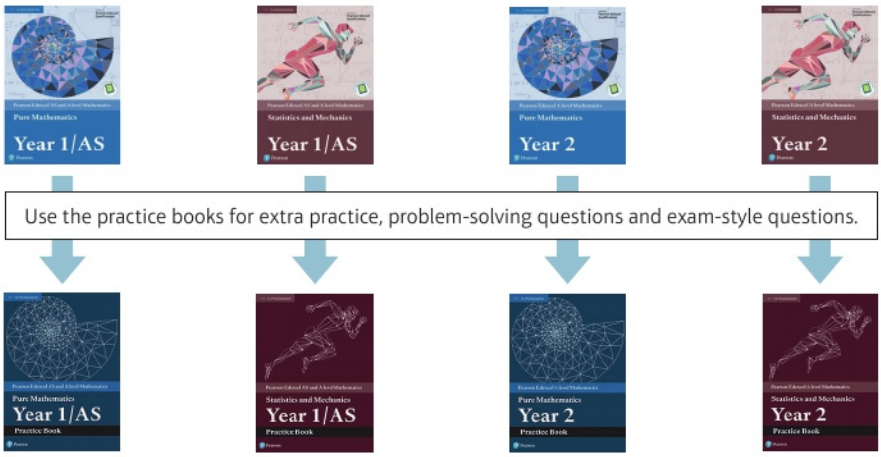
How to use this book

The Statistics and Mechanics Year 2 Practice Book is designed to be used alongside your Pearson Edexcel A level Mathematics Statistics and Mechanics Year 2 textbook. It provides additional practice, including problem-solving and exam-style questions, to help make sure you are ready for your exam.

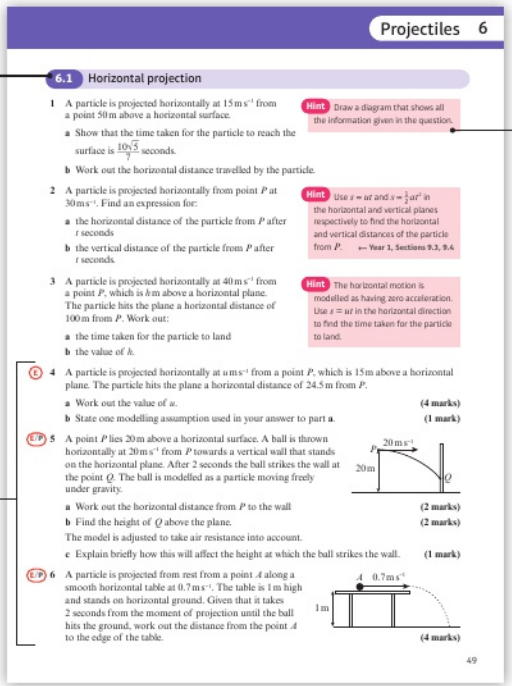
- The chapters and exercises in this practice book match the chapters and sections in your textbook, so you can easily locate additional practice for any section in the textbook.
- Each chapter finishes with problem-solving practice questions at three different difficulty levels.
- An Exam question bank at the end of the book provides mixed exam-style questions to help you practise selecting the correct mathematical skills and techniques.



Finding your way around the book

One-to-one match between exercises in this practice book and sections in your textbook.

Use the exam-style questions in every exercise to check that you are working at exam standard.



Projectiles 6

6.1 Horizontal projection

1 A particle is projected horizontally at 15 m s^{-1} from a point 50 m above a horizontal surface.

Hint Draw a diagram that shows all the information given in the question.

a Show that the time taken for the particle to reach the surface is $\frac{10\sqrt{5}}{3}$ seconds.

b Work out the horizontal distance travelled by the particle.

2 A particle is projected horizontally from point P at 30 m s^{-1}. Find an expression for:

Hint Use $s = ut$ and $s = \frac{1}{2}at^2$ in the horizontal and vertical planes respectively to find the horizontal and vertical distances of the particle from P. ← Year 1, Sections 9.3, 9.4

a the horizontal distance of the particle from P after t seconds

b the vertical distance of the particle from P after t seconds.

3 A particle is projected horizontally at 40 m s^{-1} from a point P, which is h m above a horizontal plane. The particle hits the plane a horizontal distance of 100 m from P. Work out:

Hint The horizontal motion is modelled as having zero acceleration. Use $s = ut$ in the horizontal direction to find the time taken for the particle to land.

a the time taken for the particle to land

b the value of h.

E 4 A particle is projected horizontally at u m s^{-1} from a point P, which is 15 m above a horizontal plane. The particle hits the plane a horizontal distance of 24.5 m from P.

a Work out the value of u. **(4 marks)**

b State one modelling assumption used in your answer to part **a**. **(1 mark)**

E/P 5 A point P lies 20 m above a horizontal surface. A ball is thrown horizontally at 20 m s^{-1} from P towards a vertical wall that stands on the horizontal plane. After 2 seconds the ball strikes the wall at the point Q. The ball is modelled as a particle moving freely under gravity.

a Work out the horizontal distance from P to the wall **(2 marks)**

b Find the height of Q above the plane. **(2 marks)**

The model is adjusted to take air resistance into account.

c Explain briefly how this will affect the height at which the ball strikes the wall. **(1 mark)**

E/P 6 A particle is projected from rest from a point A along a smooth horizontal table at 0.7 m s^{-1}. The table is 1 m high and stands on horizontal ground. Given that it takes 2 seconds from the moment of projection until the ball hits the ground, work out the distance from the point A to the edge of the table.

(4 marks)

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Hints in each exercise remind you of the key skills, formulae or techniques for that section. If you need more help, look at the corresponding section of your textbook.

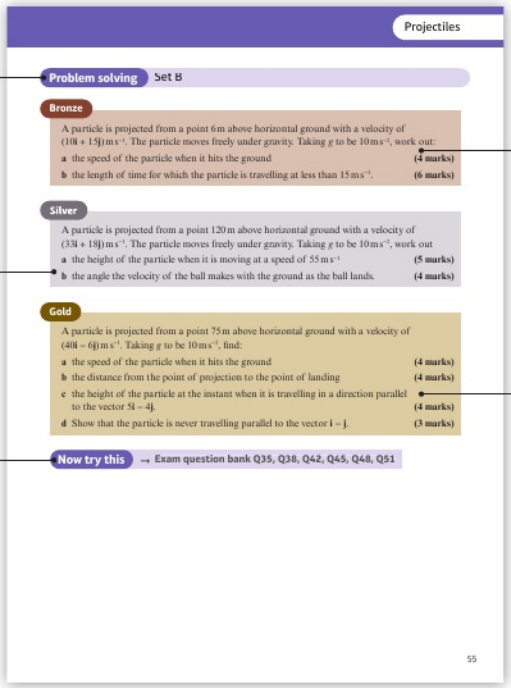
Exam-style questions are flagged with (E) and have marks allocated to them.

Problem-solving questions are flagged with (P)

Each chapter ends with two sets of exam-style problem-solving questions which draw on material from throughout the chapter and from earlier chapters.

Silver questions are more challenging, and provide less scaffolding. If you're struggling with the Silver question, try the Bronze question first.

You can find more exam-style questions on this chapter in the Exam question bank.



Projectiles

Problem solving Set B

Bronze

A particle is projected from a point 6 m above horizontal ground with a velocity of $(10\mathbf{i} + 15\mathbf{j})\,\text{m s}^{-1}$. The particle moves freely under gravity. Taking g to be $10\,\text{m s}^{-2}$, work out:

a the speed of the particle when it hits the ground **(4 marks)**

b the length of time for which the particle is travelling at less than $15\,\text{m s}^{-1}$. **(6 marks)**

Silver

A particle is projected from a point 120 m above horizontal ground with a velocity of $(33\mathbf{i} + 18\mathbf{j})\,\text{m s}^{-1}$. The particle moves freely under gravity. Taking g to be $10\,\text{m s}^{-2}$, work out

a the height of the particle when it is moving at a speed of $55\,\text{m s}^{-1}$ **(5 marks)**

b the angle the velocity of the ball makes with the ground as the ball lands. **(4 marks)**

Gold

A particle is projected from a point 75 m above horizontal ground with a velocity of $(40\mathbf{i} - 6\mathbf{j})\,\text{m s}^{-1}$. Taking g to be $10\,\text{m s}^{-2}$, find:

a the speed of the particle when it hits the ground **(4 marks)**

b the distance from the point of projection to the point of landing **(4 marks)**

c the height of the particle at the instant when it is travelling in a direction parallel to the vector $5\mathbf{i} - 4\mathbf{j}$. **(4 marks)**

d Show that the particle is never travelling parallel to the vector $\mathbf{i} - \mathbf{j}$. **(3 marks)**

Now try this → Exam question bank Q35, Q38, Q42, Q45, Q48, Q51

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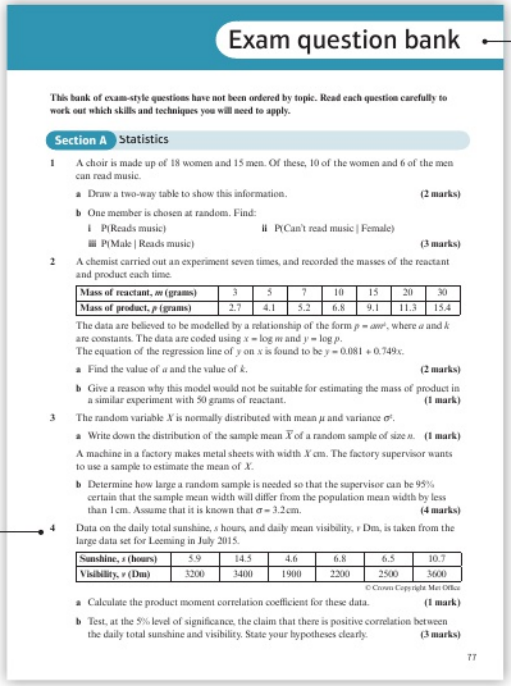
Bronze questions might have more steps to lead you through the technique, or require a more straightforward application of the skills from that chapter.

Gold questions involve tricky problem-solving elements, or might require you to think more creatively. If you can answer the Gold questions then you can be confident that you are ready to tackle the hardest exam questions.



Exam question bank

This bank of exam-style questions have not been ordered by topic. Read each question carefully to work out which skills and techniques you will need to apply.

Section A Statistics

1 A choir is made up of 18 women and 15 men. Of these, 10 of the women and 6 of the men can read music.

a Draw a two-way table to show this information. **(2 marks)**

b One member is chosen at random. Find:

i P(Reads music) ii P(Can't read music | Female)

iii P(Male | Reads music) **(3 marks)**

2 A chemist carried out an experiment seven times, and recorded the masses of the reactant and product each time.

Mass of reactant, m (grams)	3	5	7	10	15	20	30
Mass of product, p (grams)	2.7	4.1	5.2	6.8	9.1	11.3	15.4

The data are believed to be modelled by a relationship of the form $p = am^k$, where a and k are constants. The data are coded using $x = \log m$ and $y = \log p$.
The equation of the regression line of y on x is found to be $y = 0.081 + 0.749x$.

a Find the value of a and the value of k. **(2 marks)**

b Give a reason why this model would not be suitable for estimating the mass of product in a similar experiment with 50 grams of reactant. **(1 mark)**

3 The random variable X is normally distributed with mean μ and variance σ^2.

a Write down the distribution of the sample mean $\bar{X}$ of a random sample of size n. **(1 mark)**

A machine in a factory makes metal sheets with width X cm. The factory supervisor wants to use a sample to estimate the mean of X.

b Determine how large a random sample is needed so that the supervisor can be 95% certain that the sample mean width will differ from the population mean width by less than 1 cm. Assume that it is known that $\sigma = 3.2$ cm. **(4 marks)**

4 Data on the daily total sunshine, s hours, and daily mean visibility, v Dm, is taken from the large data set for Leeming in July 2015.

Sunshine, s (hours)	5.9	14.5	4.6	6.8	6.5	10.7
Visibility, v (Dm)	3200	3400	1900	2200	2500	3600

a Calculate the product moment correlation coefficient for these data. **(1 mark)**

b Test, at the 5% level of significance, the claim that there is positive correlation between the daily total sunshine and visibility. State your hypotheses clearly. **(3 marks)**

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One challenge of the exam is that you aren't usually told which techniques or strategies you need to apply to a particular question. The questions in the Exam question bank are not ordered by topic, so you need to choose the appropriate mathematical skills.

There are a lot more questions in the Exam question bank than there will be on your exam paper. Don't try and tackle them all at once, but make sure you try some of the trickier questions from the end of each section of the question bank.

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1.1 Exponential models

1 Data are coded using $Y = \log y$ and $X = x$ to give a linear relationship.

The equation of the regression line for the coded data is $Y = 0.7 + 1.2X$.

a State whether the relationship between y and x is of the form $y = ax^n$ or $y = kb^x$.

b Write down the relationship between y and x and find the values of the constants.

Hint Use the change of variable to find an expression for $\log y$ in terms of x. Then use the fact that $10^{\log y} = y$ and the laws of indices to find the form of the relationship.

← Pure Year 1, Section 14.8

2 Data are coded using $Y = \log y$ and $X = \log x$ to give a linear relationship.

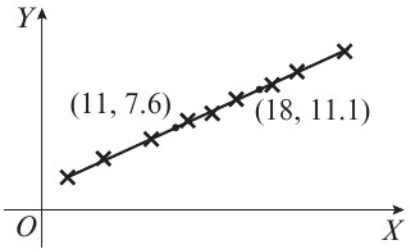
The equation of the regression line for the coded data is $Y = 1.5 + 0.8X$.

a State whether the relationship between y and x is of the form $y = ax^n$ or $y = kb^x$.

b Write down the relationship between y and x and find the values of the constants.

Hint Use the change of variable to find an expression for $\log y$ in terms of $\log x$.

Ⓟ **3** The scatter diagram shows the relationship between two sets of coded data, X and Y, where $X = x$ and $Y = \log y$. The regression line of Y on X is shown, and passes through the points (11, 7.6) and (18, 11.1).



The relationship between the original data sets is modelled by an equation of the form $y = kb^x$. Find the values of k and b.

Ⓟ **4** The table shows the populations of the ten most populated districts of England. R is the population ranking and P is the population in 1000s.

R	1	2	3	4	5	6	7	8	9	10
P	1137	785	578	561	546	535	524	496	492	459

a Plot a scatter diagram showing $\log P$ against $\log R$.

b Comment on the correlation between $\log P$ and $\log R$.

c State whether your answer to part **b** supports the fact that the original data can be modelled by a relationship of the form $P = aR^n$.

The data are coded using the change of variable $x = \log R$, $y = \log P$.

The regression line of y on x is found to be $y = 3.00 - 0.355x$.

d Find an equation for P in terms of R, giving your answer in the form $P = aR^n$, where a and n are constants to be found.

(P) **5** Specific humidity can be expressed as the grams of water vapour per kilogram of atmospheric gas. The table shows the temperature, in °C, and the specific humidities recorded at different atmospheric temperatures.

Temperature, t (°C)	−40	−30	−20	−10	0	10	20	30	40
Specific humidity, h (g/kg)	0.1	0.3	0.75	2	3.5	7	14	26.5	47

a Plot a scatter diagram showing $\log h$ against t.

b Comment on the correlation between $\log h$ and t.

c Use your answer to part **a** to explain why an equation of the form $h = kb^t$, where k and b are constants, is likely to be a good model for the relationship between t and h.

d The regression line of $\log h$ on t is given as $\log h = 0.476 + 0.0327t$. Determine the values of the constants k and b in the equation given in part **c**.

6 Data are collected on the amounts of water, W million gallons per day, used to generate electricity, E million kilowatt hours, from a sample of hydroelectric generators. The data are coded using $x = \log W$ and $y = \log E$. It is found that a linear relationship exists between x and y and that the equation of the regression line of y on x is $y = 1.02x - 1.22$.

Find an equation for E in terms of W, giving your answer in the form $E = aW^n$, where a and n are constants to be found.

7 Data are collected on the lengths, l m, and masses, m g, of a sample of rainbow trout from a river. The data are coded using $x = l$ and $y = \log m$. It is found that a linear relationship exists between x and y and that the equation of the regression line of y on x is $y = 3.27x + 1.50$.

Find an equation to describe the relationship between m and l, giving your answer in the form $m = kb^l$, where k and b are constants to be found.

(E/P) **8** The table shows some data collected on the amounts of nitrogen, kg per hectare, added to an agricultural field and the harvest of grain, kg per hectare.

Nitrogen input, N (kg/ha)	101	142	169	255	325	408	481	564	601	635
Harvest of grain, H (kg/ha)	8600	10900	12500	16200	19100	22800	24400	27600	28700	29200

The data are coded using $x = \log N$ and $y = \log H$.

The regression line of y on x is found to be $y = 2.6 + 0.67x$.

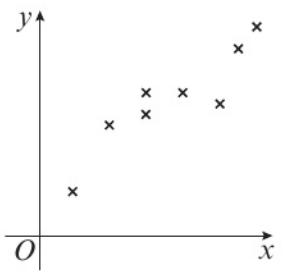
a Given that the data can be modelled by an equation of the form $y = ax^n$, where a and n are constants, find the values of a and n. **(3 marks)**

b Explain why this model is not reliable for estimating the harvest of a field with a nitrogen input of 900 kg/ha. **(1 mark)**

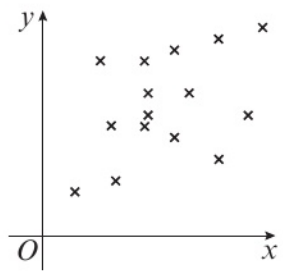
1.2 Measuring correlation

1 Suggest a value of r for each of these scatter diagrams.

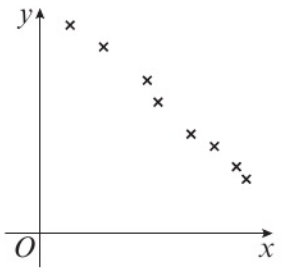
a



b



c



Hint The product moment correlation coefficient, r, measures linear correlation. The closer r is to -1 or 1, the stronger the negative or positive linear correlation, respectively.

2 The table shows 10 observations from a bivariate data set.

x	36	14	72	6	54	94	23	66	87	41
y	78	67	281	29	210	312	84	237	305	160

a State what is measured by the product moment correlation coefficient.

b Find the value of the product moment correlation coefficient between x and y.

Hint Use your calculator to find the product moment correlation coefficient. Give your answer to 4 decimal places.

3 The table shows the velocity of a river current at different depths within the channel.

Depth, d (m)	0.79	1.4	2.62	0.61	1.8	3.41	1.01	2.23	0.21	3.23
Velocity, v (ms^{-1})	0.43	0.42	0.18	0.34	0.35	0.07	0.42	0.28	0.47	0.12

a Use your calculator to find the value of the product moment correlation coefficient between d and v.

b Use your answer to part **a** to describe the correlation between the depth and velocity of the river based on this sample.

(P) **4** The table shows the populations of the ten most populated districts of England. R is the population ranking and P is the population in 1000s.

R	1	2	3	4	5	6	7	8	9	10
P	1137	785	578	561	546	535	524	496	492	459

The data are coded using $x = \log R$, $y = \log P$.

Calculate the product moment correlation coefficient for the coded data.

E/P **5** The table shows the masses of household waste, W, for ten towns with population P.

Population, P (1000s)	87	9	73	62	7	44	53	98	29	35
Waste, W (tonnes 1000s)	95	19	58	41	14	37	47	82	24	26
log W										

The data are coded using $x = P$ and $y = \log W$.

a Copy and complete the table showing the values of $\log W$. **(2 marks)**

b Calculate the product moment correlation coefficient for the coded data. **(1 mark)**

c With reference to your answer to part **b**, state whether an exponential model is a good fit for these data. **(2 marks)**

The equation of the regression line of y on x is found to be $y = 1.15 + 0.0085x$.

d Find an expression for W in terms of P, giving your answer in the form $W = ab^P$, where a and b are constants to be found. **(3 marks)**

E/P **6** The table shows the engine sizes, E litres, and fuel economies, M miles per gallon, for a sample of ten cars.

Engine size, E (litres)	3.2	3.2	2.8	2.5	2.8	1.8	5	6	2.3	3.5
Fuel economy, M (mpg)	24	26	26	27	26	29	21	19	30	23
log E										
log M										

The data are coded using $x = \log E$ and $y = \log M$.

a Copy and complete the table showing the values of $\log E$ and $\log M$. **(3 marks)**

b Calculate the product moment correlation coefficient for the coded data. **(1 mark)**

c With reference to your answer to part **b**, state whether a model of the form $y = kx^n$, where k and n are constants, is a good fit for these data. **(2 marks)**

The equation of the regression line of y on x is found to be $y = 1.59 - 0.383x$.

d Determine the values of k and n. **(3 marks)**

E **7** From the large data set, the daily mean air temperatures, t °C, and daily mean visibilities, v Dm, were recorded for Camborne on seven consecutive days in August 1987.

Temp, t (°C)	16.8	16.7	15.8	14.4	13.9	14.0	13.8
Visibility, v (Dm)	2100	700	2500	6300	6200	4700	3900

a Calculate the product moment correlation coefficient for these data. **(1 mark)**

b With reference to your answer to part **a**, comment on the suitability of a linear regression model for these data. **(2 marks)**

E **8** From the large data set, the daily maximum gusts, g kn and daily mean visibilities, v Dm, were recorded for Leuchars on seven consecutive days in July 1987.

Maximum gust, g (kn)	19	18	17	24	16	n/a	27
Visibility, v (Dm)	3100	3100	3100	4800	5100	1300	3200

a State the meaning of n/a in the table. **(1 mark)**

b Calculate the product moment correlation coefficient for these data. State how you have dealt with the data entry marked n/a. **(2 marks)**

Jamie states that because the value of r is close to 0, there is no relationship between daily maximum gust and daily mean visibility.

c Comment on Jamie's statement. **(1 mark)**

1.3 Hypothesis testing for zero correlation

1 A population of students each took two different tests. A sample of 30 students was taken from the population and their scores on the two tests were recorded. A product moment correlation coefficient of 0.4125 was calculated.

Test whether or not this shows evidence of positive correlation between the test scores:

a at the 5% level

b at the 1% level.

Hint You need to use a one-tailed test with $H_0: \rho = 0$, $H_1: \rho > 0$.

Use the table of critical values on page 92. ← **Year 1, Section 7.1**

2 A biologist takes a sample of 12 otters and records their masses and lengths. The product moment correlation coefficient of the data is found to be 0.62.

Test whether or not this shows evidence of correlation between the mass and length:

a at the 10% level

b at the 2% level.

Hint You need to use a two-tailed test with $H_0: \rho = 0$, $H_1: \rho \neq 0$.

Remember to halve the probability in each tail.

3 The table shows the engine sizes, E litres, and fuel economies, M miles per gallon, for a sample of six cars.

Engine size, E (litres)	3.8	5.7	2.4	3.4	2.2	2.4
Fuel economy, M (mpg)	30	28	30	28	34	33

a Calculate the product moment correlation coefficient for these data, correct to 3 significant figures.

b For these data, test $H_0: \rho = 0$ against $H_1: \rho \neq 0$, using a 5% significance level.

(P) **4** Each year for eight years, an oil company recorded the number of barrels of oil it produced that year and the total number of active wells it was using by the end of the year.

Year	2010	2011	2012	2013	2014	2015	2016	2017
Wells	46	157	237	337	419	501	601	657
Barrels of oil (millions)	5	21	17	20	25	28	24	30

The company believes that the number of active wells and the number of barrels of oil produced in a year are positively correlated and can be modelled by a linear relationship. Using a 1% significance level, test to see if there is evidence that the company's beliefs are correct.

(E) **5** A city council collects data on the population densities in different areas of the city, in people per hectare, and the distances of those areas from the city centre, in km. It calculates the product moment correlation coefficient between the two sets of data and finds it to be –0.51.

Given that the council collected data from 24 sample areas, test, at the 1% level of significance, the claim that there is a negative correlation between population density and the distance from the city centre. State your hypotheses clearly. **(3 marks)**

(P) **6** A zoologist believes that there is positive correlation between the lengths and masses of rainbow trout in a river. She decides to carry out a hypothesis test to see if there is evidence for her claim. She takes a random sample of 12 rainbow trout and finds that the correlation coefficient is 0.7.

Given that this result provided the zoologist with sufficient evidence to reject her null hypothesis, suggest the least possible significance level for the zoologist's test.

(P) **7** Data on the daily mean temperature and the rainfall is taken from the large data set for Perth in August 2015. A meteorologist finds that the product moment correlation coefficient for these data is –0.833. Given that the researcher tests for negative correlation at the 0.5% level of significance, and concludes that the value is significant, find the smallest possible sample size.

(E) **8** The Mercalli scale and the Richter scale are both used to measure earthquakes. The Mercalli scale is based on structural damage to buildings and the Richter scale is based on the amplitude of the ground vibrations. The table shows data from eight earthquakes.

Mercalli intensity, *M*	9	7	8	6	5	8	6	8
Richter magnitude, *R*	6.3	5.3	6.0	5.5	4.0	5.4	4.7	7.1

a Calculate, to 3 decimal places, the product moment correlation coefficient between M and R. **(1 mark)**

It is suggested that there is linear correlation between the structural damage and ground vibrations caused by an earthquake.

b Test this suggestion at the 1% significance level, stating your null and alternative hypotheses clearly. **(3 mark)**

E **9** An employee at a weather centre believes that there is positive correlation between the daily mean windspeed, w knots, and the daily maximum gust, g knots.

The employee takes a sample of data from Heathrow in May 2015.

Maximum windspeed, w (kn)	9	10	14	12	14	9	13	5
Maximum gust, g (kn)	21	22	41	24	30	25	29	19

a Calculate the product moment correlation coefficient for these data. **(1 mark)**

b Test, at the 2.5% level of significance, the employee's claim. State your hypotheses clearly. **(3 marks)**

Problem solving

Bronze

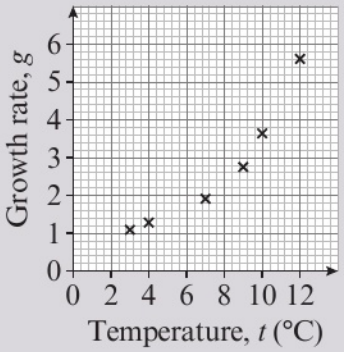
The table shows some data collected on body weights (in kilograms) and corresponding brain weights (in grams) for seven terrestrial mammals.

Body weight, x (kg)	53.6	0.01	250	519	3.38	0.55	1.45
Brain weight, y (g)	450	0.48	510	680	42.5	3.2	14.9

Test, at the 0.5% level of significance, whether these results show evidence of a positive linear relationship between x and y. State your hypotheses clearly. **(4 marks)**

Silver

A biologist collected data on the temperature, t °C, of a colony of bacteria and its growth rate g. The data are shown on the scatter diagram.



a With reference to the scatter diagram, explain why a linear regression model may not be suitable for the relationship between t and g. **(1 mark)**

The table shows the data collected.

Temperature, t (°C)	3	4	7	9	10	12
Growth rate, g	1.09	1.28	1.91	2.75	3.64	5.61

The data are coded using $x = t$ and $y = \log g$.

b Stating your hypothesis clearly, test, at the 5% significance level, whether the product moment correlation coefficient for the coded data is greater than 0. **(5 marks)**

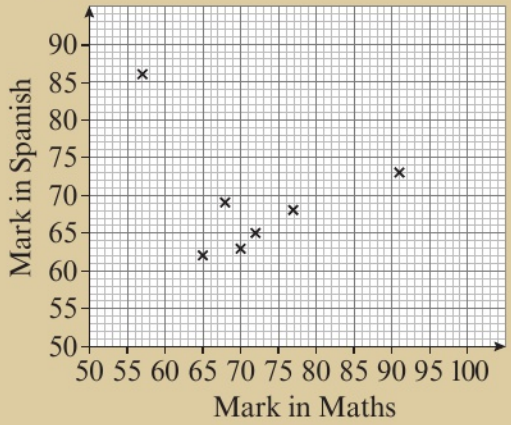
Gold

Seven students take a maths test and a Spanish test. The maths test is marked out of 100 and the Spanish test is marked out of 75.

The table shows the marks attained by the seven students in both tests.

Student	*A*	*B*	*C*	*D*	*E*	*F*	*G*
Maths	70	72	68	65	57	77	91
Spanish	63	65	69	62	86	68	73

A graph of the data is shown.



An outlier is defined as a data value that is more than 2 standard deviations from the mean.

a Show that the data for student *E* is an outlier. **(3 marks)**

A teacher claims that the results of the two tests are linearly correlated and wishes to use a hypothesis test to support this claim.

b By carrying out suitable tests at the 10% significance level, determine whether the removal of the data for student *E* will affect the teacher's conclusion. You should state the hypotheses for any tests, and label your working clearly. **(7 marks)**

c Give a reason to support the removal of the data for student *E*, and hence write a short conclusion based on your answer to part **b**. **(2 marks)**

Now try this → **Exam question bank Q2, Q4, Q7, Q10, Q14, Q17**

2.1 Set notation

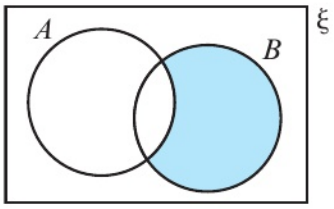
1 Use set notation to describe the shaded area in each of these Venn diagrams:

Hint The intersection of A and B is written as $A \cap B$.

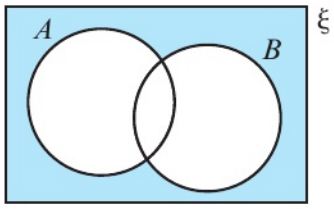
The union of A and B is written as $A \cup B$.

The complement of A is written as A'.

a



b



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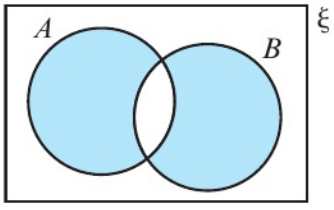
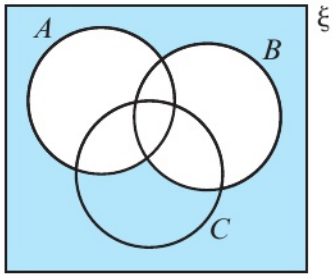
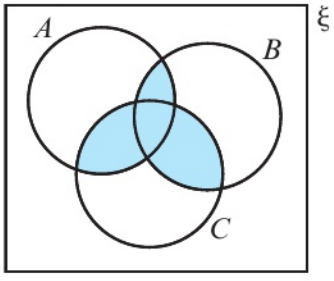


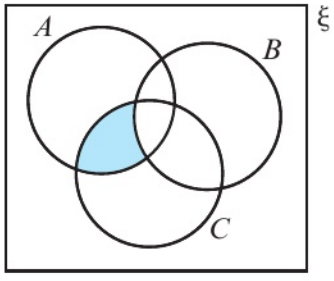
d



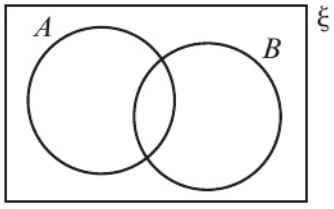
e



f



2 On separate copies of this Venn diagram, shade:

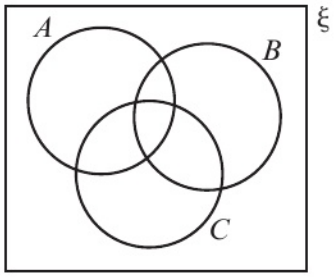


a $A \cap B'$

b $A' \cup B'$

c $(A \cap B) \cup (A' \cap B')$

3 On separate copies of this Venn diagram, shade:

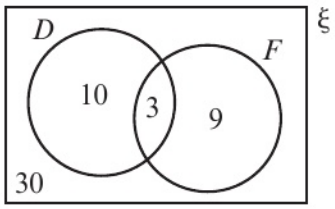


a $A \cup (B \cap C)$

b $(A \cup B) \cap C'$

c $(A \cup (B \cap C'))'$

4 A card is chosen at random from a pack of 52 playing cards. D is the event 'the card chosen is a diamond' and F is the event 'the card chosen is a Jack, Queen or King'.



The Venn diagram shows the number of outcomes for each event.

Find:

a $P(F)$ **b** $P(D)$ **c** $P(D \cap F)$

d $P(D \cup F)$ **e** $P(F')$ **f** $P(D' \cap F)$

5 A and B are two events. $P(A) = 0.4$, $P(B) = 0.3$ and $P(A \cap B) = 0.2$.

Find:

a $P(A \cup B)$ **b** $P(A')$

c $P(A' \cap B)$ **d** $P(A' \cap B')$

Hint Draw a Venn diagram showing the probabilities for each event. Remember that all the probabilities on your Venn diagram must add up to 1.

6 An insurance company surveys the behaviour of people who visit its website on one day. The results are that 40% of visitors purchase car insurance, 25% of visitors purchase home insurance and 45% of visitors purchase neither.

a Draw a Venn diagram to illustrate these percentages.

A visitor is chosen at random from those surveyed.

b Find the probability that the visitor purchased:

i car and home insurance

ii car insurance but not home insurance

iii home insurance but not car insurance.

(P) **7** A roulette wheel has 37 slots labelled 0 to 36. Slot 0 is green.

Half of the slots numbered 1 to 36 are red and half are black.

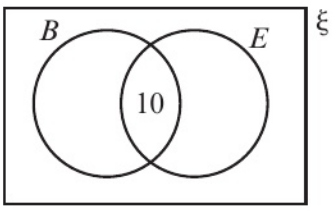
There are 10 black numbers that are even.

A ball is spun and lands in one of the slots.

B is the event that the ball lands in a black slot.

E is the event that the ball lands on an even number.

a Copy and complete the Venn diagram showing the numbers of possible outcomes.



b Find:

i $P(B \cup E)$ **ii** $P(B' \cap E)$ **iii** $P((B \cap E)')$

E/P **8** Twenty mathematicians from a university are attending a conference.

The tables show which of three seminars, A, B and C, each mathematician will be attending.

	A	B	C
Agnesi	✓		
Birman			✓
Cartwright		✓	
Darden	✓	✓	
Easley	✓		
Faddeeva		✓	✓
Germain		✓	
Hypatia	✓		
Inniss			✓
Johnson	✓		

	A	B	C
Kovalevskaya			
Lovelace			✓
Mirzakhani	✓		
Noether		✓	
Owens		✓	
Paulson	✓		
Quinn	✓	✓	
Robinson		✓	✓
Somerville	✓		
Tinney			

A mathematician is chosen at random from the twenty attendees.

A, B and C are the events that the mathematician attends each of the three seminars.

a Draw a Venn diagram showing the probabilities of each event. **(3 marks)**

b Show that:

i A and C are mutually exclusive

ii B and C are independent. **(3 marks)**

c Find:

i $P(A' \cap C')$

ii $P(A \cup (B' \cup C))$

iii $P((A \cup C)' \cap B)$ **(3 marks)**

2.2 Conditional probability

1 The two-way table shows the languages studied by 80 students in a sixth-form.

	French	**Spanish**	**Total**
Male	24	18	42
Female	16	22	38
Total	40	40	80

Hint The probability that a student is studying French given that they are male is written as P(French | Male). There are 42 males in total, and 24 of them are studying French.

Find:

a P(Male) **b** P(French | Male) **c** P(Male | Spanish) **d** P(Spanish | Female)

2 A green spinner and a yellow spinner each have four equally likely outcomes, numbered 1 to 4. The two spinners are spun at the same time, and the product of the numbers shown, X, is recorded.

Hint Draw your sample space diagram as a two-way table with 4 row headings and 4 column headings.

a Draw a sample space diagram for X.

b Find:

i $P(X = 8)$ **ii** $P(X = 6 | \text{Green spinner is } 2)$ **iii** $P(\text{Yellow spinner is } 2 | X = 4)$

3 Two fair six-sided dice are thrown and the larger number, Y, is recorded.

a Draw a sample space diagram to illustrate the possible outcomes.

b Find $P(Y = 4)$.

c Given that the first dice shows a 2, find the probability that $Y = 2$.

d Given that $Y = 6$, find the probability that the second dice shows a 3.

4 A fitness club has two types of membership, full and weekend.

The club has 120 members, of whom 65 are full members.

The table shows the exercise classes attended by the members.

	Kickboxing	**Yoga**
Full	36	42
Weekend	33	31

A member is chosen at random.

a Find:

i $P(\text{Weekend} | \text{Yoga})$ **ii** $P(\text{Full} | \text{Kickboxing})$ **iii** $P(\text{Yoga} | \text{Full})$

b Decide whether taking yoga and kickboxing are mutually exclusive.

Give a reason for your answer.

5 An integer between 1 and 20 inclusive is selected at random.
Given that it is not a prime number, find the probability that it is a square number.

E/P **6** A group of 140 sixth-form students are asked about their A level subjects. 82 study humanities subjects (H) and 64 study languages (L). Of those who study humanities, 36 also study languages.

a Draw a two-way table to show this information. **(2 marks)**

b One student is chosen at random. Find:

i $P(L')$

ii $P(H' \cap L)$

iii $P(H | L)$

iv $P(L | H')$ **(4 marks)**

c Decide whether the events L and H are independent. Give a reason for your answer. **(2 marks)**

E/P **7** A hotel had 540 guests during one month. Of these guests, 290 were male. 320 of the guests had the full breakfast and 160 of the guests had the continental breakfast. Of the remaining guests, 20 are female guests who did not have breakfast. 75 male guests had the continental breakfast.

One guest is chosen at random. Given that:

M is the event that a guest is male

C is the event that a guest had the continental breakfast

F is the event that a guest had the full breakfast

Find:

a $P(C' \cap F')$ **(1 mark)**

b $P(M \mid C')$ **(1 mark)**

c $P(F \mid M)$ **(1 mark)**

d $P((C' \cap F') \mid M)$ **(1 mark)**

E/P **8** The eating tastes of 60 cats are recorded. There are 38 adult cats, of which 11 like Furina, 17 like Purskers and the rest like Whilix. Of the kittens, 7 like Furina, 8 like Whilix and the rest like Purskers.

a Draw a two-way table to show this situation. **(2 marks)**

One cat is chosen at random.

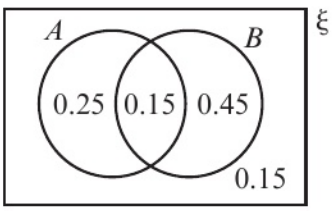
b Find:

i P(Adult cat | Furina) **ii** P(Kitten | Whilix) **iii** P(Purskers | Kitten) **(3 marks)**

c Given that one-third of the cats who like Furina are male, two-thirds of the cats who like Purskers are male and half of the cats who like Whilix are male, find the probability that a randomly chosen cat is male. **(2 marks)**

2.3 Conditional probabilities in Venn diagrams

1 The Venn diagram shows the probabilities for two events, A and B.



Find:

a $P(A \cup B)$ **b** $P(A \mid B)$ **c** $P(B' \mid A)$ **d** $P(B \mid A \cup B)$

Hint You can find conditional probabilities from a Venn diagram by considering the section of the Venn diagram that corresponds to the restricted sample space. For example, to find $P(A \mid B)$, just consider the circle B.

2 C and D are two events such that $P(C) = 0.6$, $P(D) = 0.5$ and $P(C \cap D) = 0.3$.

a Draw a Venn diagram showing the probabilities for events C and D.

b Find:

i $P(C \cup D)$ **ii** $P(C|D)$

iii $P(D|C')$ **iv** $P(D'|C)$

Hint To find $P(D|C')$ only consider the area outside the circle C.

3 S and T are two events such that $P(S) = 0.25$ and $P(T) = 0.8$.

Given that S and T are independent,

a draw a Venn diagram showing the probabilities for events S and T

b find:

i $P(S \cap T)$ **ii** $P(S|T')$ **iii** $P(T|S)$ **iv** $P(S' \cup T|T)$

4 A bag contains 150 counters. Some of the counters are black (B) and some of the counters are triangular (T). The bag contains 50 black counters, 75 triangular counters and 25 black triangular counters. A counter is picked from the bag at random. Find:

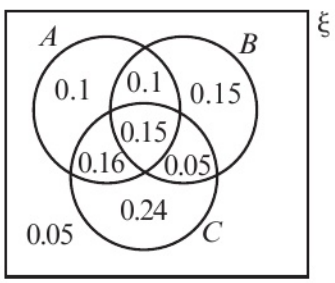
a $P(B' \cap T)$ **b** $P(T|B)$ **c** $P(B|T')$ **d** $P(B|B \cup T)$

5 A group of 50 students are asked whether they like burgers or curries. 36 said they liked burgers (B) and 24 said they liked curries (C), but 10 said they liked neither.

One student is chosen at random. Find:

a $P(B \cap C)$ **b** $P(C|B)$ **c** $P(B|C)$ **d** $P(B'|C')$

6 The Venn diagram shows the probabilities of three events, A, B and C.



Find:

a $P(A|B)$ **b** $P(B'|C)$

c $P((A \cup C)|B')$ **d** $P(A|(B' \cap C'))$

E/P **7** From the large data set, the daily total rainfalls, daily mean temperatures and daily total hours of sunshine were recorded for the 30 days of September in Hurn in 2015.

The event R is that the daily total rainfall is greater than the monthly mean.

The event S is that the daily total hours of sunshine is greater than the monthly mean.

The event T is that the daily mean temperature is greater than the monthly mean.

The table summarises the data.

Event	Number of days
only R	3
R and T but not S	4
only S	12
S and T but not R	4
only T	5
not R, S or T	2

a Draw a Venn diagram to represent this information. **(3 marks)**

b One day is selected at random. Given that only one of the monthly means has been exceeded, find the probability that the daily total rainfall has exceeded its monthly mean. **(2 marks)**

c Determine whether events S and T are statistically independent. **(3 marks)**

E/P **8** Three events, A, B and C, are such that A and B, A and C, and B and C are independent.

$P(A) = 0.4$, $P(B) = 0.1$, $P(C) = 0.25$ and $P(A \cap B \cap C) = 0.01$.

a Draw a Venn diagram to show the probabilities for events A, B and C. **(4 marks)**

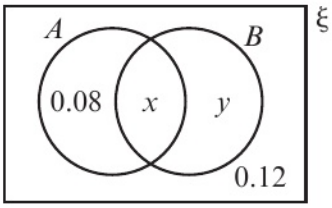
b Find:

i $P(A \mid C)$

ii $P(B' \mid A')$

iii $P(C \mid (A \cup B))$ **(3 marks)**

E/P **9** The Venn diagram shows the probabilities for two events, A and B. Given that $P(A \mid B) = P(A)$, find the values of x and y. **(3 marks)**



2.4 Probability formulae

1 A and B are two events.

$P(A) = 0.5$, $P(B) = 0.3$ and $P(A \cup B) = 0.6$.

Find:

a $P(A \cap B)$ **b** $P(A')$ **c** $P(A' \cup B)$ **d** $P(A \cup B')$

Hint $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

2 X and Y are two events.

$P(X) = 0.45$, $P(Y) = 0.6$ and $P(X \cup Y) = 0.75$.

a Use the probability addition formula to find $P(X \cap Y)$.

b Using a Venn diagram, or otherwise, find:

i $P(X' \cap Y')$ **ii** $P(Y \mid X)$ **iii** $P(Y \mid X')$

c State, with a reason, whether events X and Y are statistically independent.

3 A and B are two events.

$P(A) = 0.4$, $P(B) = 0.7$ and $P(A \mid B) = 0.5$.

Find:

a $P(A \cap B)$ **b** $P(B \mid A)$ **c** $P(A \cup B)$

Hint $P(B \mid A) = \dfrac{P(B \cap A)}{P(A)}$

$P(B \cap A) = P(B \mid A) \times P(A)$

(P) **4** E and F are two events where $P(E) = x$, $P(F) = 1.5x$, $P(E \cap F) = 0.3$ and $P(E' \cap F') = 0.05$.

a Use this information to write $P(E \cup F)$ in terms of x.

b Hence show that $x = 0.5$.

c Find:

i $P(F)$

ii $P(E \cap F')$

iii $P(E|F')$

5 Some students were asked whether their families owned pets. 40% said they owned a dog and 25% said they owned a cat. Half of the students said they owned either a dog or a cat or both. Find the probability that a randomly chosen student owns a cat but not a dog.

6 60% of visitors at an outdoor activity park go abseiling and 30% go canoeing. Given that a visitor goes canoeing, there is a 50% chance that they will go abseiling. A visitor is chosen at random.

a Find the probability that they:

i went canoeing but not abseiling

ii didn't go abseiling or canoeing.

b Given that the visitor chosen went abseiling, find the probability that they went canoeing.

7 C and D are two events where $P(C|D) = 0.2$, $P(C|D') = 0.6$ and $P(D) = 0.5$. Find:

a $P(C \cap D)$ **b** $P(C \cap D')$ **c** $P(C)$

d $P(D|C)$ **e** $P(D'|C)$ **f** $P(D'|C')$

(E/P) **8** Given that $P(A) = 0.5$, $P(B) = 0.35$ and $P(A \cap B) = 0.24$, find:

a $P(A \cup B)$ **(2 marks)**

b $P(A'|B)$ **(2 marks)**

The events B and C are mutually exclusive and the events A and C are independent.

$P(A \cap C) = 0.14$

c Find $P(C)$. **(2 marks)**

d Draw a Venn diagram to illustrate the events A, B and C, showing the probabilities for each region. **(4 marks)**

e Find $P((A \cup C')')$. **(2 marks)**

(E/P) **9** Three events A, B and C are such that $P(A) = 0.5$, $P(B) = 0.4$, $P(C) = 0.2$ and $P(A \cap B) = 0.1$.

Given that A and C are mutually exclusive and that B and C are independent, find:

a $P(B \cap C)$ **(1 mark)**

b $P(B|A)$ **(1 mark)**

c $P(A'|B)$ **(1 mark)**

d $P((B \cap C)|A')$ **(1 mark)**

E/P **10** The probability that Zahira gets home from work before 6 pm is 0.4.

The probability that she goes to the gym in the evening is 0.6.

The probability that she gets home from work after 6 pm and does not go to the gym is 0.35.

Let H represent the event that Zahira gets home after 6 pm and G represent the event that she goes to the gym in the evening.

Find:

a $P(G \cup H)$ **(2 marks)**

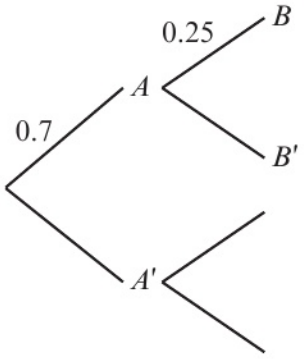
b $P(G \cap H')$ **(2 marks)**

c $P(H \mid G)$ **(3 marks)**

d State, with a reason, whether or not events G and H are independent. **(2 marks)**

2.5 Tree diagrams

1 A and B are two events such that $P(B \mid A) = 0.25$, $P(B \mid A') = 0.6$ and $P(A) = 0.7$.



a Copy and complete the tree diagram to represent this information.

b Find:

i $P(A \cap B)$ **ii** $P(A' \cap B')$

iii $P(B' \mid A)$ **iv** $P(A \mid B)$

2 A packet of 32 bulbs contains 18 tulips and 14 daffodils. A gardener chooses a bulb at random; the type of bulb is recorded and the bulb is not replaced. A second bulb is chosen and the type recorded.

a Draw a tree diagram to represent this information.

Find the probability the gardener chooses:

b two tulip bulbs

c one tulip and one daffodil

d two daffodils given that he chooses at least one daffodil.

Hint Since the bulbs are removed and not replaced, the total number of bulbs is reduced to 31 for the second pick.

P **3** Max has two bags. Bag A contains the same number of black counters and white counters. Bag B contains twice as many black counters as white counters.

Max chooses one of the bags at random and chooses a counter.

a Find the probability he chooses a black counter.

b Given that he chooses a white counter, find the probability that he chose from bag A.

(P) **4** Dev goes to gym every morning before work. He always uses either the treadmill or the exercise bike. If he uses the treadmill one day, the probability he uses the treadmill the next day is 0.5. If he uses the exercise bike one day, the probability he uses the exercise bike the next day is 0.6.

Dev uses the treadmill on Monday. Given that he also uses the treadmill on Thursday, find the probability that he used the exercise bike on Tuesday.

(E/P) **5** A fish tank contains 5 rainbowfish and 6 goldfish. A fish is selected at random from the tank and its type is recorded. The fish is moved to another tank. A second fish is selected at random and its type is recorded.

a Draw a tree diagram to represent the information. **(3 marks)**

Find the probability that:

b the second fish selected is a rainbowfish **(2 marks)**

c both fish selected are rainbowfish, given that the second fish is a rainbowfish. **(2 marks)**

(E/P) **6** During a football season, players A, B and C take all of the penalties for a team. Player A takes 35% of the penalties, player B takes 50% and the rest are taken by player C. Players A, B and C score penalties with probabilities 0.7, 0.8 and 0.6 respectively.

a Draw a tree diagram to represent this information. **(3 marks)**

b Find the probability that a randomly selected penalty is:

i taken by player B and missed **ii** scored. **(5 marks)**

c Given that a randomly selected penalty is missed, find the probability that it was taken by player C. **(3 marks)**

(E/P) **7** A factory produces microchips and 95% of the microchips have no defects.

A quality control test is applied to help determine whether or not a microchip is defective.

If a microchip has no defects, it passes the quality control test with probability 0.95.

If a microchip has defects, it passes the quality control with probability 0.08.

a Draw a tree diagram to represent this information. **(3 marks)**

b A microchip is selected at random and tested for defects. Find the probability that it passes the quality control test. **(3 marks)**

c A microchip is selected at random and tested for defects. Given that it fails the quality control test, find the probability that the microchip has no defects. **(2 marks)**

d Comment on the effectiveness of the quality control test. **(1 mark)**

(E/P) **8** Candidates taking a professional exam either pass, fail or are retested, with probabilities 0.1, 0.4 and 0.5 respectively. When a candidate is retested for the first time, the three possible outcomes and their probabilities remain the same as for the original test. When a candidate is retested for the second time there are just two possible outcomes: pass or fail, with probabilities 0.2 and 0.8 respectively.

a Draw a probability tree diagram to illustrate the outcomes. **(3 marks)**

b Find the probability that a randomly selected candidate passes. **(2 marks)**

c Find the probability that a randomly selected candidate is retested at least once, given that this candidate passes. **(3 marks)**

Problem solving Set A

Bronze

Bag A contains 7 black counters and 9 white counters.

Bag B contains 5 black counters and 6 white counters.

A counter is chosen at random from bag A and its colour is recorded. The counter is then added to bag B. A counter is then chosen at random from bag B and its colour is recorded.

a Draw a tree diagram to represent this information. **(3 marks)**

Find the probability of choosing:

b a black counter from bag B **(2 marks)**

c a white counter from bag A, given that a white counter is chosen from bag B. **(2 marks)**

Silver

A bag contains 11 red dice and 10 blue dice.

The red dice are all six-sided and fair.

The blue dice are also six-sided but are biased so that they are three times as likely to land on 4 as on any other outcome.

A dice is chosen at random from the bag and thrown.

Give that the score on the dice is 4, find the probability that the dice is blue. **(5 marks)**

Gold

A bag contains red and blue counters. There are three times as many blue counters as red.

Each counter is numbered 1, 2, 3, 4 or 5.

If a red counter is chosen at random, the number shown on the counter, X, has probability distribution

$$\mathrm{P}(X = x) = \begin{cases} \frac{x}{15} & x = 1, 2, 3, 4, 5 \\ 0 & \text{otherwise} \end{cases}$$

If a blue counter is chosen at random, the number shown on the counter, Y, has probability distribution

$$\mathrm{P}(Y = y) = \begin{cases} \frac{4 - y}{15} & y = 1, 2, 3 \\ a & y = 4 \\ b & y = 5 \\ 0 & \text{otherwise} \end{cases}$$

where a and b are constants.

A counter is chosen at random from the bag and the number shown on the counter is recorded.

a Find $\mathrm{P}(1)$. **(2 marks)**

b Given that $\mathrm{P}(\text{Blue} \mid 4) = \frac{27}{43}$, find the values of a and b. **(4 marks)**

Problem solving Set B

Bronze

Yvonne and Zoe both work for the same sales department.

Y is the event that Yvonne does not meet her weekly sales target.

Z is the event that Zoe does not meet her weekly sales target.

$P(Y) = 0.15$, $P(Y \cap Z) = 0.05$ and $P(Y' \cap Z') = 0.75$.

a For a randomly selected week, find the probability that:

i at least one of Yvonne or Zoe do not meet their weekly sales target

ii Zoe does not meet her weekly sales target. **(3 marks)**

b Given that Zoe does not meet her weekly sales target, find the probability that Yvonne does not meet her target. **(3 marks)**

The sales manager suspects that Yvonne not meeting her sales target and Zoe not meeting her sales target are linked in some way.

c Determine whether Y and Z are statistically independent. **(2 marks)**

d Comment on the manager's suspicion in the light of your calculation in part **d**. **(1 mark)**

Silver

The events A and B are independent and such that $P(A) = 3P(B)$ and $P(A \cup B) = \frac{7}{12}$

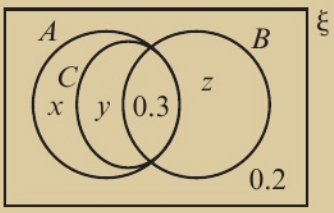
a Show that $P(B) = \frac{1}{6}$ **(5 marks)**

b Find $P(A \cap B)$. **(3 marks)**

c Find $P(B|A')$. **(2 marks)**

Gold

The events A, B and C are shown on this Venn diagram.



Given that $P(A|B) = 0.6$ and $P(C|A) = 0.75$, find the values of x, y and z. **(7 marks)**

Now try this → **Exam question bank Q1, Q5, Q9, Q12, Q16, Q18, Q20**

3.1 The normal distribution

1 State, with a reason, whether or not the normal distribution might be suitable for modelling each of the following:

Hint The normal distribution is used to model continuous data that take values grouped around a central value.

a X, the number of children in a randomly chosen family

b Y, the time taken for a particular employee to drive to work each day using the same route

c R, the reaction time recorded by a person in an experiment

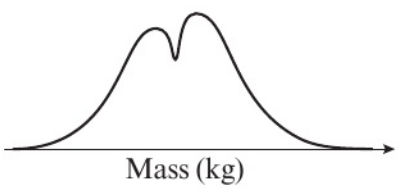
d L, the length of a particular breed of earthworm selected at random in a particular area.

2 The scores on an IQ test, X, scored by a population of adults are normally distributed with mean 100 and standard deviation 15. Sketch the distribution of X.

Hint The normal distribution curve is symmetrical about the mean and has points of inflection at $\mu + \sigma$ and $\mu - \sigma$. The horizontal axis is an asymptote at each end of the curve.

3 The masses, in kg, of a group of 200 people is shown.

State, with reasons, why the normal distribution is not a suitable model for this data.



4 The volume of soap dispensed by a soap-dispenser on each press, X ml, is modelled as $X \sim N(7, 0.9^2)$.

State the proportion of presses that dispense:

a between 6.1 ml and 7.9 ml

b between 5.2 ml and 8.8 ml.

5 The masses, M kg, of a population of badgers are modelled as $M \sim N(5, \sigma^2)$.

If 95% of the badgers have masses between 2.6 kg and 7.4 kg, find σ^2.

6 The weights of steel sheets produced by a factory are modelled as $W \sim N(\mu, 2.2^2)$.

If 16% of steel sheets weigh more than 34.7 kg, find μ.

(P) **7** The masses, Y grams, of a brand of chocolate bar are modelled as $Y \sim N(\mu, \sigma^2)$.

If 84% of the chocolate bars weigh more than 40.5 g and 2.5% of the chocolate bars weigh more than 45 g, find μ and σ.

8 The thicknesses of table cloths, T, produced by a loom are modelled as a normal distribution with mean 2.5 mm and standard deviation 0.15 mm. Find:

a $P(T < 2.5)$ **b** $P(2.35 < T < 2.5)$ **c** $P(T > 2.8)$

(P) **9** From the large data set, the daily mean temperatures, T °C, for Heathrow during May 1987 are shown below.

14.6	8.8	7.2	7.3	10.1	11.9	12.2
12.1	15.2	11.1	10.6	12.7	8.9	10.0
9.5	9.3	10.9	10.7	11.9	9.5	10.4
9.7	9.8	13.1	12.6	15.9	14.9	13.5
14.2	16.8	16.2				

a Use the temperature data to find:

i $P(T < 11.6)$ **ii** $P(9 < T < 14.2)$ **iii** $P(6.5 < T < 16.8)$

The data has mean 11.6 and standard deviation 2.6.

b Use your results from part **a** to decide whether the data can be modelled using the random variable $T \sim N(11.6, 2.6^2)$.

3.2 Finding probabilities for normal distributions

1 The random variable $X \sim N(40, 4^2)$.

Find:

a $P(X < 43)$ **b** $P(X > 36)$ **c** $P(X \geqslant 38.6)$

Hint Use the normal cumulative distribution function on your calculator to find probabilities from a normal distribution.

2 The random variable $X \sim N(50, 25)$.

Find:

a $P(X > 58)$ **b** $P(X \leqslant 45)$ **c** $P(45 < X < 57)$

Hint To find $P(X > 58)$ on your calculator, choose an upper limit that is at least 5 standard deviations away from the mean, such as 100 or 500.

3 The random variable $Y \sim N(36, 36)$.

Find:

a $P(Y < 30)$ **b** $P(30 < Y < 40)$

c $P(Y > 31.7)$

Hint Make sure you enter the standard deviation into your calculator, and not the variance.

4 The random variable $X \sim N(22, 8)$.

Find:

a $P(X > 20)$ **b** $P(X < 18)$ **c** $P(20.5 < X < 21.8)$

5 The random variable $M \sim N(10, 1.2^2)$.

a Find:

i $P(M > 11)$ **ii** $P(M < 11)$

b Calculate the sum of your answers to parts **a i** and **ii** and comment on your answer.

6 The random variable $T \sim N(5.2, 0.5)$.

a Find $P(T < 4.8)$.

b Without further calculation, write down $P(T > 4.8)$.

(P) **7** The random variable $Y \sim N(62, 5^2)$. Find:

a $P(Y < 58 \text{ or } Y > 65)$ **b** $P(Y < 60 \text{ or } 65 < Y < 70)$

(E) **8** The lengths of screws, L mm, produced by a machine are modelled as $L \sim N(100, 2.5^2)$.

a Find:

i $P(L > 104)$

ii $P(L < 99)$ **(2 marks)**

Five screws are selected at random.

b Find the probability that all of the screws are longer than 99 mm. **(2 marks)**

(E/P) **9** A reading test is given to a group of year 2 students. The scores on the test are found to be normally distributed with $X \sim N(80, 12^2)$.

a Find:

i $P(X > 90)$

ii $P(X < 65)$ **(2 marks)**

A class of 25 year 2 students take the test. If three or more students score less than 60 then the class will have to retake the test.

b Find the probability that the class will have to retake the test. **(4 marks)**

(E/P) **10** The amount of jam, J grams, in each jar produced by a factory is modelled as $J \sim N(454, 5.6^2)$. A jar is selected at random.

a Find the probability that the jar contains more than 460 g of jam. **(1 mark)**

The jars pass a quality control test if they contain between 452 g and 458 g of jam.

b Find the probability that a jar will pass the quality control test. **(2 marks)**

A sample of 18 jars is taken.

c Find the probability that at least 15 pass the quality control test. **(4 marks)**

(E/P) **11** The heights of a group of female athletes are modelled by a normal distribution with mean 165 cm and standard deviation 4.8 cm. The weights of this group of athletes are modelled by a normal distribution with mean 65 kg and standard deviation 6.7 kg.

a Assuming that for these athletes height and weight are independent, find the probability that a randomly chosen athlete is taller than 170 cm and weighs more than 68 kg. **(4 marks)**

b Comment on the assumption that height and weight are independent. **(1 mark)**

3.3 The inverse normal distribution function

1 The random variable $X \sim N(40, 3^2)$. Find the value of a, to 2 decimal places, such that:

a $P(X < a) = 0.25$ **b** $P(X < a) = 0.75$

c $P(X > a) = 0.4$

Hint Use the inverse normal distribution function on your calculator. In part **c**, use that fact that $P(X > a) + P(X < a) = 1$ to find the area to the **left** of a before using your calculator.

2 The random variable $X \sim N(15, 2^2)$. Find the value of a, to 2 decimal places, such that:

a $P(X < a) = 0.2$ **b** $P(X > a) = 0.75$

c $P(13 \leqslant X \leqslant a) = 0.4$ **d** $P(a < X < 18) = 0.43$

Hint Use the inverse normal distribution function on your calculator.
In part **c**, use that fact that
$P(13 \leqslant X \leqslant a) = P(X \leqslant a) - P(X < 13)$

3 The random variable $X \sim N(25, 15)$.

a Find the value of a and the value of b such that:

i $P(X < a) = 0.303$ **ii** $P(X > b) = 0.303$

b Find $P(b < X < a)$.

4 The random variable $Y \sim N(80, 25)$.

a Find the value of a and the value of b such that:

i $P(Y < a) = 0.212$ **ii** $P(Y < b) = 0.885$

b Find $P(a < Y < b)$.

(P) **5** The random variable $X \sim N(100, 36)$.

a Find values of a and b such that $P(a < X < b) = 0.6$.

b Explain briefly why there are other values of a and b with this property.

6 The distances travelled to work, D km, by the employees at a large company are normally distributed with $D \sim N(25, 7^2)$.

For this population, find:

a the upper quartile **b** the 30th percentile **c** the value of Q_2.

(E) **7** A packing plant fills bags with cement. The weight X kg of a bag of cement can be modelled by a normal distribution with mean 60 kg and standard deviation 3 kg.

a Find the value of a such that $P(X > a) = 0.4$. **(1 mark)**

b Find the interquartile range of the scores. **(2 marks)**

(E/P) **8** The weights of bags of popcorn, W g, produced by a factory are normally distributed with $W \sim N(175, 10^2)$.

a Find the value of b such that $P(Y < b) = 0.85$. **(1 mark)**

b Find the 20% to 80% interpercentile range of masses. **(2 marks)**

E/P **9** The heights of seedlings are normally distributed with mean 100 mm and standard deviation 6.4 mm.

The probability that a randomly selected seedling is at least $(100 + k)$ mm high is 0.1379.

Find, to the nearest integer, the value of k. **(2 marks)**

3.4 The standard normal distribution

1 For the standard normal distribution $Z \sim N(0, 1^2)$, find:

a $P(Z < 1.88)$ **b** $P(Z > 1.25)$

c $P(Z < -0.77)$ **d** $P(-1.64 < Z < 0)$

e $P(-2.55 < Z < -0.78)$ **f** $P(-1.15 < Z < 1.15)$

Hint The standard normal distribution has mean 0 and standard deviation 1.

2 For the standard normal distribution $Z \sim N(0, 1^2)$, find values of a such that:

a $P(Z < a) = 0.9177$ **b** $P(Z > a) = 0.0351$

c $P(Z > a) = 0.8315$ **d** $P(0 < Z < a) = 0.1700$

e $P(0 < Z < a) = 0.4970$ **f** $P(-a < Z < a) = 0.60$

Hint For part **f**, use the fact that the normal distribution is symmetrical about its mean.

3 The random variable $X \sim N(1.4, 0.25^2)$. For each of the following values of X, write down the corresponding value of the standardised normal distribution, $Z \sim N(0, 1^2)$.

a $x = 1.2$ **b** $x = 0.625$ **c** $x = 0.64$ **d** $x = 1.981$

Hint If $X \sim N(\mu, \sigma^2)$, and the values of X are coded using $Z = \frac{X - \mu}{\sigma}$, then $Z \sim N(0, 1^2)$.

4 The normal distribution $X \sim N(124, 10^2)$. Write in terms of $\Phi(z)$:

a $P(X < 136)$ **b** $P(X > 140)$ **c** $P(120 < X < 132)$

E **5** **a** Use the percentage points table to find a value of z such that $P(Z > z) = 0.05$. **(1 mark)**

A university offers academic awards to the top 5% of students taking an examination.

b Given that the scores can be modelled using a normal distribution with mean 65 and standard deviation 7.2, use your answer to part **a** to find the score necessary to receive an award. **(2 marks)**

E **6** **a** Use the percentage points table to find a value of z such that $P(Z < z) = 0.2$. **(1 mark)**

An electronics store offers a 'basic' insurance policy for laptops. The policy should be suitable for 20% of the lowest priced laptops sold in the store.

b Given that the prices of laptops can be modelled using a normal distribution with mean £177 and standard deviation £33, use your answer to part **a** to find the maximum price of a laptop for which the 'basic' insurance policy is suitable. Give your answer correct to the nearest pound. **(2 marks)**

E 7 **a** Use the percentage points table to find the values of z that correspond to the 20% to 80% interpercentile range. **(2 marks)**

A pottery student makes plates that are intended to be 20 cm in diameter. The actual diameter of each finished plate, D cm, can be modelled as $D \sim N(20, 1.5^2)$. The pottery student will be able to sell any plates that fall in the 20% to 80% interpercentile range.

b Use your answer to part **a** to find the largest and smallest diameters of plates which the student will be able to sell. **(2 marks)**

3.5 Finding μ and σ

1 The random variable $X \sim N(\mu, 4^2)$ and $P(X < 26) = 0.7734$.

Find the value of μ.

Hint You need to use the standard normal distribution.
Find the value of z such that $P(Z < z) = 0.7734$
Then use the coding formula $z = \frac{x - \mu}{\sigma}$ to find μ.

2 The random variable $X \sim N(17, \sigma^2)$ and $P(X > 20) = 0.1151$.

Find the value of σ.

3 The random variable $Y \sim N(\mu, 30)$ and $P(Y < 28) = 0.0721$.

a Find the value of μ. **b** Find $P(Y < 40)$

4 The random variable $Y \sim N(50, \sigma^2)$ and $P(Y > 40) = 0.6554$.

a Find the value of σ. **b** Find $P(45 < Y < 50)$

P **5** The random variable $X \sim N(\mu, \sigma^2)$.

Given that $P(X < 25) = 0.9401$ and $P(X < 22) = 0.8130$, find the values of μ and σ.

Hint Use the two probabilities given to form two equations involving μ and σ, then solve these equations simultaneously.

P **6** The random variable $Y \sim N(\mu, \sigma^2)$.

Given that $P(Y < 5) = 0.1817$ and $P(Y > 11) = 0.0345$, find:

a the value of μ and the value of σ **b** $P(4 < Y < 8)$

E/P **7** An angler is investigating the lengths, X cm, of trout caught at a particular location. She knows that the lower quartile of the lengths is 24.9 cm and the upper quartile of the lengths is 26.1 cm. She assumes that the lengths will be normally distributed.

a Give two reasons why this assumption might be valid. **(2 marks)**

b Find the mean and standard deviation of the distribution of lengths. **(3 marks)**

c Find the probability that a trout caught at random has a length between 24 cm and 25 cm. **(1 mark)**

P **8** The random variable $X \sim N(0, \sigma^2)$.

Given that $P(-5 < X < 5) = 0.75$, find the value of σ.

E/P **9** The random variable $X \sim N(1.45, \sigma^2)$.

Given that $P(X > 2a) = 0.3$ and $P(X < a) = 0.6$, find the value of σ and the value of a.

(6 marks)

E/P **10** The scores, X points, of a group of learner drivers in the hazard perception section of their theory test is modelled as a normal distribution with mean μ and standard deviation 7.2. A learner driver needs to score more than 44 points to pass the hazard perception test. Given that 39% of learner drivers pass the hazard perception test, find:

a the value of μ **(2 marks)**

b $P(48 < X < 53)$ **(1 mark)**

Three learner drivers are chosen at random.

c Find the probability that all of the drivers score more than 50 points. **(3 marks)**

E/P **11** The diameters of bolts, D mm, made by a particular machine are modelled using a normal distribution with mean 12.5 mm. Given that 80% of bolts have a diameter less than 13 mm, find:

a the standard deviation of the diameter **(2 marks)**

b the proportion of bolts with diameters between 12.4 mm and 12.6 mm. **(1 mark)**

A bolt is accepted if its diameter is between 12.3 mm and 12.7 mm.
A sample of 10 bolts is taken.

c Find the probability that at least 5 bolts will be accepted. **(3 marks)**

E/P **12** The masses of adult squirrels are found to be normally distributed with mean μ and standard deviation σ. Given that 1 in 3 squirrels have a mass less than 455 g and 1 in 5 have a mass greater than 500 g,

a sketch a diagram to represent this information **(2 marks)**

b find the value of μ and the value of σ **(6 marks)**

c find the 10th to 90th percentile range. **(2 marks)**

3.6 Approximating a binomial distribution

1 For each of the following binomial random variables, X:

i state, with reasons, whether X can be approximated by a normal distribution

ii if appropriate, write down the normal approximation to X in the form $N(\mu, \sigma^2)$, giving the values of μ and σ.

Hint When n is large and p is close to 0.5, you can approximate a binomial distribution $B(n, p)$ with a normal distribution $N(np, np(1-p))$.

a $X \sim B(250, 0.9)$ **b** $X \sim B(12, 0.48)$ **c** $X \sim B(120, 0.52)$

d $X \sim B(420, 0.45)$ **e** $X \sim B(600, 0.54)$ **f** $X \sim B(800, 0.74)$

2 The random variable $X \sim B(300, 0.47)$.
Use a suitable approximation to estimate:

a $P(X \leqslant 140)$

b $P(120 \leqslant X \leqslant 160)$

c $P(X = 150)$

Hint X is a discrete distribution so you need to use a continuity correction. If the binomial random variable X is approximated with the normal random variable Y, then $P(X \leqslant 140) \approx P(Y < 140.5)$

3 The random variable $X \sim B(250, 0.52)$. Use a suitable approximation to estimate:

a $P(X > 120)$ **b** $P(115 < X < 150)$ **c** $P(X = 125)$

4 A fair dice is thrown 125 times. Use a suitable approximation to estimate the probability of obtaining more than 60 even scores.

E **5** Students taking an intensive driving course pass their driving test with a 56% success rate.

A random sample of 15 students is taken and X pass their driving test.

a Find $P(X = 10)$ **(1 mark)**

A second random sample of 250 students is taken.

b Using a suitable approximation, find the probability that fewer than 130 students in the sample will pass their driving test. **(3 marks)**

c The probability that at least q students pass their test is 0.1. Find q. **(3 marks)**

E **6** It is known that 47% of patients calling a GP surgery are given a next day appointment.

A random sample of 80 patients is taken.

a Find the probability that exactly 40 patients are given a next day appointment. **(1 mark)**

b Find the percentage error when using a normal approximation to calculate the probability that exactly 40 patients are given a next day appointment. **(4 marks)**

E/P **7** It is suggested to a student that the random variable $X \sim B(220, 0.55)$ can be approximated by a normal random variable Y.

The student is asked to use this approximation to find an estimate for $P(X \leqslant 110)$.

The student's attempt is shown here.

$Y \sim N(\mu, \sigma^2)$

$\mu = np = 220 \times 0.55 = 121$

$\sigma = \sqrt{np(1-p)} = \sqrt{220 \times 0.55 \times (1 - 0.55)} = \sqrt{54.45}$

$Y \sim (121, 54.45)$

$P(X \leqslant 110) \approx P(Y \leqslant 110) = 0.068$

a Identify the error made by the student. **(1 mark)**

b Find the correct estimate. **(1 mark)**

Use the normal approximation to the binomial distribution to find estimates for:

c $P(X > 130)$ **(1 mark)**

d $P(105 < X < 120)$ **(1 mark)**

3.7 Hypothesis testing with the normal distribution

1 The distances, X metres, that Michael can hit a golf ball are normally distributed with mean 200 and standard deviation 15. Michael wants to test, at the 5% significance level, whether a new golf club has increased the distance that he can hit a golf ball.

Michael uses the new club 50 times and records the distances that he hits the ball.

a Write down a suitable test statistic.

b Write down two suitable hypotheses.

c Write down the distribution of the sample mean.

d Find the critical region for the test statistic.

Hint The sample mean has distribution $\overline{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$

2 A sample of size 30 is taken from a normal distribution $X \sim N(\mu, 5.7^2)$ and is found to have a sample mean of $\bar{x} = 16.6$.

The sample is used to test $H_0: \mu = 18$ against $H_1: \mu < 18$ at the 10% level of significance.

a Explain the condition under which the null hypothesis would be rejected.

b Find $P(\overline{X} < 16.6)$ and state the result of the hypothesis test.

3 The times a mobile phone battery lasts before needing to be recharged are assumed to be normally distributed with a mean of 45 hours and a standard deviation of 6 hours.

The manufacturers of the mobile phone claim to have developed a new battery with a longer battery life. The manufacturers test a sample of 50 batteries to determine whether this is true.

a Find, at the 5% level, the critical region for this test, stating your hypotheses clearly.

The mean battery life for the sample of 50 batteries was found to be 50 hours.

b Comment on this observation in light of the critical region.

Hint You may assume that the standard deviation of the battery lives is unchanged.

E **4** **a** Explain the difference between a one-tailed hypothesis test and a two-tailed hypothesis test for the mean of a normal distribution. **(1 mark)**

A sample of size 25 is taken from a normal distribution $X \sim N(\mu, 1.1^2)$, and is used to test $H_0: \mu = 34$ against $H_1: \mu \neq 34$ at the 1% significance level.

b Find the critical region for the test statistic and state the result of the hypothesis test. **(3 marks)**

The sample is found to have a mean of $\bar{x} = 33.7$.

c Comment on this observation in light of your answer to part **b**. **(1 mark)**

E **5** An automated pottery wheel is used to make bowls. The diameters of the bowls, D mm, are normally distributed with mean 200 mm and standard deviation 6 mm. After the wheel is repaired, a random sample of 20 bowls is taken and their diameters are measured to see if the mean diameter has altered.

The mean of the sample is found to be 201.5 mm.

Test, at the 10% significance level, whether there is evidence of a change in the mean diameter of bowls produced by the automated pottery wheel. **(4 marks)**

E **6** The masses of gentoo penguins on an island are found to be normally distributed with mean 5.9 kg and standard deviation 0.84 kg. A zoologist believes gentoo penguins in captivity have larger masses.

A random sample of 16 gentoo penguins are chosen from a zoo. The mean mass of the sample is found to be 6.6 kg.

Test, at the 1% level, whether this sample provides sufficient evidence to support the zoologist's claim. **(4 marks)**

E/P **7** The waiting times at a doctor's surgery are assumed to be normally distributed with mean 12 minutes and standard deviation 3.6 minutes.

A new patient check-in system is introduced to reduce waiting times. After the system is introduced the surgery manager wants to determine if the waiting times have been reduced.

The surgery manager takes a sample of 25 patient times.

a State suitable null and alternative hypotheses to test whether the new check-in system has reduced waiting times. **(1 mark)**

b Determine the maximum mean waiting time the surgery manager could observe in order to reject H_0 at the 1% significance level. **(4 marks)**

In fact, the surgery manager decides to test at the 10% significance level and observes a mean waiting time for the sample of 12.3 minutes.

c Explain briefly why the surgery manager can draw a conclusion without making any further calculations, and state this conclusion. **(2 marks)**

Problem solving Set A

Bronze

From experience, a heptathlete knows that she is able to throw her javelin a distance of 45 m or more once in every 5 attempts.

Given that the distances, X m, that the javelin is thrown can be assumed to be normally distributed with $X \sim N(40, \sigma^2)$,

a draw a sketch to show the distribution of X, labelling the area of the region represented by $X > 45$ **(2 marks)**

b find, to 2 decimal places, the value of σ. **(2 marks)**

The heptathlete throws her javelin.

c Calculate the probability that she throws it further than 42 m. **(2 marks)**

Silver

A machine cuts strips of metal to length L cm, where L is normally distributed with $L \sim N(\mu, \sigma^2)$.

Given that 20% of the cut lengths are less than 19.50 cm and 5% of the cut lengths are greater than 20.76 cm,

a find the value of μ and the value of σ. **(6 marks)**

Strips with lengths between 19.75 cm and 20.5 cm are graded as 'firsts', and can be sold for a higher price.

b Find the proportion of strips produced which can be graded as 'firsts'. **(2 marks)**

Gold

In a 'test-your-strength' game at a fairground, the players must hit a button with a hammer to make a pointer rise up a 2-metre column.

The game operator knows that players win no prize with probability $\frac{1}{5}$ and a small prize with probability $\frac{3}{4}$

The operator assumes that the heights, X cm, reached by players can be modelled using a normal distribution.

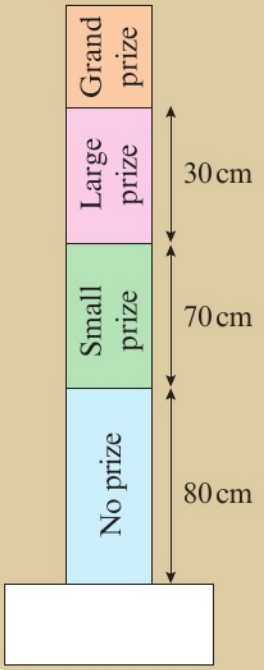
a Using this assumption, find the mean and standard deviation of this distribution. **(3 marks)**

On one day, 300 people play the game.

b Estimate the number of large prizes the fairground owner can expect to have to give out. **(2 marks)**

The fairground owner states that a normal distribution is not a suitable model as in reality it is not possible for players to reach heights above 2 m.

c With reference to your answer to part **a**, state, with justification, whether you agree with the fairground owner's statement. **(2 marks)**



Problem solving Set B

Bronze

Applicants for a job are given an aptitude test. The percentage, $X\%$, achieved by a randomly chosen applicant is modelled as $X \sim N(50, 10^2)$.

Applicants are scored as 'outstanding' if they achieve 65% or more.

A group of 8 applicants are assessed on one day.

a Find the probability that:

i at least one is scored as outstanding

ii exactly two are scored as outstanding

iii fewer than three are scored as outstanding. **(6 marks)**

An interviewer claims that the model is unsuitable as, according to the model, an applicant could score more than 100%.

b Evaluate the interviewer's claim. **(1 mark)**

Silver

The scores, X points, of a group of learner drivers in the theory test is modelled as a normal distribution with mean 38 and standard deviation 6.4. A learner driver needs to score more than 43 points to pass the test.

a Find:

i $P(X > 43)$ **ii** $P(X > 45)$ **(2 marks)**

b Explain briefly why $P(X > 43 \text{ and } X > 45) = P(X > 45)$. **(1 mark)**

A learner driver is chosen at random.

c Given that the driver passed the test, find the probability that the driver scored more than 45 points. **(2 marks)**

Gold

Anton is modelling the lifetime of low energy lightbulbs in his house. He finds that with normal usage, the lifetime, X days, of a particular type of bulb can be modelled as $X \sim N(720, 150^2)$.

Anton's dining room light fixture uses 6 of these bulbs.

Anton installs 6 new bulbs in this light fixture.

a Find the probability that at least 5 of them will last for more than 700 days. **(4 marks)**

After 400 days, none of the bulbs has failed.

b Find the probability that all 6 bulbs will continue to last for at least another 300 days. **(5 marks)**

After these 400 days, Anton decides to replace 4 of the bulbs at random with new bulbs.

c Find the new probability that all 6 bulbs will last for at least another 300 days. **(3 marks)**

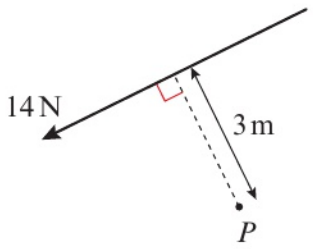
Now try this → **Exam question bank Q3, Q6, Q8, Q11, Q13, Q15, Q19**

4.1 Moments

1 Find the moment of the force acting about the point P.

Hint You should include the direction of rotation when you describe the moment of the force.

a



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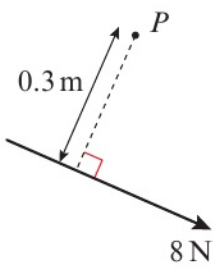
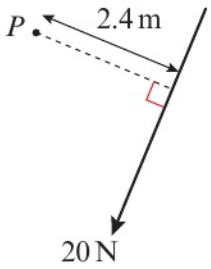


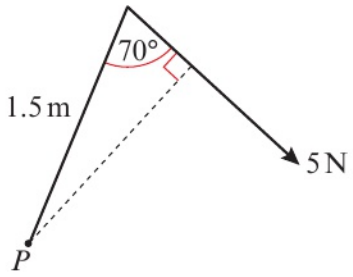
c



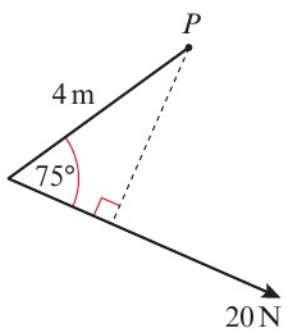
2 Find the moment of the force acting about the point P.

Hint Multiply the magnitude of the force by the perpendicular distance from the axis of rotation.

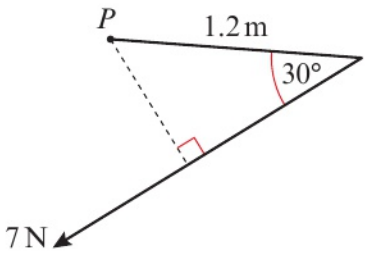
a



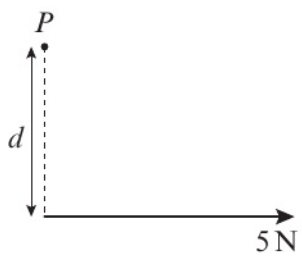
b



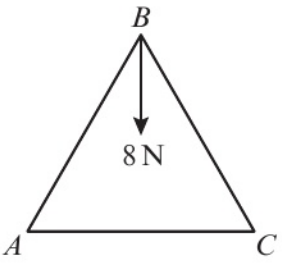
c



3 A force of 5 N acts about the point P as shown in the diagram.
Given that the force produces a moment of 1.5 N m in the anticlockwise direction, work out the distance d.



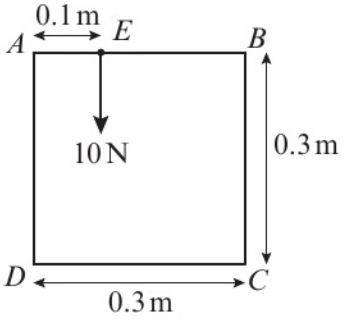
4 ABC is a lamina in the shape of an equilateral triangle of side 20 cm. A force of 8 N acts vertically downwards at B, as shown in the diagram.



Work out the moment of this force about:

a A **b** C

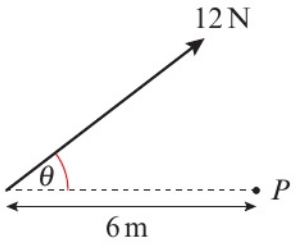
5 $ABCD$ is a lamina in the shape of a square of side 0.3 m. A force of 10 N acts vertically downwards at the point E that lies on the line AB, 0.1 m from A.



Work out the moment of this force about:

a A **b** C

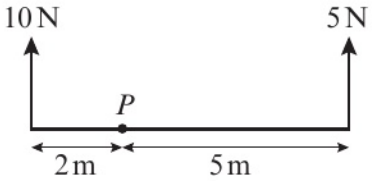
6 A force of 12 N produces a moment of 3600 N cm in the clockwise direction about the point P as shown in the diagram. Work out the size of angle θ.



E/P **7** A mass hangs from a horizontal wooden beam AB of length 0.6 m. The mass is fixed at a point C which lies on the beam between A and B. Given that the moment of the weight about A is twice the moment about B, work out the length AC. **(4 marks)**

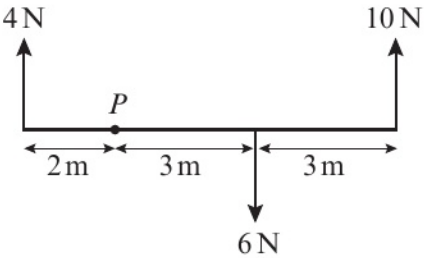
4.2 Resultant moments

1 The diagram shows the forces acting on a light rod.
Work out the resultant moment acting about the point P.



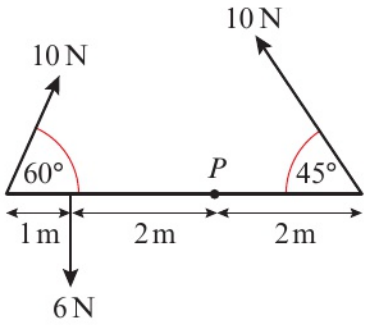
Hint Find the anticlockwise moment caused by the 5 N force and subtract the clockwise moment caused by the 10 N force.

2 The diagram shows the forces acting on a light rod.
Work out the resultant moment acting about the point P.



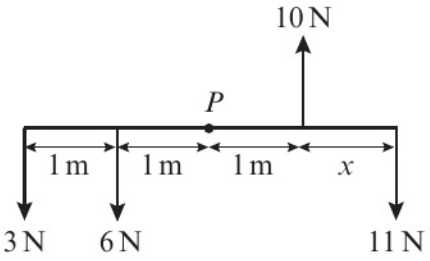
Hint Set clockwise as the positive direction, so that anticlockwise moments will be negative. Then add all the moments together. If the answer is negative, the resultant moment will act in the anticlockwise direction.

3 The diagram shows the forces acting on a light rod. Calculate the resultant moment acting about the point P.

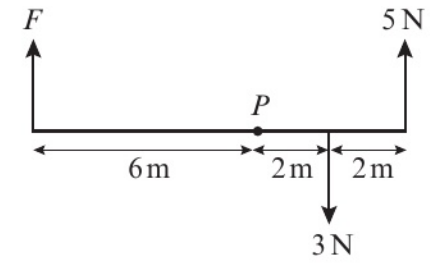


Hint First work out the perpendicular distance of each force from P.

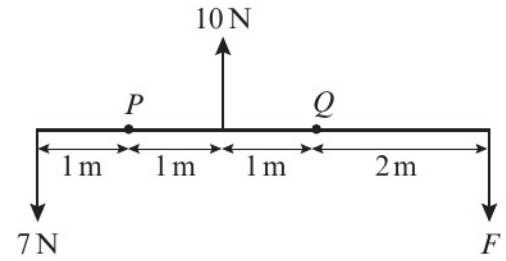
4 The diagram shows the forces acting on a light rod. Given that the resultant moment acting about P is 0, work out the length x.



E **5** The diagram shows the forces acting on a light rod. Given that the resultant moment acting about P is 5 N m in the clockwise direction, work out the value of F. **(4 marks)**



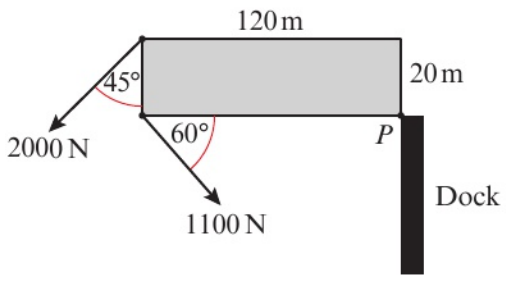
E **6** The diagram shows the forces acting on a light rod. Given that the resultant moment acting about Q is twice the resultant moment acting about P, and that both act in the clockwise direction, work out the value of F. **(3 marks)**



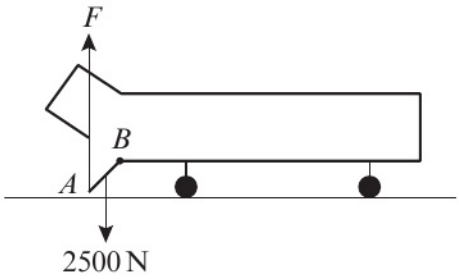
E/P **7** A barge is tethered to a dock at the point P and is being moved into position by two tugs. The forces exerted by the two tugs are shown in the diagram.

a Calculate the resultant moment acting about P. **(4 marks)**

b State one modelling assumption that you have used. **(1 mark)**



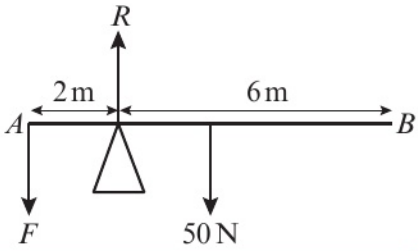
E/P **8** The loading door, AB, on a transport aeroplane is to be closed. The door is 4 m long and has a weight of 2500 N. The door is angled at 45° to the horizontal and hinged at B. A force of F N acts vertically upwards at A, as shown in the diagram.



By modelling the door as a uniform rod, show that in order for the door to move, F must be greater than 1250 N. **(3 marks)**

4.3 Equilibrium

1 A uniform rod of length 8 m and weight 50 N rests on a support that is 2 m from A. The rod is held in a horizontal position by a force F that acts vertically downwards at A.



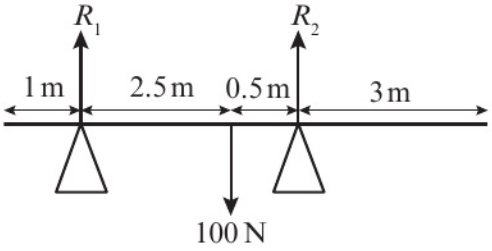
a Work out the magnitude of F.

Hint The rod is in equilibrium so the resultant moment is zero. This means the sum of the clockwise moments equals the sum of the anticlockwise moments.

b Work out the reaction at the support.

Hint Resolve vertically to obtain $R = F + 50$.

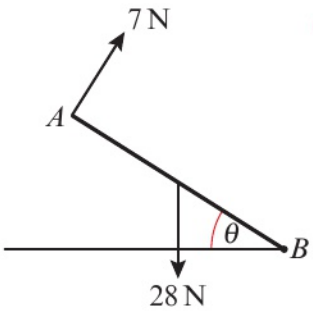
2 A uniform rod of length 7 m and weight 100 N rests horizontally on two supports.



Work out the values of the reactions R_1 and R_2 acting at the supports.

Hint Take moments about the point that makes the calculations as simple as possible.

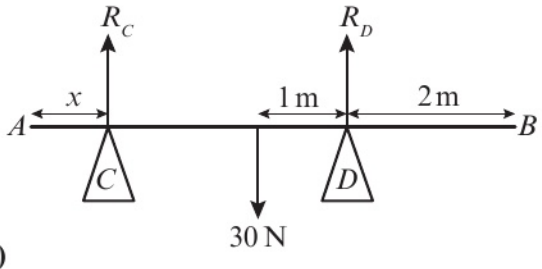
3 A uniform rod AB, of length 2 m and weight 28 N, is hinged at B, and is held in equilibrium by a force of 7 N acting perpendicularly to the rod at A. The rod makes an angle of θ with the horizontal, as shown in the diagram. Find the size of angle θ.



Hint By taking moments about B you can ignore the reaction at the hinge.

(P) **4** A uniform rod AB of mass m kg and length l m, is held horizontally in equilibrium by two vertical strings attached at points A and C where $AC : CB = 3 : 1$. Find, in terms of m, the tensions T_A and T_C in the strings attached to the rod at A and C respectively.

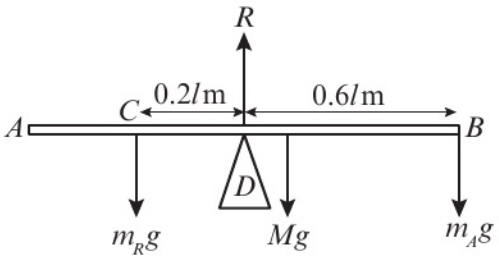
(E/P) **5** A uniform rod AB, of weight 30 N and length 6 m, sits horizontally in equilibrium on two supports at points C and D as shown in the diagram.



The reactions of the supports on the rod at points C and D are R_C and R_D respectively.
Given that $R_C = 4R_D$, work out the length x.

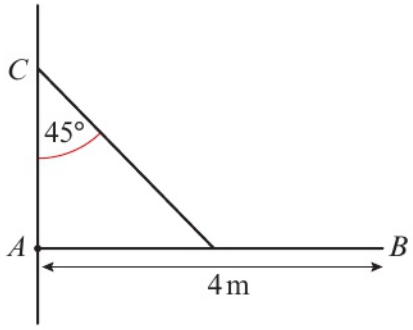
(4 marks)

(E/P) **6** A seesaw is modelled as a uniform rod AB of mass M kg and length l m. It rests on a support at D where $AD = 0.4l$ m. Ruairi, who has mass m_R kg, sits $0.2l$ m from the support at point C and Athena, who has mass m_A kg, sits at B as shown in the diagram.



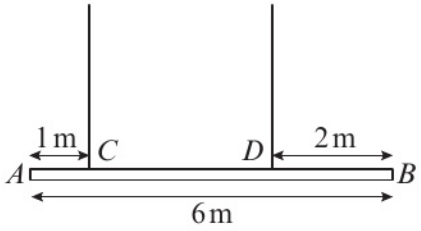
Given that the seesaw rests horizontally in equilibrium, show that $M = 2(m_R - 3m_A)$. **(3 marks)**

(E/P) **7** A uniform rod AB, of length 4 m and mass 10 kg, is hinged to a vertical wall at A. It is held in the horizontal position by a rope that is attached at one end to the midpoint of the rod and at the other end to the point C that lies vertically above A. The rope makes an angle of 45° with the vertical, as shown on the diagram.



Work out the tension in the rope. **(3 marks)**

(E/P) **8** A plank AB has mass 10 kg and length 6 m. It is held in equilibrium in a horizontal position by two ropes attached at the points C and D, where $AC = 1$ m and $DB = 2$ m.



The plank can be modelled as a uniform rod and the ropes as light strings.

a Work out:

i the tension in the rope at C

ii the tension in the rope at D. **(4 marks)**

A mass of 5 kg is placed on the plank at a distance x m from C, between C and D. The mass can be modelled as a particle.

Given that the plank remains in equilibrium in the horizontal position,

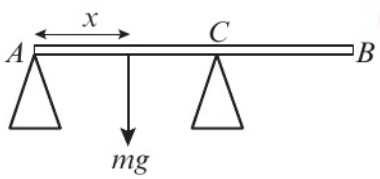
b find, in terms of x, an expression for the tension in the rope at D. **(3 marks)**

The rope at D will break if the tension exceeds $10g$ N. The rope at C cannot break.

c Work out the range of possible values of x. **(3 marks)**

4.4 Centres of mass

1 A non-uniform plank AB, of length 3 m, sits horizontally on supports at A and C, where $AC = \frac{3}{5}AB$. The reactions at A and C are equal and the centre of mass of the plank lies a distance of x m from A. Work out the distance x.



Hint The plank is non-uniform so the centre of mass is not at the midpoint of AB.

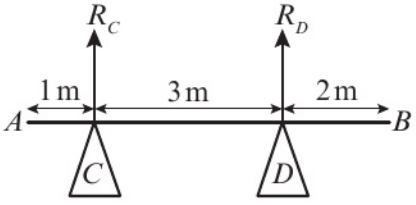
2 A non-uniform rod AB, of length 8 m and weight 2 N, rests on two supports positioned at A and B. The centre of mass of the rod lies 5.6 m from A.

a Draw a diagram to represent this situation. Include all the information you have been given.

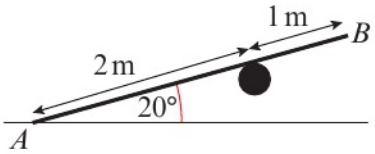
b Find the reactions acting on the two supports.

Hint Take moments about A to find the reaction at B, and vice versa.

3 A non-uniform rod AB, of length 6 m, sits horizontally on two supports at C and D as shown in the diagram. The centre of mass of the rod lies 2 m from A. A mass, m, is loaded onto the rod at B, and the reactions at the two supports are now equal. Find, in terms of m, the mass of the rod.



E/P **4** A non-uniform rod AB, of length 3 m and weight 25 N, rests on a smooth peg that supports the rod 2 m from A. The end A rests on horizontal ground such that the rod is angled at 20° to the horizontal.

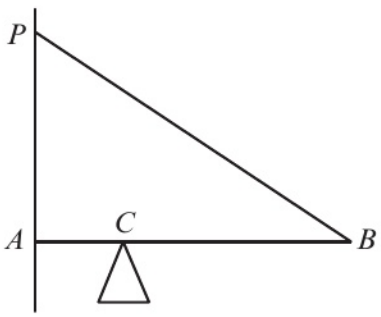


Given that the centre of mass of the rod lies 1.25 m from A, work out the magnitude of the reaction of the peg on the rod. **(4 marks)**

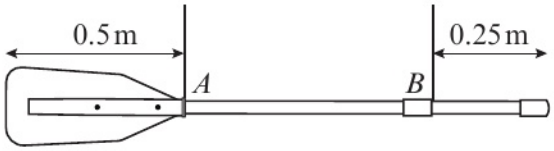
E/P **5** A non-uniform platform AB is used to stand on to clean windows on a skyscraper. It is 4 m long and has a weight of 200 N. The platform is suspended at each end by vertical ropes so that it hangs in horizontal equilibrium. A window cleaner of weight 600 N stands on the platform 1.5 m from A. Given that the tension in the rope at A is 450 N, work out the distance of the centre of mass of the platform from B. **(4 marks)**

E/P **6** A non-uniform rod AB, of length 3 m and weight 100 N, rests against a smooth vertical wall and on a support at the point C, where $AC = 1$ m. The rod is held in horizontal equilibrium by a rope connected to the rod at B, and to a fixed point P on the wall that lies 2 m vertically above A.

Given that the tension in the rope is 100 N, work out the distance of the centre of mass of the rod from A. **(4 marks)**



E/P **7** A rowing club has an oar hanging above its entrance door. The oar is suspended by two vertical ropes and held in the horizontal position as shown.



The oar is 1.5 m long and has a mass of 5 kg. It is modelled as a uniform rod and the ropes are modelled as light inextensible strings.

a Using this model, calculate:

i the tension in the rope at A

ii the tension in the rope at B. **(4 marks)**

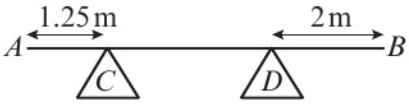
b Comment on the assumption that the oar can be modelled as a uniform rod. **(1 mark)**

The oar is now modelled as a non-uniform rod. The tension in the rope at A is four times the tension in the rope at B.

c Work out the distance of the centre of mass of the oar from B. **(3 marks)**

4.5 Tilting

1 A uniform beam AB, of mass 4 kg and length 5 m, rests horizontally on two supports positioned at C and D where AC is 1.25 m and DB is 2 m.



When a mass of m kg is placed at B, the beam is on the point of tipping about D.
Work out the value of m.

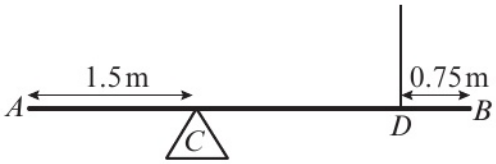
Hint When the beam is on the point of tipping about D, the reaction at C is zero.

2 A uniform beam AB, of mass m kg and length 3 m, rests horizontally on two supports positioned at C and D, where AC is 1.25 m and DB is 0.75 m.

When a mass of 3 kg is placed 0.5 m from A, the beam is on the point of tipping about C.
Work out the value of m.

Hint Take moments about C.

3 A uniform beam AB, of mass 20 kg and length 4 m, rests on a support positioned at C where AC is 1.5 m. It is held horizontal by a rope positioned vertically at D where DB is 0.75 m.



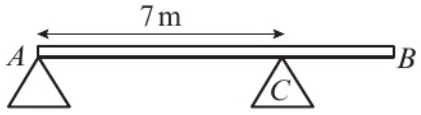
a Find the tension in the rope.

When a mass of 10 kg is placed x m from A, the beam is on the point of tipping about C.

b Work out the value of x.

Hint When the beam is on the point of tipping, the tension in the rope is zero.

E/P **4** A uniform plank AB, of mass 80 kg and length 10 m, rests on supports positioned at A and C where AC is 7 m.



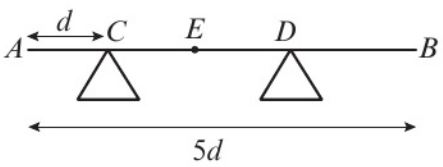
A man walks along the plank to the point D where the plank starts to tip. Given that $AD = 9$ m, calculate the mass of the man. **(4 marks)**

E/P **5** A non-uniform plank AB, of mass 120 kg and length 8 m, rests on supports positioned at A and C where AC is 5 m. The centre of mass of the plank lies $(3 + x)$ m from A. A mass of 150 kg is placed x m from C causing the plank to be on the point of tipping. Work out the value of x. **(5 marks)**

E/P **6** A uniform plank AB, of length 6 m, hangs in equilibrium in a horizontal position from two vertical ropes attached to the plank at C and D where AC is 2 m and DB is 1 m. When a weight of 300 N is attached at A, the plank is on the point of tipping.

The weight at A is removed and a different weight, W N, is attached at B. Given that the plank is on the point of tipping, work out the value of W. **(6 marks)**

E/P **7** A non-uniform rod AB, of mass $2m$ and length $5d$, rests horizontally in equilibrium on supports at C and D where $AC = d$ and $DB = 2d$. The centre of mass of the rod is at the point E. A particle of mass m is placed on the rod at B causing the rod to be on the point of tipping about D.

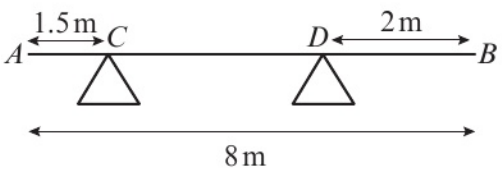


a Show that $ED = d$ **(2 marks)**

The particle is moved from B to the midpoint of the rod and the rod remains in equilibrium.

b Find, in terms of m, the magnitude of the normal reaction between the support at D and the rod. **(2 marks)**

E/P **8** A uniform rod AB of length 8 m and mass m kg rests horizontally in equilibrium on supports at C and D where $AC = 1.5$ m and $DB = 2$ m. A mass of 1.5 kg is placed on the rod 0.5 m from A, causing the rod to be on the point of tipping about C.



a Work out the mass of the rod, m. **(2 marks)**

The 1.5 kg mass is now moved to a position between D and B, at a distance of x m from D. Given that the rod remains in equilibrium,

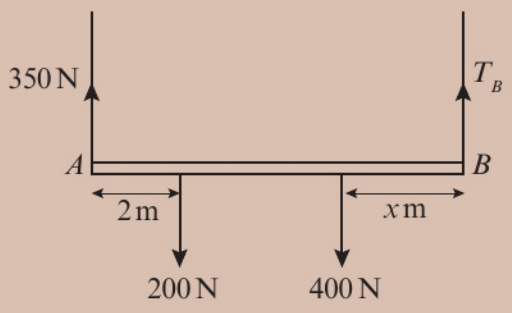
b find the range of possible values of x. **(2 marks)**

Problem solving Set A

Bronze

A non-uniform platform AB is 6 m long and has a weight of 400 N. Its centre of mass lies x m from B. It is suspended at each end by ropes so that it is horizontal. When a weight of 200 N is placed on the platform 2 m from A, the tension in the rope at A is 350 N.

Work out the value of x. **(3 marks)**



Silver

A non-uniform platform AB is 10 m long and has a weight of 900 N. It is suspended at each end by ropes so that it is horizontal. A weight of 600 N is placed on the platform 4 m from B. The tension in the rope at A is twice the tension in the rope at B.

Find:

a the tension in each rope **(2 marks)**

b the distance of the centre of mass of the platform from A. **(2 marks)**

Gold

A non-uniform platform AB is x m long and has a weight of W N. It is suspended at each end by ropes so that it is horizontal. A weight of $0.8W$ N is placed on the platform $0.25x$ m from B. The tension in the rope at A is half the tension in the rope at B.

Calculate the distance of the centre of mass of the platform from A. **(4 marks)**

Problem solving Set B

Bronze

A uniform plank AB, of weight 400 N and length 20 m, rests on supports positioned at C and D where AC is 5 m and DB is 8 m. A 500 N weight is placed on the plank at a distance x m from A.

Given that the plank remains in horizontal equilibrium, find the range of possible values of x. **(5 marks)**

Silver

A uniform plank AB, of weight 400 N and length 15 m, rests on supports positioned at C and D where AC is 5 m and BD is 2.5 m. A weight, W N, is placed 1 m from A and the plank is on the point of tilting about C.

Work out the value of W. **(4 marks)**

Gold

A uniform plank AB, of weight 400 N and length 6 m, rests on supports positioned at C and D where AC is x m and BD is $2x$ m. A weight, W N, is placed x m from B causing the plank to start to tilt.

Show that $x = \dfrac{1200}{W + 800}$ **(4 marks)**

Now try this → **Exam question bank Q21, Q24, Q28, Q31, Q37, Q41, Q50**

5.1 Resolving forces

1 Each diagram shows a box on a smooth surface acted on by a single force.

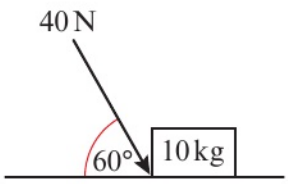
In each case, work out:

i the acceleration of the box

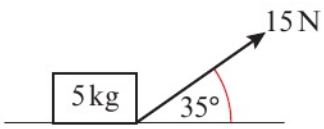
ii the normal reaction between the box and the floor.

Hint To find the normal reaction between the box and the floor, resolve vertically, considering the weight of the box and the vertical component of the force acting on the box.

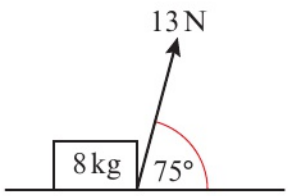
a



b



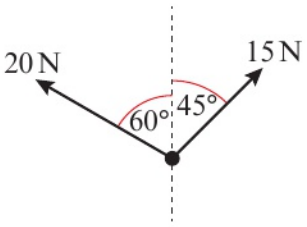
c



2 Each diagram shows a particle being acted upon by two forces. In each case, work out the magnitude and direction of the resultant force acting on the particle.

Hint You can use the triangle law for vector addition. The resultant force will be the third side of the triangle formed by adding the two given forces end-to-end.

a



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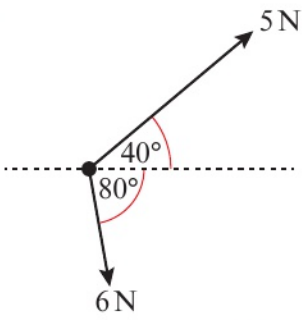
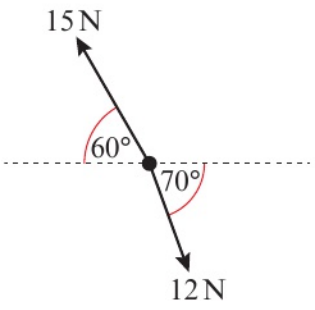


c



3 Each diagram shows a particle in equilibrium, acted upon by three forces.

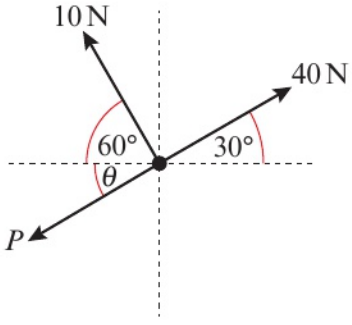
In each case, work out:

i the size of the angle θ

ii the magnitude of P.

Hint The particles are in equilibrium, so the resultant force is zero. Resolve horizontally and vertically, then solve the equations simultaneously to find P and θ.

a



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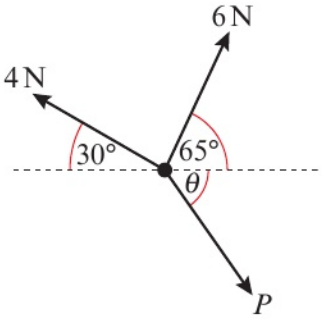
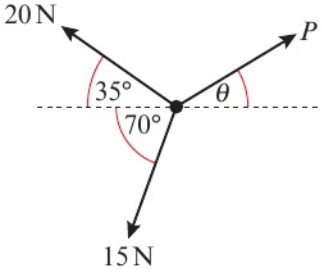
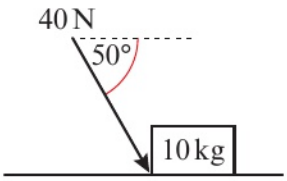


c



4 A box of mass 10 kg lies on a smooth horizontal floor. A force of 40 N is applied to the box at an angle of 50° below the horizontal, as shown in the diagram.



Work out:

a the acceleration of the box

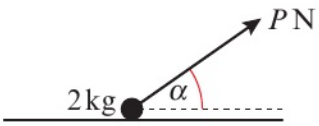
b the normal reaction between the box and the floor.

E **5** A box of mass m kg sits at rest on a smooth horizontal surface. A force of 20 N is applied to the box at an angle of 30° above the horizontal, causing the box to accelerate at $3\,\text{m s}^{-2}$.

a Work out the value of m. **(2 marks)**

b State what effect the assumption that the surface is smooth has on your calculations. **(1 mark)**

E/P **6** A particle of mass 2 kg sits at rest on a smooth horizontal surface. A force of P N is applied to the particle at an angle of α above the horizontal, where $\tan\alpha = \frac{3}{4}$



Given that the particle travels 40 m in the first 5 seconds of its motion, work out the value of P. **(4 marks)**

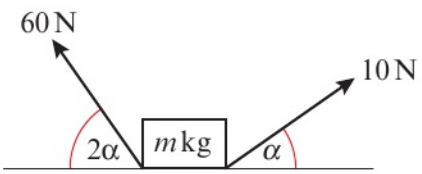
E/P **7** A box of mass 3 kg sits at rest on a smooth horizontal surface. A force of 6 N is applied to the box at an angle θ above the horizontal, reducing the normal reaction between the box and the surface by 3 N.

a Work out the size of angle θ. **(2 marks)**

An additional force P, acting at an angle of 2θ above the horizontal, is applied to the box causing the acceleration of the box to double.

b Show that $P = 6\sqrt{3}$ N. **(4 marks)**

E/P **8** A box of mass m kg lies at rest on a smooth horizontal surface. The box is acted upon by two forces of magnitudes 10 N and 60 N, acting in opposite directions at angles of α and 2α above the horizontal respectively.



Find:

a the size of α **(7 marks)**

b the least possible value of m. **(3 marks)**

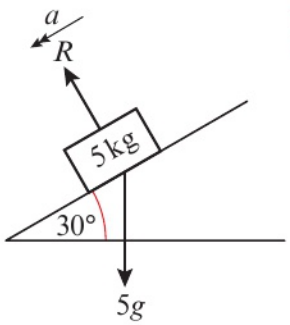
5.2 Inclined planes

1 A box of mass 5 kg slides down a smooth plane that is inclined at 30° to the horizontal.

Work out:

a the normal reaction between the box and the plane

b the acceleration of the box.



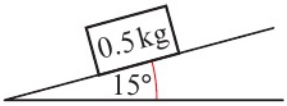
Hint The normal reaction will be equal to the component of the weight of the box that acts perpendicular to the plane. The acceleration will be due to the component of the weight of the box that acts parallel to the plane.

2 A box of mass 0.5 kg lies on a smooth plane that is inclined at 15° to the horizontal.

A force of magnitude 2 N is applied to the box parallel to and up the plane.

a Copy and complete the diagram to show all the forces acting on the box.

b Work out the acceleration of the box.



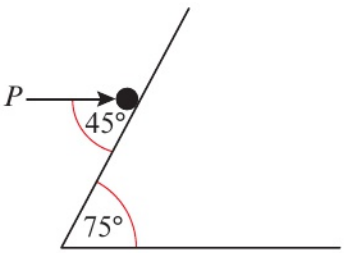
Hint There are three forces acting on the box.

3 A particle of mass 0.25 kg lies on a smooth plane that is inclined at 75° to the horizontal. It is held at rest by a force P, applied to the particle at an angle of 45° to the plane as shown in the diagram.

a Work out the magnitude of P.

The force P is removed.

b Work out the velocity of the particle after 2 seconds.

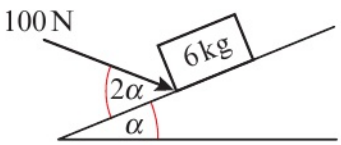


Hint For part **b**, work out the acceleration of the particle then use $v = u + at$

← Year 1, Section 9.3

E/P **4** A particle of mass m kg lies on a smooth plane that is inclined at an angle θ to the horizontal. Show that the acceleration of the particle is independent of its mass. **(2 marks)**

E/P **5** A box of mass 6 kg sits on a smooth plane that is inclined at an angle α to the horizontal where $\tan\alpha = \frac{5}{12}$. A force of 100 N is applied to the box at an angle of 2α to the plane, causing the box to accelerate from rest up the hill.



After 2 seconds the force is removed. Calculate:

a the speed of the box at the instant the force is removed **(4 marks)**

b the length of time it takes the box to come to rest from the instant the force is removed. **(4 marks)**

E/P **6** A box of mass m kg lies on a smooth plane that is inclined at 30° to the horizontal. A force of g N is applied to the box parallel to and up the plane. The box accelerates down the plane at $\frac{1}{3}g$ m s^{-2}. Work out the value of m. **(4 marks)**

E/P **7** A particle of mass 5 kg sits on a smooth plane that is inclined at 30° to the horizontal. A force of $2g$ N is applied vertically upwards to the particle. Calculate the magnitude of the acceleration of the particle and state its direction. **(6 marks)**

E/P **8** A particle of mass 2 kg sits on a smooth plane that is inclined at 20° to the horizontal.

The particle is initially at rest at point P, when a force of g N is applied to the particle parallel to and up the plane, causing it to accelerate up the plane. After 3 seconds the force is removed. Calculate how long, from the instant it first moves, it takes the particle to return to P. **(12 marks)**

5.3 Friction

1 The diagram shows a box lying on a rough horizontal plane. The box is acted upon by a horizontal force of magnitude 8 N, which causes it to accelerate.

a Work out the magnitude of the frictional force and the acceleration of the box in the case when the coefficient of friction between the box and the plane is equal to:

i 0.2 **ii** 0.35

Hint $F_{MAX} = \mu R$, where R is the normal reaction between the box and the plane, and μ is the coefficient of friction.

A student claims that in the case when $\mu = 1$, the magnitude of the frictional force will be $2g$ N and the box will remain at rest.

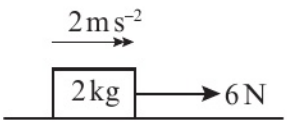
b Explain why the student is incorrect, and state the magnitude of the frictional force in this case.

Hint If the box is in equilibrium, the frictional force will only be as large as is necessary to prevent the box from moving.

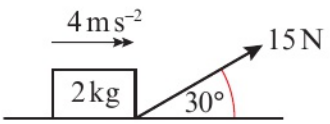
2 Each of the diagrams shows a box accelerating on a rough horizontal plane. In each case, work out the value of the coefficient of friction, μ, between the box and the plane.

Hint In parts **b** and **c** you will need to consider the vertical component of the force acting on the box when working out the normal reaction.

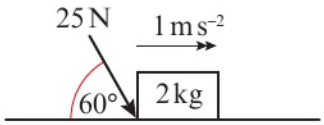
a



b

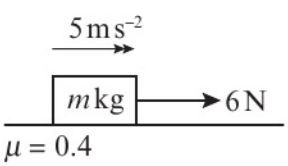


c

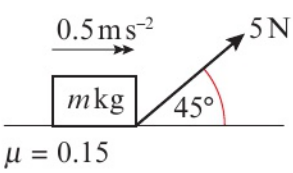


3 Each of the diagrams shows a box accelerating on a rough horizontal plane.
The coefficient of friction between the box and the plane is μ.
In each case, work out the value of m.

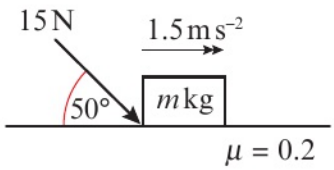
a



b



c



4 A box of mass 2 kg lies on a rough plane that is inclined at 30° to the horizontal.
The coefficient of friction between the box and the plane is 0.1. A force of 15 N is applied to the box parallel to and up the plane. Work out the acceleration of the box.

(E) **5** A block of mass 0.2 kg sits at rest on a rough horizontal surface. The coefficient of friction between the surface and the block is 0.4. A horizontal force of 4 N is applied to the block.

After 5 seconds the force is removed. Calculate:

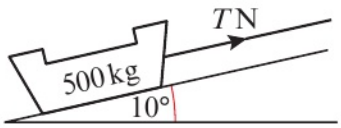
a the speed of the box at the instant the force is removed **(3 marks)**

b the length of time it takes the box to come to rest from the instant the force is removed. **(3 marks)**

(E/P) **6** A particle of mass m kg is sliding down a rough slope inclined at an angle of θ to the horizontal. The coefficient of friction between the particle and the slope is μ.

Show that the acceleration of the particle is given by $a = g(\sin\theta - \mu\cos\theta)$ **(4 marks)**

(E/P) **7** A sled of mass 500 kg sits at rest on a rough slope that is inclined at 10° to the horizontal. The coefficient of friction between the slope and the sled is 0.3.

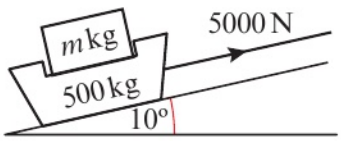


The sled is held in place by a wire attached to a winch. The tension in the wire is T N.

The sled is modelled as a particle, and the wire is modelled as a light inextensible string.

a Find the maximum possible value of T. **(3 marks)**

A load of mass m kg is added to the sled, and the tension in the wire is increased to 5000 N in order to pull the sled and load up the slope.



Given that the sled and load are pulled a distance of 100 m in 80 seconds,

b work out the value of m. **(6 marks)**

Problem solving Set A

Bronze

A box of mass 3 kg sits at rest on a rough horizontal surface. The coefficient of friction between the surface and the box is 0.15. A horizontal force of 6 N is applied to the block.

a Draw a diagram to show all the forces acting on the box. **(2 marks)**

b Calculate the distance travelled by the block in the first 6 seconds of its motion. **(5 marks)**

Silver

A block of mass 2 kg sits at rest on a rough horizontal surface. The coefficient of friction between the surface and the block is 0.25. A force of 10 N that acts at 60° above the horizontal is applied to the block.

Calculate the velocity of the block 10 seconds after it begins to move. **(5 marks)**

Gold

A block of mass m kg sits at rest on a rough horizontal surface. The coefficient of friction between the surface and the block is μ. A force of $2g$ N pulls the block at an angle of θ above the horizontal.

Show that the velocity of the block after t seconds is given by $\left(\frac{2\cos\theta + 2\mu\sin\theta}{m} - \mu\right)gt$ **(5 marks)**

Problem solving Set B

Bronze

A block of mass 0.2 kg lies on a rough slope that is inclined at 30° to the horizontal.

The coefficient of friction between the block and the slope is 0.2. A force of P N acts on the block in a direction parallel to and up the slope, causing it to accelerate up the slope.

a Draw a diagram to show all the forces acting on the box. **(2 marks)**

b Work out the exact magnitude of the frictional force. **(1 mark)**

c Given that the block accelerates up the slope at 0.2 m s^{-2}, work out the value of P. **(3 marks)**

Silver

A particle of mass m kg sits at rest on a rough slope that is inclined at an angle α to the horizontal where $\tan\alpha = \frac{3}{4}$

The coefficient of friction between the particle and the slope is 0.1. A force of magnitude F N acts on the particle in a direction parallel to and up the slope.

Given that the particle accelerates up the slope,

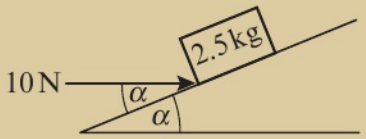
a show that $F > kmg$, where k is a constant to be determined. **(3 marks)**

Given further that $F = 50$ and that the particle accelerates up the slope at 1.1 m s^{-2},

b work out the value of m. **(3 marks)**

Gold

A box of mass 2.5 kg sits at rest on a rough slope that is inclined at an angle α to the horizontal. The coefficient of friction between the box and the slope is 0.15.



A force of magnitude 10 N acts horizontally on the box.

Given that the box is in limiting equilibrium, work out the two possible values of α. **(6 marks)**

Now try this → Exam question bank Q22, Q25, Q29, Q32, Q36, Q47

6.1 Horizontal projection

1 A particle is projected horizontally at $15\,\text{m s}^{-1}$ from a point 50 m above a horizontal surface.

a Show that the time taken for the particle to reach the surface is $\frac{10\sqrt{5}}{7}$ seconds.

b Work out the horizontal distance travelled by the particle.

Hint Draw a diagram that shows all the information given in the question.

2 A particle is projected horizontally from point P at $30\,\text{m s}^{-1}$. Find an expression for:

a the horizontal distance of the particle from P after t seconds

b the vertical distance of the particle from P after t seconds.

Hint Use $s = ut$ and $s = \frac{1}{2}at^2$ in the horizontal and vertical planes respectively to find the horizontal and vertical distances of the particle from P. ← Year 1, Sections 9.3, 9.4

3 A particle is projected horizontally at $40\,\text{m s}^{-1}$ from a point P, which is h m above a horizontal plane. The particle hits the plane a horizontal distance of 100 m from P. Work out:

a the time taken for the particle to land

b the value of h.

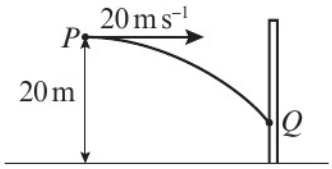
Hint The horizontal motion is modelled as having zero acceleration. Use $s = ut$ in the horizontal direction to find the time taken for the particle to land.

E **4** A particle is projected horizontally at $u\,\text{m s}^{-1}$ from a point P, which is 15 m above a horizontal plane. The particle hits the plane a horizontal distance of 24.5 m from P.

a Work out the value of u. **(4 marks)**

b State one modelling assumption used in your answer to part **a**. **(1 mark)**

E/P **5** A point P lies 20 m above a horizontal surface. A ball is thrown horizontally at $20\,\text{m s}^{-1}$ from P towards a vertical wall that stands on the horizontal plane. After 2 seconds the ball strikes the wall at the point Q. The ball is modelled as a particle moving freely under gravity.



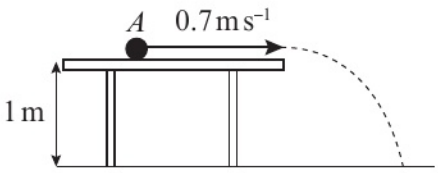
a Work out the horizontal distance from P to the wall **(2 marks)**

b Find the height of Q above the plane. **(2 marks)**

The model is adjusted to take air resistance into account.

c Explain briefly how this will affect the height at which the ball strikes the wall. **(1 mark)**

E/P **6** A particle is projected from rest from a point A along a smooth horizontal table at $0.7\,\text{m s}^{-1}$. The table is 1 m high and stands on horizontal ground. Given that it takes 2 seconds from the moment of projection until the ball hits the ground, work out the distance from the point A to the edge of the table.



(4 marks)

E/P **7** A particle of mass 0.5 kg is projected along a horizontal table with an initial velocity of $6\,\text{m s}^{-1}$.

The table has a rough surface. The coefficient of friction between the particle and the table's surface is 0.2.

After travelling for 2 seconds the particle leaves the table.

a Find the speed of the particle at the instant it leaves the table. **(3 marks)**

Given that the particle lands a horizontal distance of 0.78 m from the edge of the table,

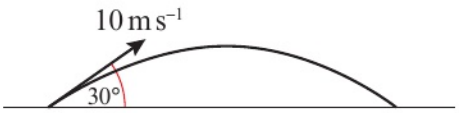
b work out the height of the table. **(3 marks)**

c How does your answer to part **b** change if the horizontal surface of the table is smooth? **(1 mark)**

6.2 Horizontal and vertical components

In these questions you may assume that **i** and **j** are unit vectors acting horizontally and vertically respectively.

1 A particle is projected from a point on a horizontal plane with an initial velocity of $10\,\text{m s}^{-1}$ at an angle of 30° above the horizontal.

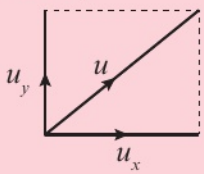


Find:

a the horizontal component of the initial velocity

b the vertical component of the initial velocity.

Hint Resolve horizontally and vertically to find the horizontal and vertical components of the initial velocity.



2 A particle is projected from a point on a horizontal plane with an initial velocity of $20\,\text{m s}^{-1}$ at an angle of 45° above the horizontal. Express the initial velocity as a vector in terms of **i** and **j**.

Hint Find the horizontal and vertical components of the initial velocity, then write them in vector form.

3 A particle is projected with an initial velocity of $(12\mathbf{i} + 5\mathbf{j})\,\text{m s}^{-1}$. Find the initial speed of the particle and its angle of projection.

Hint Speed is the magnitude of the velocity vector. ← **Year 1, Section 8.4**

E **4** A particle is projected from a point on a horizontal plane with an initial velocity of $15\,\text{m s}^{-1}$ at an angle of 25° above the horizontal.

a Find the horizontal and vertical components of the initial velocity. **(2 marks)**

b Express the initial velocity as a vector in terms of **i** and **j**. **(2 marks)**

E/P **5** A particle is projected from a tall building with an initial velocity of $8\,\text{m s}^{-1}$ at an angle of 10° below the horizontal.

a Find the horizontal and vertical components of the initial velocity. **(2 marks)**

b Express the initial velocity as a vector in terms of **i** and **j**. **(2 marks)**

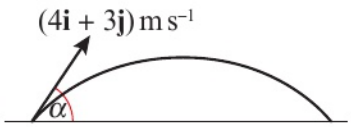
E **6** A particle is projected from horizontal ground with an initial velocity of $156\,\text{m s}^{-1}$ at an angle α above the horizontal where $\tan\alpha = \frac{12}{5}$

a Find the horizontal and vertical components of the initial velocity. **(2 marks)**

b Express the initial velocity as a vector in terms of **i** and **j**. **(2 marks)**

E **7** A particle is projected with an initial velocity of $(4\mathbf{i} + 3\mathbf{j})\,\text{m s}^{-1}$.

Find the initial speed of the particle and its angle of projection, α. **(3 marks)**



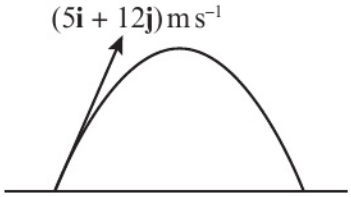
6.3 Projection at any angle

In these questions you may assume that **i** and **j** are unit vectors acting horizontally and vertically respectively.

1 A particle is projected from a point on a horizontal plane with a speed of $98\,\text{m s}^{-1}$ at an angle of 30° above the horizontal. Given that the particle moves freely under gravity, find the time of flight of the particle.

Hint You only need to consider the vertical motion of the particle.

2 A particle is projected from a point on a horizontal plane with a velocity of $(5\mathbf{i} + 12\mathbf{j})\,\text{m s}^{-1}$. The particle moves freely under gravity.



a By considering the vertical motion of the particle, find the time of flight of the particle.

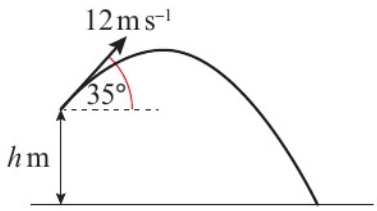
b By considering the horizontal motion of the particle, find the range of the particle.

Hint The range is the horizontal distance travelled by the particle.

3 A particle is projected from a point on a horizontal plane with a speed of $34\,\text{m s}^{-1}$ at an angle of 50° above the horizontal. Given that the particle moves freely under gravity, find the maximum height reached by the particle.

Hint The particle reaches its maximum height when the vertical component of its velocity is 0.

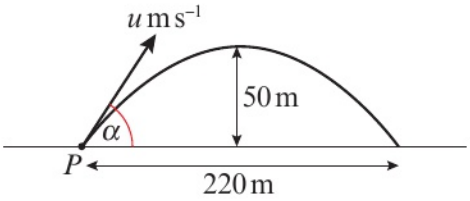
E **4** A particle is projected from a point h m above a horizontal plane with a speed of $12\,\text{m s}^{-1}$ at an angle of 35° above the horizontal. The particle moves freely under gravity and lands on the horizontal plane after 5 seconds. Find the value of h. **(3 marks)**



E **5** A particle is projected from a point 10 m above a horizontal plane with a speed of $50\,\text{m s}^{-1}$ at an angle of 20° above the horizontal. Given that the particle moves freely under gravity, find the range of the particle. **(6 marks)**

E **6** A particle is projected from a point on a horizontal plane with a velocity of $(20\mathbf{i} + 15\mathbf{j})\,\text{m s}^{-1}$. Given that the particle moves freely under gravity, find the length of time for which the particle is above 10 m. **(6 marks)**

E/P **7** A particle is projected from a point P on horizontal ground with a speed of $u\,\text{m s}^{-1}$ at an angle α above the horizontal. The maximum height reached by the particle is 50 m and the range of the particle is 220 m.



Find the value of:

a i α

ii u **(10 marks)**

b Work out the speed of the particle after 5 seconds. **(4 marks)**

E/P **8** The point P lies 200 m from the base of a vertical cliff of height 100 m. An object is projected from point P, towards the cliff, with a velocity of $(36\mathbf{i} + 48\mathbf{j})\,\text{m s}^{-1}$.

Find the distance the object lands from the edge of the cliff. **(6 marks)**

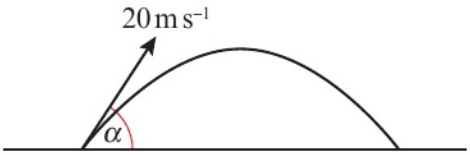
E/P **9** A rugby player is attempting to kick a penalty. In order for the penalty to be successful, the ball must pass between the posts and clear a crossbar that is 4 m high. The player kicks the ball from the ground with a speed of $25\,\text{m s}^{-1}$ at an angle of 40° above the horizontal. Assuming that the ball passes between the posts and by modelling the rugby ball as a projectile moving freely under gravity, work out the greatest distance from which the player can kick a successful penalty. **(6 marks)**

6.4 Projectile motion formulae

In these questions you may assume that **i** and **j** are unit vectors acting horizontally and vertically respectively.

P **1** A particle is projected from a point on a horizontal plane with a speed of $20\,\text{m s}^{-1}$ at an angle α above the horizontal.

Given that the particle moves freely under gravity, show that the greatest height it reaches is $\frac{200\sin^2\alpha}{g}$



Hint Consider the vertical motion of the particle and use $v^2 = u^2 + 2as$.

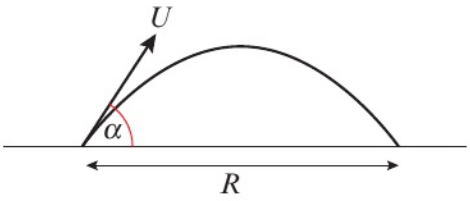
P **2** A particle is projected from a point on a horizontal plane with a speed of U at an angle α above the horizontal.

a Find an expression in terms of U, g and α for the range, R, of the particle.

b Work out the value of α that gives the greatest possible range.

Hint Consider the value of α in the range $0 \leqslant \alpha \leqslant 90°$ that maximises the value of the trigonometric function in your answer to part **a**.

(P) **3** A particle is projected from a point on a horizontal plane with a speed of U at an angle α above the horizontal. The particle moves freely under gravity, and travels a horizontal distance of R.



a Show that the maximum height h of the particle is given by $h = \frac{R\tan\alpha}{2} - \frac{gR^2}{8U^2\cos^2\alpha}$

Hint If the upwards direction is taken to be positive then the vertical acceleration is $-g$.

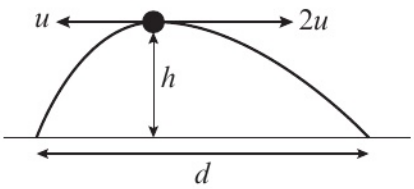
b Show that the equation found in part **a** can be rewritten as $h = \frac{R\tan\alpha}{2} - \frac{gR^2}{8U^2}(1 + \tan^2\alpha)$

Hint You'll need to use trigonometric identities covered in the Pure part of your course to answer this.

← Pure Year 2, Chapter 6

(E/P) **4** A particle A is projected from a point P on a horizontal plane with a speed of u_A at an angle α_A above the horizontal. A second particle B is then projected from P at a speed of u_B at an angle α_B above the horizontal. Find, in terms of u_A, u_B, α_A, α_B and g, the distance between the points where the particles land. **(4 marks)**

(E/P) **5** Two warning flares F_1 and F_2 are fired horizontally in opposite directions from a helicopter that is hovering stationary above level ground at a height of h. F_1 is fired with speed u and F_2 is fired with speed $2u$.



Find, in terms of u, g and h, the distance d between the points where the flares land. **(4 marks)**

(E/P) **6** A particle is projected from a point P that lies on level ground with velocity $(a\mathbf{i} + b\mathbf{j})$.

a Show that the horizontal distance travelled by the particle is $\frac{2ab}{g}$ **(6 marks)**

b Given that the particle travels a horizontal distance of 200 m and that its velocity of projection is parallel to the vector $12\mathbf{i} + 5\mathbf{j}$, find the initial speed of the particle. **(4 marks)**

(E/P) **7** A ball is thrown from a point P on horizontal ground with a speed of U at an angle α above the horizontal. Given that the ball is modelled as a particle moving freely under gravity, show that the horizontal distance travelled by the ball is $\frac{U^2\sin 2\alpha}{g}$ **(6 marks)**

(E/P) **8** A particle is projected from a point P 15 m above a horizontal plane with a speed of U at an angle α above the horizontal. The particle moves freely under gravity, and just passes over a wall of height 25 m that is a horizontal distance of 40 m from P.

a Show that $\frac{800g}{U^2}(1 + \tan^2\alpha) - 40\tan\alpha + 10 = 0$ **(6 marks)**

b Given that $U = 40\,\text{m s}^{-1}$, work out two possible sizes of angle α. **(6 marks)**

Problem solving Set A

In these questions, **i** and **j** are the unit vectors horizontally and vertically respectively.

Bronze

A ball is projected from a point on a horizontal plane with a speed of $2U$ at an angle α above the horizontal. By modelling the ball as a particle moving freely under gravity,

a show that the time between the instant the ball is projected and the instant it lands on the plane is given by $\frac{4U\sin\alpha}{g}$ **(4 marks)**

b Hence show that the range of the particle is given by $\frac{4U^2\sin 2\alpha}{g}$ **(4 marks)**

Silver

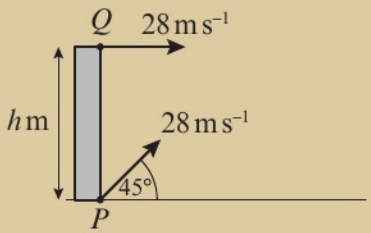
A particle is projected from a point P on a horizontal plane with a speed of U at an angle θ above the horizontal, where $\tan\theta = \frac{3}{4}$. When the particle is a horizontal distance x from P, its height above P is h.

a Show that $h = ax - b\left(\frac{gx^2}{U^2}\right)$, where a and b are constants to be determined. **(6 marks)**

b Given that $U = 35\,\text{m s}^{-1}$, find the exact speed of the particle at the point where $h = \frac{x}{2}$ **(4 marks)**

Gold

An arrow is fired horizontally with a speed of $28\,\text{m s}^{-1}$ from a point Q at the top of a vertical tower of height h m. Another arrow is simultaneously fired from a point P at the base of the tower with a speed of $28\,\text{m s}^{-1}$ at an angle of 45° above the horizontal.



The ground is modelled as a horizontal plane and the two arrows are modelled as particles travelling in the same vertical plane.

a Given that the paths of the arrows intersect, find the maximum value of h. **(4 marks)**

A student claims that this means that arrows will collide in mid-air.

b Explain why the student is incorrect. **(1 mark)**

c Find, in terms of h, the horizontal distance from the tower to the point where the paths of the arrows intersect. **(6 marks)**

Problem solving Set B

Bronze

A particle is projected from a point 6 m above horizontal ground with a velocity of $(10\mathbf{i} + 15\mathbf{j})\,\text{m s}^{-1}$. The particle moves freely under gravity. Taking g to be $10\,\text{m s}^{-2}$, work out:

a the speed of the particle when it hits the ground **(4 marks)**

b the length of time for which the particle is travelling at less than $15\,\text{m s}^{-1}$. **(6 marks)**

Silver

A particle is projected from a point 120 m above horizontal ground with a velocity of $(33\mathbf{i} + 18\mathbf{j})\,\text{m s}^{-1}$. The particle moves freely under gravity. Taking g to be $10\,\text{m s}^{-2}$, work out

a the height of the particle when it is moving at a speed of $55\,\text{m s}^{-1}$ **(5 marks)**

b the angle the velocity of the ball makes with the ground as the ball lands. **(4 marks)**

Gold

A particle is projected from a point 75 m above horizontal ground with a velocity of $(40\mathbf{i} - 6\mathbf{j})\,\text{m s}^{-1}$. Taking g to be $10\,\text{m s}^{-2}$, find:

a the speed of the particle when it hits the ground **(4 marks)**

b the distance from the point of projection to the point of landing **(4 marks)**

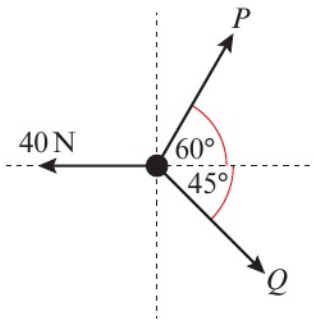
c the height of the particle at the instant when it is travelling in a direction parallel to the vector $5\mathbf{i} - 4\mathbf{j}$. **(4 marks)**

d Show that the particle is never travelling parallel to the vector $\mathbf{i} - \mathbf{j}$. **(3 marks)**

Now try this → **Exam question bank Q35, Q38, Q42, Q45, Q48, Q51**

7.1 Static particles

1 A particle is held in static equilibrium by the forces shown in the diagram.



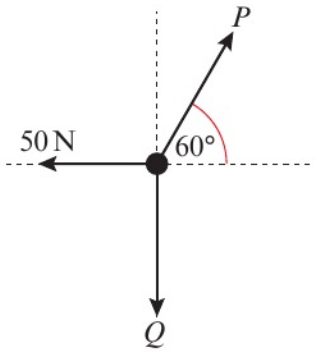
a Resolve the components in the x-direction.

b Resolve the components in the y-direction.

c Find the magnitudes of P and Q.

Hint Solve your equations from parts **a** and **b** simultaneously.

2 A particle is held in equilibrium by the forces shown in the diagram.

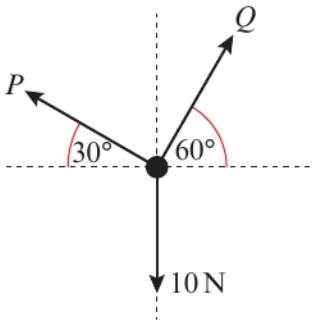


a Draw a triangle of forces diagram.

b Find the magnitudes of the forces P and Q.

Hint Use trigonometry to find the values of P and Q.

3 A particle is held in equilibrium by the forces shown in the diagram.

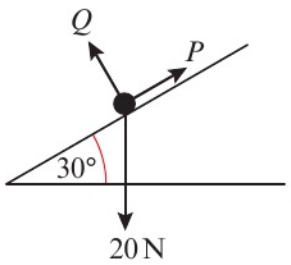


Find the magnitudes of the forces P and Q.

Hint Resolve horizontally and vertically then solve the resulting equations simultaneously.

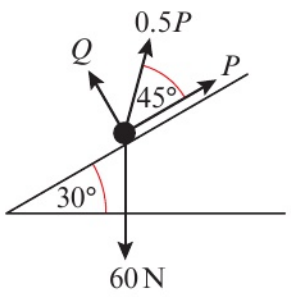
← Section 5.1

4 A particle is held in equilibrium on a slope by the forces shown in the diagram.



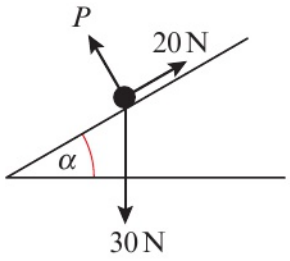
Find the magnitudes of the forces P and Q.

5 A particle is held in equilibrium on a slope by the forces shown in the diagram.



Find the magnitudes of the forces P and Q.

6 A particle is held in equilibrium on a slope by the forces shown in the diagram.



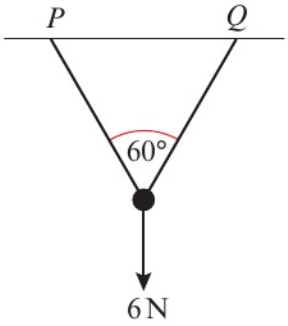
Given that the slope is angled at α to the horizontal, find:

a the size of angle α

b the magnitude of P.

7.2 Modelling with statics

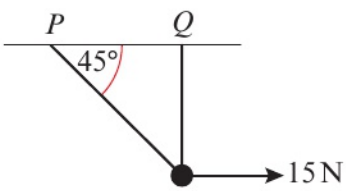
1 A smooth bead of weight 6 N is threaded on a light inextensible string that is attached to points P and Q as shown.



Work out the tension in the string.

Hint Because the bead is smooth, the tension in the string is the same on either side of the bead.

2 A smooth bead is threaded on a light inextensible string that is attached to points P and Q as shown. The bead is held in equilibrium vertically below Q by a horizontal force of 15 N.



Hint Copy the diagram and show all the forces acting on the bead.

a Find the tension in the string.

b Work out the weight of the bead.

c How have you used the fact that the bead is smooth in your calculations?

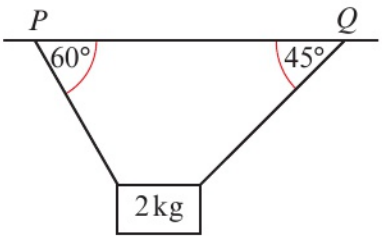
3 A box of mass 5 kg rests on a smooth plane that is inclined at an angle α to the horizontal, where $\tan\alpha = \frac{5}{12}$. A force of P N acts in a direction parallel to the plane and holds the box in equilibrium.

Hint Resolve parallel and perpendicular to the slope to find the normal reaction.

a Find the normal reaction between the box and the plane.

b Work out the value of P.

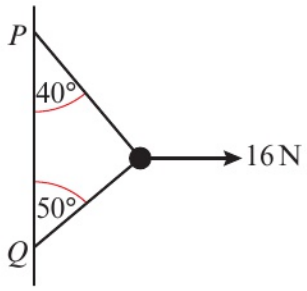
(E) **4** A sign of mass 2 kg hangs in equilibrium as shown on the diagram.



It is attached to the points P and Q by light inextensible strings.

Work out the tension in each of the strings. **(5 marks)**

E **5** A smooth bead is threaded on a light inextensible string that is attached to a vertical wall at the points P and Q as shown in the diagram. The bead is held in equilibrium by a horizontal force of magnitude 16 N.



a Find the tension in the string. **(2 marks)**

b Work out the mass of the bead. **(2 marks)**

c State how the tension in the string would be affected if the string were not modelled as being light. Justify your answer. **(2 marks)**

E/P **6** A particle of mass 0.2 kg rests on a smooth plane that is inclined at an angle α to the horizontal. The particle is held in equilibrium by a horizontal force of magnitude 2 N.

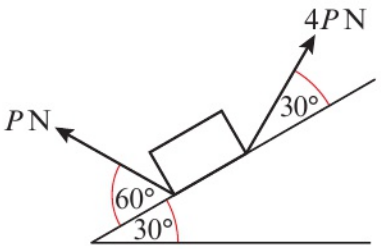
a Find the size of angle α. **(3 marks)**

b State how your answer to part **a** would change if the particle was heavier. **(1 mark)**

E/P **7** A box of mass m kg rests on a smooth plane that is inclined at an angle α to the horizontal. A force of $2g$ N acts in a direction parallel to and up the plane and holds the box in equilibrium.

Show that $\alpha = \arcsin\left(\frac{2}{m}\right)$ **(2 marks)**

E/P **8** A box of weight 20 N rests on a smooth slope that is inclined at an angle of 30° to the horizontal. The box is held in equilibrium by a force of magnitude $4P$ N that acts at an angle of 30° to the slope and a second force of magnitude P N that acts at an angle of 60° to the slope, as shown in the diagram.



Show that the normal reaction between the box and the slope is 7.65 N to 3 significant figures. **(8 marks)**

7.3 Friction and static particles

1 A box of mass 2 kg rests on a rough horizontal plane. The coefficient of friction between the box and the plane is $\mu = 0.2$. A force P acts on the box in a direction parallel to the plane.

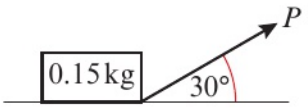
a Draw a diagram to show the forces acting on the box.

b Given that the box remains at rest, find the maximum magnitude of P.

Hint Use $F \leq \mu R$ to work out the magnitude of the frictional force.

← Section 5.3

2 A box of mass 0.15 kg rests on a rough horizontal plane. The coefficient of friction between the box and the plane is 0.25. A force P acts on the box at an angle of 30° to the plane.

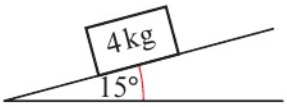


a Complete the diagram showing all the forces acting on the box.

b Given that the box remains at rest, find the maximum magnitude of P.

Hint First resolve vertically to write the normal reaction in terms of P.

3 A box of mass 4 kg rests on a rough plane that is inclined at 15° to the horizontal. The coefficient of friction between the box and the plane is μ.



Hint When the box is in limiting equilibrium, it is on the point of slipping, and $F = \mu R$.

Given that the box is in limiting equilibrium, find:

a the normal reaction between the box and the plane

b the value of μ.

E/P **4** A box of mass 2.5 kg rests on a rough plane that is inclined at 18° to the horizontal. The coefficient of friction between the box and the plane is 0.3. A force of P N acts on the box parallel to and up the plane. Given that the box is in limiting equilibrium, find:

a the normal reaction between the box and the plane **(2 marks)**

b the size of the maximum force that can be applied to the box without causing it to move. **(3 marks)**

E/P **5** A block of mass m kg rests on a rough plane that is inclined at 30° to the horizontal. The coefficient of friction between the box and the plane is 0.4. A force of magnitude 5 N acts on the block parallel to and up the plane.

Given that the box is in limiting equilibrium, find the two possible values of m. **(5 marks)**

E/P **6** A particle of mass 1 kg rests in limiting equilibrium on a rough slope that is inclined at an angle of α to the horizontal. A force of magnitude g N acts on the particle parallel to and down the slope.

Given that the particle remains in equilibrium, show that the minimum value of the coefficient of friction between the box and the plane is

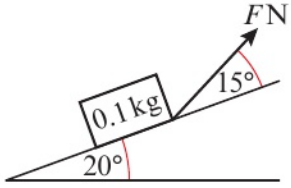
$$\mu = \frac{1 + \sin\alpha}{\cos\alpha}$$ **(4 marks)**

E/P **7** A boat of mass 250 kg is held at rest on a slipway that is angled at 25° to the horizontal by a rope that acts parallel to and up the slope. The tension in the rope is P N. The coefficient of friction between the slipway and the boat is 0.15. The boat can be modelled as a particle and the rope as a light inextensible string.

a Given that the boat is on the point of slipping down the slope, work out the value of P. **(4 marks)**

b Comment on the validity of modelling the boat as a particle in this situation. **(1 mark)**

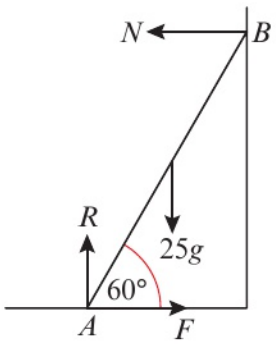
E/P **8** A block of mass 0.1 kg rests on a slope that is inclined at 20° to the horizontal. The coefficient of friction between the block and the slope is 0.25. The block is held in equilibrium by a force of magnitude F N that acts up the slope and at an angle of 15° to the slope.



By modelling the block as a particle, find the range of possible values of F. **(8 marks)**

7.4 Static rigid bodies

1 A uniform ladder AB, of mass 25 kg and length 6 m, rests with end A on rough horizontal ground and end B on a smooth vertical wall. The angle between the ladder and the ground is 60°.



Given that the ladder is in limiting equilibrium, find:

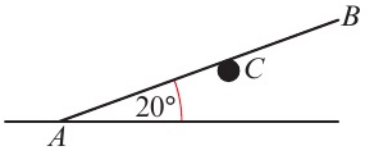
a the normal reaction, N, between the ladder and the wall in terms of F

b the normal reaction between the ladder and the ground.

c By taking moments about the base of the ladder, work out the magnitude of N.

Hint For part **a**, resolve horizontally, and for part **b** resolve vertically.

2 A uniform rod AB, of mass 20 kg and length 5 m, rests with end A on rough horizontal ground. The rod is supported by a smooth peg at the point C where $AC = 3.5$ m. The angle between the rod and the ground is 20°. The rod is in limiting equilibrium.



a Copy and complete the diagram to show the forces acting on the rod.

b Work out the normal reaction between the rod and the peg.

c Find the coefficient of friction between the rod and the ground.

d State how you have used the fact that the peg is smooth in your calculations.

Hint Take moments about A to find the normal reaction at the peg.

3 A uniform ladder AB, of mass 10 kg and length 3 m, rests with end A on rough horizontal ground and end B on a smooth vertical wall.
The angle between the ladder and the ground is 60°. The ladder is in limiting equilibrium.

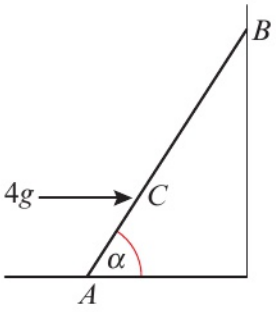
a Draw a diagram to show the forces acting on the ladder.

b Work out the magnitude of the normal reaction between the ladder and the wall.

c Find the coefficient of friction between the ladder and the ground.

Hint Resolve horizontally and vertically then take moments about A.

E/P **4** A uniform ladder AB, of length 4 m and mass 12 kg, rests with end A on smooth horizontal ground and the end B against a smooth vertical wall. The ladder is kept in equilibrium by a horizontal force of magnitude $4g$ N acting at a point C on the ladder, where $AC = 1$ m.



The angle between the ladder and the floor is α.

a By modelling the ladder as a uniform rod, find the size of angle α. **(4 marks)**

b Comment on the modelling assumption that the ground and the wall are smooth. **(1 mark)**

The ground and the wall are now modelled as rough surfaces, and the rod is assumed to be in limiting equilibrium.

c Without further calculation, state whether the size of angle α will increase or decrease. **(1 mark)**

E/P **5** A uniform ladder AB, of length 6 m and mass 24 kg, rests in equilibrium with end A on rough horizontal ground and end B against a smooth vertical wall. The ladder is inclined at an angle α to the horizontal. When a builder of mass 72 kg stands at a point C on the ladder, where $AC = 4$ m, the ladder is at the point of slipping. Given that the coefficient of friction between the ladder and the ground is 0.3, find:

a the size of the frictional force of the ground acting on the ladder **(3 marks)**

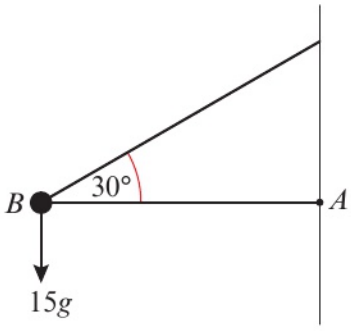
b the size of angle α. **(6 marks)**

E/P **6** A uniform plank AB, of weight W N and length 6 metres, rests on a smooth peg at a point C. The plank rests in limiting equilibrium with end A on rough horizontal ground, where the coefficient of friction between the plank and the ground is 0.8. The plank is inclined at an angle α to the horizontal, where $\tan\alpha = 0.75$. Given that the magnitude of the normal reaction of the ground on the plank at A is 80 N, find:

a the magnitude of the normal reaction between the plank and the peg **(5 marks)**

b the value of W. **(3 marks)**

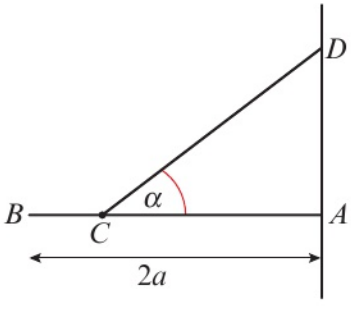
E/P **7** A uniform rod AB, of length 4 m and mass 25 kg, is smoothly hinged at the point A, which lies on a vertical wall. A particle of mass 15 kg is suspended from B. The rod is kept in a horizontal position by a light inextensible string, which makes an angle of 30° with the rod, as shown in the diagram.



a Work out the tension in the string. **(2 marks)**

b Calculate the magnitude and direction of the reaction at the hinge. **(8 marks)**

E/P **8** A uniform rod AB, of mass M and length $2a$, rests with its end A against a rough vertical wall. The rod is held in a horizontal position by a rope. One end of the rope is attached to the rod at a point C that lies $\frac{1}{2}a$ from B, and the other end is attached to the wall at a point D that lies vertically above A. The coefficient of friction between the wall and the rod is 0.4 and the angle between the rope and the rod is α.



a Show that the tension in the rope is $T = \dfrac{2Mg}{3\sin\alpha}$ **(3 marks)**

b Given that the rod is in limiting equilibrium, show that the magnitude of the horizontal component of the force exerted on the rod by the wall at A is $\dfrac{2Mg}{3\tan\alpha}$ and find the magnitude of the vertical component of this force. **(3 marks)**

c Find the size of angle α. **(3 marks)**

d Explain how your answer to part **c** would change if the coefficient of friction between the rod and the wall were greater than 0.4. **(1 mark)**

7.5 Dynamics and inclined planes

1 A box of mass 5 kg is pushed up a rough surface, which is inclined at 30° to the horizontal, by a force of 120 N that acts in the direction parallel to and up the surface. The coefficient of friction between the box and the surface is 0.25.

a Draw a force diagram to represent this situation.

b Find the normal reaction between the box and the surface.

c Work out the acceleration of the box.

Hint Use $F = ma$ to set up an equation of motion for the box.

2 A block of mass 1 kg is held at rest on a rough surface that is inclined at 60° to the horizontal. The coefficient of friction between the block and the surface is 0.4.

The block is released from rest.

a Calculate the frictional force between the block and the surface, stating in your answer in which direction it acts.

b Show that the block will begin to slide down the slope.

c Work out the acceleration of the block.

Hint Frictional forces always act so as to oppose the direction of motion. ← Section 5.3

3 A particle of mass m kg is being pulled up a rough surface, which is inclined at 45° to the horizontal, by a force of 20 N, which acts parallel to the surface. The coefficient of friction between the particle and the surface is 0.2. Given that the particle accelerates at $2\,\text{m}\,\text{s}^{-2}$, find the value of m.

Hint To find m write an equation of motion for the particle.

E/P **4** A box of mass 0.3 kg is held at rest on a rough surface that is inclined at 30° to the horizontal. The coefficient of friction between the box and the surface is 0.15. The box is released from rest. Work out the time taken for the box to travel 5 m. **(6 marks)**

E/P **5** A particle of mass 4 kg, initially at rest, is pulled up a rough surface, which is inclined at 25° to the horizontal, by a force of 50 N that acts at an angle of 30° to the surface.

The coefficient of friction between the particle and the surface is 0.1.

a Work out the acceleration of the particle. **(6 marks)**

After 3 seconds the force is removed.

b Work out the time taken for the particle to come to rest. **(8 marks)**

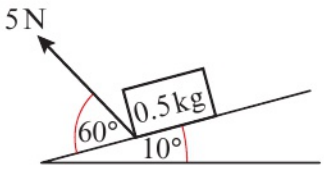
c Show clearly whether the particle will remain at rest or begin to slide back down the slope. **(3 marks)**

E/P **6** A particle of mass 4 kg is released from rest on a rough slope that is inclined at 35° to the horizontal. After 3 seconds the particle has travelled 4 m. Work out the coefficient of friction between the particle and the slope. **(8 marks)**

E/P **7** A particle of mass 2 kg is moving up a rough slope inclined at 5° to the horizontal with constant acceleration. After 2 seconds the particle has a velocity of 4 m s^{-1}. After 6 seconds the particle has come to instantaneous rest. **(6 marks)**

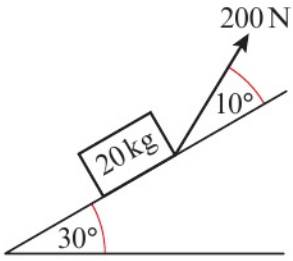
Work out the coefficient of friction between the particle and the slope.

E/P **8** A particle of mass 0.5 kg, initially at rest, is pulled down a rough surface, which is inclined at 10° to the horizontal, by a force of 5 N that acts at an angle of 60° to the surface. The coefficient of friction between the particle and the surface is 0.75.



Find the time taken for the velocity of the particle to reach 30 m s^{-1}. **(6 marks)**

E/P **9** A sled of mass 20 kg, initially at rest at point A, is being pulled by a rope up a rough plane, which is inclined at 30° to the horizontal, to point B. The tension in the rope is constant, equal to 200 N, and acts at an angle of 10° to the plane. The coefficient of friction between the sled and the plane is 0.3. After 10 seconds of pulling, the rope snaps when the sled is 60 m from B.



Determine whether the sled reaches point B. You must justify your answer. **(12 marks)**

7.6 Connected particles

1 A car of mass 1000 kg is towing a trailer of mass 200 kg up a straight hill that is angled at 10° to the horizontal. The car engine exerts a constant force of 10 kN. The car and the trailer are connected by a tow-bar. The total resistances to motion of the car and the trailer are modelled as being constant and of magnitudes 200 N and 150 N respectively.

a Draw a force diagram to represent this situation.

b By modelling the car and the trailer as particles and the tow-bar as a light inextensible rod, work out:

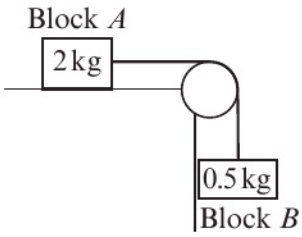
i the acceleration of the car and trailer

ii the tension in the tow-bar.

Hint To work out the acceleration of the car and trailer, consider them together.

c State how you have used the fact that the tow-bar is inextensible in your calculations.

2 A block A of mass 2 kg rests on a rough horizontal table and is attached to one end of a light inextensible string. The string passes over a small smooth pulley fixed at the edge of the table. The other end of the string is attached to a block B of mass 0.5 kg, which hangs freely below the pulley. The coefficient of friction between block A and the table is 0.2.



The system is released from rest with the string taut.

a Copy and complete the diagram to show the forces acting on the blocks.

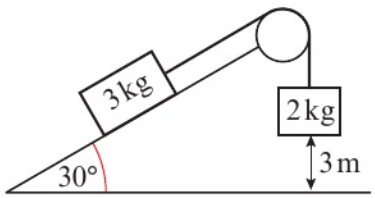
b Find the acceleration of block A.

c Work out the tension in the string.

d State how you have used the fact that the pulley is smooth in your calculations.

Hint To work out the acceleration, write equations of motion for the two blocks, then solve the equations simultaneously.

3 A 3 kg mass lies on a smooth plane that is inclined at 30° to the horizontal. It is attached to a 2 kg mass that hangs freely at a height of 3 m above the ground from a light inextensible string that passes over a small smooth pulley.



The system is released from rest with the string taut, and in the subsequent motion the 3 kg mass does not reach the pulley.

a Work out the tension in the string when the system is released.

b Find the acceleration of the system.

c Find the time taken for the 2 kg mass to reach the ground.

Hint You can work out either the acceleration or the tension first.

E/P **4** A van of mass 1200 kg is towing a trailer of mass 250 kg up a straight hill that is inclined at 12° to the horizontal. The van engine exerts a constant force of 12 kN and the van and trailer are accelerating at 5 m s^{-2}. The resistances to motion of the van and trailer are modelled as being constant.

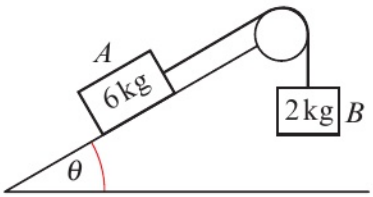
a Work out the total resistances to motion of the van and trailer. **(3 marks)**

b Given that the resistance to motion of the van is three times the resistance to motion of the trailer, work out the tension in the tow-bar. **(2 marks)**

When the van and trailer are travelling at 8 m s^{-1}, the tow-bar snaps.

c Assuming that the resistance to motion remains unchanged, work out the time taken for the trailer to come to rest. **(4 marks)**

E/P **5** Box A of mass 6 kg rests on a rough plane which is inclined at an angle θ to the horizontal, where $\tan\theta = 0.75$. The coefficient of friction between box A and the plane is μ. Box A is attached to one end of a light inextensible string, which passes over a smooth pulley. The string is attached to a second box, B, of mass 2 kg, which hangs freely below the pulley, as shown in the diagram.



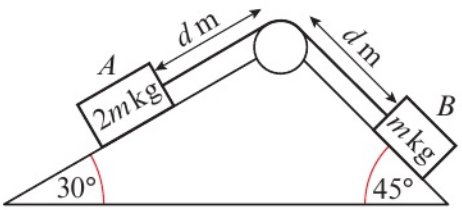
The system is released from rest.

a Given that box A begins to slide down the slope, find the range of possible values of μ. **(3 marks)**

b Given further that $\mu = 0.2$, find the time taken for box A to reach a speed of $5\,\text{m s}^{-1}$. **(4 marks)**

c State two assumptions you have made in order to answer part **b**. You may not include any modelling assumptions given in the question. **(2 marks)**

E/P **6** Block A of mass $2m$ kg and block B of mass m kg sit on smooth inclined planes angled at 30° and 45° to the horizontal respectively. The blocks are attached at either end of a light inextensible string. The string passes over a small smooth pulley at the top of the planes. Both blocks are initially at rest and at a distance of d m from the pulley.



The blocks are released from rest.

a Show that block B begins to slide up the slope. **(3 marks)**

b Show that the time taken for block B to hit the pulley is $\sqrt{\dfrac{12d}{(2-\sqrt{2})g}}$ **(9 marks)**

Problem solving Set A

Bronze

A particle of mass 6 kg is at rest on a rough slope that is inclined at 15° to the horizontal. The particle is projected up the slope with initial speed $10\,\text{m s}^{-1}$. The coefficient of friction between the particle and the slope is 0.35.

a Find the magnitude of the frictional force acting on the particle. **(3 marks)**

b Work out the time taken for the particle to come to rest. **(2 marks)**

c State how your answer to part **b** would change if the coefficient of friction between the particle and the slope were less than 0.35. **(1 mark)**

Silver

A particle of mass 2 kg is at rest on a rough slope that is inclined at an angle α to the horizontal. The particle is projected up the slope with initial speed $20\,\text{m s}^{-1}$. The coefficient of friction between the particle and the slope is 0.2.

Given that the particle comes to rest in limiting equilibrium, find:

a the size of angle α **(3 marks)**

b the time taken for the particle to come to rest. **(4 marks)**

c State, with a reason, how the size of α would differ if the coefficient of friction between the particle and the slope were greater than 0.2. **(2 marks)**

Gold

A particle of mass 3 kg is at rest on a rough slope that is inclined at α to the horizontal. The particle is projected up the slope with initial speed $15\,\text{m s}^{-1}$. The coefficient of friction between the particle and the slope is 0.15.

a Given that the particle comes to instantaneous rest and then immediately starts to slide back down the slope, find the range of possible sizes of α. **(3 marks)**

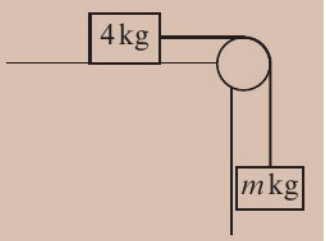
b Given further that $\alpha = 20°$, find the total time taken from the time the particle is projected to when it returns to its starting position. **(7 marks)**

c State, with reasons, what effect increasing the mass of the particle would have on your answers to parts **a** and **b**. **(2 marks)**

Problem solving Set B

Bronze

A box of mass 4 kg rests on a rough horizontal table and is attached to one end of a light inextensible string. The string passes over a small smooth pulley fixed at the edge of the table. The other end of the string is attached to a box of mass m kg which hangs freely below the pulley. The coefficient of friction between the box and the table is 0.375. The system is released from rest with the string taut, causing the boxes to accelerate at $1.2\,\text{m s}^{-2}$.



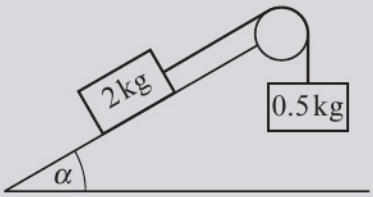
a Work out the tension in the string. **(3 marks)**

b Work out the value of m. **(2 marks)**

c State how you have used the fact that the pulley is smooth in your calculations. **(1 mark)**

Silver

A block of mass 2 kg rests on a smooth slope that is inclined at an angle α to the horizontal. It is attached to one end of a light inextensible string. The string passes over a small smooth pulley fixed at the top of the slope. The other end of the string is attached to a mass of 0.5 kg which hangs freely below the pulley. The system is released from rest with the string taut, causing the boxes to accelerate at $0.8\,\text{m}\,\text{s}^{-2}$.



a Work out the two possible sizes of angle α. **(5 marks)**

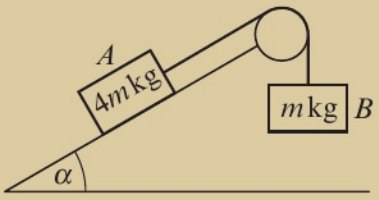
b Given that α takes the lesser of these two sizes, find the magnitude of the force exerted on the pulley by the string. **(4 marks)**

c State how your answer to part **b** would differ if α took the greater of its two possible sizes. **(1 mark)**

Gold

Block A of mass $4m$ kg is held at rest on a rough slope that is inclined at an angle α to the horizontal, where $\tan\alpha = \frac{5}{12}$. It is attached to one end of a light inextensible string.

The string passes over a small smooth pulley fixed at the top of the slope. The other end of the string is attached to a block B of mass m kg which hangs freely below the pulley. The coefficient of friction between block A and the slope is μ.



The system is released from rest. Given that block A begins to slide down the slope,

a find the range of possible values of μ. **(4 marks)**

b Given that $\mu = 0.1$, find the acceleration of the system. **(8 marks)**

c State, with reasons, the effect that doubling m would have on:

i the acceleration of the system

ii the tension in the string. **(3 marks)**

Now try this → **Exam question bank Q26, Q34, Q40, Q43, Q46, Q49, Q52**

8.1 Vectors in kinematics

In these questions, **i** and **j** are the unit vectors due east and north respectively.

1 A particle starts from the point with position vector $(4\mathbf{i} + 2\mathbf{j})$ m and moves with constant velocity $(\mathbf{i} - 3\mathbf{j})\,\text{m s}^{-1}$. Work out the position of the particle after 5 seconds.

Hint Use $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$ to find the position of the particle.

2 A particle starts from the point with position vector $(-6\mathbf{i} + \mathbf{j})$ m and moves with constant velocity $(2\mathbf{i} + \mathbf{j})\,\text{m s}^{-1}$.

a Work out the position of the particle after 2 seconds.

b Find the time when the particle is due north of the origin.

Hint When the particle is due north of the origin the **i** component of its displacement will be zero.

3 A particle passes a point P with velocity $(30\mathbf{i} + 20\mathbf{j})\,\text{m s}^{-1}$ and moves with constant acceleration $(10\mathbf{i} + 20\mathbf{j})\,\text{m s}^{-2}$. Work out:

a the displacement of the particle 2 seconds after it passes P

b its distance from P at this time.

Hint Use $\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$. Distance is the magnitude of the displacement vector.

4 A particle has velocity $(3\mathbf{i} - 2\mathbf{j})\,\text{m s}^{-1}$ and moves with constant acceleration $(3\mathbf{i} + \mathbf{j})\,\text{m s}^{-2}$. Work out the speed of the particle and the bearing on which it is travelling after 3 seconds.

Hint Use $\mathbf{v} = \mathbf{u} + \mathbf{a}t$. To work out the bearing draw a triangle showing the velocity vector and work out the angle clockwise from north to the velocity vector.

5 A runner runs on a horizontal playing field. At time $t = 0$, the runner is at a fixed point O and has a velocity of $(-8\mathbf{i} + 3\mathbf{j})\,\text{m s}^{-1}$. After 8 seconds, the runner has a velocity of $(4\mathbf{i} - 5\mathbf{j})\,\text{m s}^{-1}$. By modelling the runner as a particle moving with constant acceleration, work out the acceleration of the runner.

6 At time $t = 0$ a toy car has velocity $(a\mathbf{i} + b\mathbf{j})\,\text{m s}^{-1}$. It accelerates at $(3\mathbf{i} + \mathbf{j})\,\text{m s}^{-2}$ for 5 seconds, after which time it has velocity $(4\mathbf{i} - 13\mathbf{j})\,\text{m s}^{-1}$.

a Work out the values of a and b.

b Calculate the distance travelled by the car.

(E) **7** An aeroplane starts from the point P with position vector $\begin{pmatrix}1540\\2500\end{pmatrix}$ m relative to a fixed origin.

The aeroplane moves with constant velocity $\begin{pmatrix}-220\\-400\end{pmatrix}\,\text{m s}^{-1}$. Find:

a the position vector of the aeroplane after t seconds **(1 mark)**

b the distance of the aeroplane from the origin after 2 seconds **(2 marks)**

c the time when the aeroplane is due south of the origin. **(2 marks)**

E **8** A toy train moves with constant acceleration. At time $t = 0$ seconds, the train has velocity $\begin{pmatrix} 2 \\ -5 \end{pmatrix}$ m s^{-1}, and at time $t = 5$ seconds, it has velocity $\begin{pmatrix} 14 \\ 5 \end{pmatrix}$ m s^{-1}.

a Find the initial speed of the train. **(2 marks)**

b Calculate the acceleration of the train. **(2 marks)**

Given that the initial position of the toy train relative to a fixed origin is $(-\mathbf{i} + 8\mathbf{j})$ m,

c find the position vector of the train after 6 seconds. **(2 marks)**

E/P **9** At time $t = 0$, two walkers, A and B, have position vectors relative to a fixed origin given by $\begin{pmatrix} 4 \\ -6 \end{pmatrix}$ km and $\begin{pmatrix} 18 \\ 12 \end{pmatrix}$ km respectively. A walks with velocity $\begin{pmatrix} 1.5 \\ 2 \end{pmatrix}$ km h^{-1} and B walks with velocity $\begin{pmatrix} -2 \\ -2.5 \end{pmatrix}$ km h^{-1}.

a Show that the two walkers will meet. **(3 marks)**

b Work out the position vector of the point where they meet. **(2 marks)**

E/P **10** A particle P of mass 2 kg moves under the action of a constant force of $\mathbf{F}$ N.
At time $t = 0$ seconds, the particle is at a point A and is travelling with velocity $(5\mathbf{i} + 10\mathbf{j})$ m s^{-1}.
At time $t = 3$ seconds, the particle is at a point B and is travelling with velocity $(14\mathbf{i} - 2\mathbf{j})$ m s^{-1}.

a Find $\mathbf{F}$, giving your answer in the form $a\mathbf{i} + b\mathbf{j}$. **(5 marks)**

At the instant that P passes point B, the force is removed.

b Find the time that further elapses before the particle is directly due east of point A. **(5 marks)**

8.2 Vector methods with projectiles

In these questions, you may assume that **i** and **j** are unit vectors acting horizontally and vertically respectively, and that $g = 9.8$ m s^{-2}.

1 A particle is projected from a point on a horizontal plane with an initial velocity of $\begin{pmatrix} 15 \\ 25 \end{pmatrix}$ m s^{-1}.

The particle moves freely under gravity. Find:

a the velocity of the particle 2 seconds after it is projected

b its speed at this time.

Hint Use $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ with $\mathbf{a} = -9.8\mathbf{j}$ m s^{-2}

2 At time $t = 0$, a ball is projected from a point P, which has position vector $\begin{pmatrix} 6 \\ 6 \end{pmatrix}$ m relative to a fixed origin O. Immediately after projection, the ball has velocity $\begin{pmatrix} 12 \\ 19 \end{pmatrix}$ m s^{-1}.

The ball is modelled as a particle moving freely under gravity.

a Find an expression for the displacement of the ball from P at time t, giving your answer in the form $\begin{pmatrix} x \\ y \end{pmatrix}$ m, where x and y are functions of t.

Hint Use $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ to find the displacement.

b Hence find an expression for the position vector of the ball relative to O.

3 At time $t = 0$, an object is projected from a point A which has position vector $\begin{pmatrix} 0 \\ 15 \end{pmatrix}$ m relative to a fixed origin O that lies on a horizontal plane. Immediately after projection, the object has velocity $\begin{pmatrix} 23 \\ 13 \end{pmatrix}$ m s^{-1}. The object moves freely under gravity before landing at the point B.

a Find an expression for the position vector of the object relative to O.

b Work out:

i the time taken for the object to reach B

ii the distance OB.

Hint When the object reaches B, the **j**-component of the position vector is zero.

4 A particle is projected from a point on level ground with initial velocity $(8\mathbf{i} + 7\mathbf{j})$ m s^{-1}.

Work out the greatest height reached by the particle.

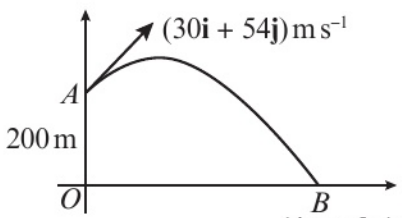
E/P **5** A particle is projected with velocity $\begin{pmatrix} a \\ b \end{pmatrix}$ m s^{-1} from a point O that lies on a horizontal plane. The particle moves freely under gravity.

The particle lands after 5 seconds at the point B where $OB = 120$ m. Work out:

a the value of a and the value of b **(3 marks)**

b the greatest height reached by the particle. **(2 marks)**

E **6** A particle is projected with velocity $(30\mathbf{i} + 54\mathbf{j})$ m s^{-1} from a point A with position vector $200\mathbf{j}$ m relative to a fixed origin O, which is at sea level.



The particle moves freely under gravity until it enters the sea at the point B. Work out:

a the time taken for the particle to enter the sea **(4 marks)**

b the distance AB. **(3 marks)**

E/P **7** A particle is projected with velocity $(13\mathbf{i} + 20\mathbf{j})$ m s^{-1} from a point O that lies on horizontal ground. The particle moves freely under gravity until it lands on a platform. The platform is at a height of 20 m above the ground. Work out:

a the horizontal distance travelled by the particle **(4 marks)**

b the speed with which the particle strikes the platform. **(3 marks)**

E/P **8** A particle is projected with velocity $(p\mathbf{i} + q\mathbf{j})$ m s^{-1} from the origin O that lies on horizontal ground. The particle moves freely under gravity until it lands on the ground 6 seconds later at an angle of α to the horizontal, where $\tan\alpha = \frac{12}{5}$

Work out the values of p and q. **(5 marks)**

8.3 Variable acceleration in one dimension

1 A particle is moving in a straight line. At time t seconds, the displacement, s m, of the particle from the origin is given by $s = 2\sin \pi t + t^2$. Find:

a the velocity of the particle when $t = 2$

b the acceleration of the particle when $t = 5$.

Hint Differentiate displacement to find velocity: $v = \frac{ds}{dt}$

Differentiate velocity to find acceleration: $a = \frac{dv}{dt}$

← Year 1, Section 11.2

2 A particle starts at rest at the origin, O, and moves in a straight line. At time t seconds, the acceleration, $a\,\text{m s}^{-2}$, of the particle is given by $a = e^t + t^2$. Find:

a an expression for the velocity of the particle

b the displacement of the particle from the origin when $t = 3$.

Hint To find the velocity, integrate the expression for the acceleration. Then use the initial condition that $v = 0$ when $t = 0$ to find the value of the constant of integration.

E/P **3** A particle moves in a straight line such that at time t seconds, its displacement s m from a fixed point O is given by:

$$s = \begin{cases} 2t + 4^t - 4, & 0 \leqslant t \leqslant 2 \\ t(t+6), & t > 2 \end{cases}$$

Work out:

a the velocity of the particle when $t = 1.5$ **(3 marks)**

b the acceleration of the particle when $t = 5$. **(2 marks)**

E/P **4** A particle is moving in a straight line. At time t seconds, its acceleration is given by

$$a = \frac{2}{t^2} - \cos 2\pi t\,\text{m s}^{-2},\ t \geqslant 1$$

Given that when $t = 1$, $v = 0$ and $s = 0$, find an expression for the displacement of the particle from the origin at time t seconds. **(8 marks)**

E/P **5** A particle moves in a straight line, passing point O at $t = 0$ seconds.

At time t seconds, $t \geqslant 0$, its velocity is given by $v = kt\cos t\,\text{m s}^{-1}$.

Given that when $t = \pi$ the particle has acceleration $0.5\,\text{m s}^{-2}$, work out:

a the value of k **(3 marks)**

b an expression for the displacement of the particle from O **(5 marks)**

c the displacement of the particle from O when $t = \pi$. **(1 mark)**

E/P **6** A particle P of mass 2 kg moves in a straight line under the action of a single variable force of magnitude F N. At time t seconds, where $t \geqslant 0$, the displacement, s m, of P from a fixed point O is given by $s = 1 - e^{-kt}$. Given that at $t = 0$ the velocity of the particle is $2\,\text{m s}^{-1}$, work out:

a the value of k **(3 marks)**

b the value of F when $t = 0.5$. **(5 marks)**

E/P **7** A particle is moving in a straight line. At time t seconds, the displacement, s m, of the particle from the origin is given by $s = \sqrt{t^2 + 15}$, where $0 \leqslant t \leqslant 7$.

a Find an expression for the velocity of the particle. **(3 marks)**

b Show that the velocity of the particle is increasing on the interval $0 \leqslant t \leqslant 7$, and hence state the maximum speed of the particle. **(5 marks)**

8.4 Differentiating vectors

In these questions, you may assume that **i** and **j** are the unit vectors due east and north respectively.

1 At time t seconds, the position vector $\mathbf{r}$ m of a particle relative to a fixed origin is given by

$\mathbf{r} = 3t^2\mathbf{i} - 5\sqrt{t}\,\mathbf{j}$, $t \geqslant 0$

a Work out the velocity of the particle when $t = 9$.

b Calculate the speed and direction in which the particle is moving when $t = 1$.

Hint Use $\mathbf{v} = \frac{d\mathbf{r}}{dt}$. You need to differentiate the **i**-component and the **j**-component separately.

2 At time t seconds the velocity $\mathbf{v}\,\text{m s}^{-1}$ of a particle P is given by

$\mathbf{v} = \cos t\mathbf{i} - 5t\mathbf{j}$, $t \geqslant 0$

Work out:

a the acceleration of P when $t = \pi$

b the direction in which P is accelerating at $t = \frac{\pi}{2}$

Hint Use $\mathbf{a} = \frac{d\mathbf{v}}{dt}$

3 At time t seconds a particle P has position vector $\mathbf{r}$ m relative to a fixed origin, where

$\mathbf{r} = t(t + 1)\mathbf{i} + (t^3 - 1)\mathbf{j}$, $t \geqslant 0$

Work out:

a the distance of P from the origin when $t = 2$

b the acceleration of P when $t = 4$, giving your answer in the form $(a\mathbf{i} + b\mathbf{j})\,\text{m s}^{-2}$.

Hint The distance is the magnitude of the displacement vector.

E/P **4** At time t seconds a particle P has position vector $\mathbf{r}$ m relative to a fixed origin, where

$\mathbf{r} = e^{3t}\mathbf{i} + \sin 2\pi t\,\mathbf{j}$, $t \geqslant 0$

Work out the acceleration of the particle when $t = 2$. **(6 marks)**

E/P **5** A particle P is moving in a plane. At time t seconds, it has a position vector relative to a fixed origin O of $\mathbf{r} = \left(4t\mathbf{i} - \frac{3}{t+1}\mathbf{j}\right)$ m, $t \geqslant 0$.

Work out:

a the time when the particle is directly south-east of O **(3 marks)**

b the speed of the particle at this time. **(3 marks)**

E/P **6** A particle P is initially at a fixed origin O. P moves such that, at time t seconds, it has a position $\mathbf{r}$ m relative to O, where $\mathbf{r} = (2 - 3t^2)\mathbf{i} - t(t^3 - 4)\mathbf{j}$, $t \geqslant 0$.

Work out:

a the initial speed of P **(3 marks)**

b the time when P is moving parallel to $\mathbf{i}$ **(2 marks)**

c the displacement of P when it is moving parallel to $\mathbf{i}$. **(2 marks)**

E/P **7** At time t seconds a particle P has displacement $\mathbf{r}$ m relative to a fixed origin, where

$\mathbf{r} = 3e^{2t(t+1)}\mathbf{i} - e^{t}\mathbf{j}$, $t \geqslant 0$

a Find the speed of P when $t = 0.2$. **(4 marks)**

b Work out the acceleration of P giving your answer in vector form. **(6 marks)**

E/P **8** A particle P moves under the action of a single force $\mathbf{F}$ N.

At time t seconds, where $t \geqslant 0$, the position vector of the particle $\mathbf{r}$ m, relative to a fixed origin, is given by $\mathbf{r} = 5t(t+1)\mathbf{i} + (3 - t^2)\mathbf{j}$.

a Show that P moves with constant acceleration. **(5 marks)**

b Given that the mass of P is 5 kg, find the magnitude of $\mathbf{F}$. **(3 marks)**

8.5 Integrating vectors

In these questions, you may assume that $\mathbf{i}$ and $\mathbf{j}$ are the unit vectors due east and north respectively.

1 A particle is moving in a plane so that at time t seconds, its acceleration $\mathbf{a}$ m s^{-2} is given by

$\mathbf{a} = 5t\mathbf{i} + 2\mathbf{j}$, $t \geqslant 0$

When $t = 2$, the velocity of the particle is $(8\mathbf{i} + 2\mathbf{j})$ m s^{-1}.

a Find an expression for the velocity of the particle at time t, giving your answer in the form $(f(t)\mathbf{i} + g(t)\mathbf{j})$ m s^{-1}.

b Calculate the speed of the particle at $t = 3$.

Hint $\mathbf{v} = \int \mathbf{a}\, dt$

Integrate and include a vector constant of integration, $\mathbf{c}$. Then use the condition given in the question to find $\mathbf{c}$.

2 A particle is moving in a plane such that, at time t seconds, it has velocity $(3\mathbf{i} - 2t^2\mathbf{j})$ m s^{-1}. When $t = 0$, the displacement of the particle relative to a fixed origin O is $(-\mathbf{i} + 3\mathbf{j})$ m.

Find the displacement of the particle at time t.

Hint Use $\mathbf{r} = \int \mathbf{v}\, dt$.

3 A particle is moving in a plane so that at time t seconds, its acceleration $\mathbf{a}\,\text{m s}^{-2}$ is given by

$\mathbf{a} = (1 - 6t)\mathbf{i} + 3\mathbf{j}$, $t \geqslant 0$

When $t = 0$ the particle has velocity $(\mathbf{i} + 2\mathbf{j})\,\text{m s}^{-1}$ and the position of the particle relative to a fixed origin O is $(4\mathbf{i} + \mathbf{j})$ m. Find:

a the displacement of the particle from O at time t

b the distance of the particle from O when $t = 3$.

Hint Integrate **a** to find the velocity first, then integrate again to find the displacement.

E/P **4** A particle is initially at rest at a fixed origin O. The particle moves in a plane such that its acceleration, $\mathbf{a}\,\text{m s}^{-2}$, at time t seconds is given by $\cos \pi t\mathbf{i} + 2\sin \pi t\mathbf{j}$, $t \geqslant 0$.

Find:

a the velocity of the particle when $t = 3$ **(5 marks)**

b the position vector of the particle when $t = 4$. **(5 marks)**

E/P **5** At time t seconds a particle is moving in a plane with acceleration $(3t\mathbf{i} - 2\mathbf{j})\,\text{m s}^{-2}$, where $t \geqslant 0$. When $t = 0$, the velocity of the particle is $2\mathbf{i}\,\text{m s}^{-1}$. Find:

a the velocity of the particle at time t seconds **(3 marks)**

b the speed of the particle when $t = 2.4$ **(3 marks)**

c the direction in which the particle is moving when $t = 4.8$. **(3 marks)**

E/P **6** At time t seconds a particle, P is moving in a plane with velocity $\mathbf{v}\,\text{m s}^{-1}$, where

$\mathbf{v} = (4t + 1)\mathbf{i} - 2k\mathbf{j}$, $t \geqslant 0$

When $t = 0$, the displacement of the particle is $\mathbf{j}$ m.

a Find an expression for the displacement of the particle. **(3 marks)**

A second particle, Q, is also moving in a plane. Its velocity is $(5\mathbf{i} - 3t\mathbf{j})\,\text{m s}^{-1}$, where $t \geqslant 0$. When $t = 0$, the displacement of Q is $2\mathbf{j}$ m.

Given that the particles collide, find:

b the value of k **(3 marks)**

c the position vector of the point of collision. **(3 marks)**

E/P **7** A particle, P is moving in a plane with acceleration $\mathbf{a}\,\text{m s}^{-2}$, where

$\mathbf{a} = (3t - 1)\mathbf{i} + \mathrm{e}^{t}\mathbf{j}$, $t \geqslant 0$

When $t = 0$, the velocity of the particle is $(7\mathbf{i} - 4\mathbf{j})\,\text{m s}^{-1}$.

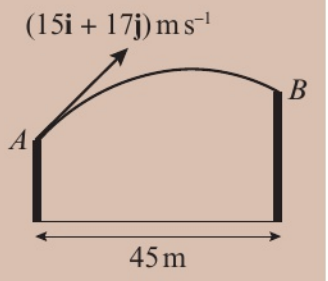
Show that the particle never moves in a direction parallel to $\mathbf{j}$. **(6 marks)**

Problem solving Set A

Bronze

For this question take $g = 10\,\text{m s}^{-2}$.

A particle is projected with a velocity of $(15\mathbf{i} + 17\mathbf{j})\,\text{m s}^{-1}$ from a point A at the top of a building that stands on a horizontal plane. The particle moves freely under gravity until it lands at point B on top of a second building that stands on the same horizontal plane. The horizontal distance between the two buildings is 45 m.



Find:

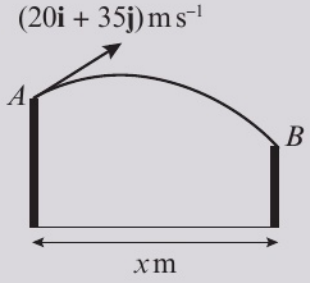
a the velocity of the particle when it strikes B, in the form $(a\mathbf{i} + b\mathbf{j})\,\text{m s}^{-1}$ **(4 marks)**

b the difference in height between the two buildings. **(3 marks)**

Silver

For this question, take $g = 10\,\text{m s}^{-2}$.

A particle is projected with a velocity of $(20\mathbf{i} + 35\mathbf{j})\,\text{m s}^{-1}$ from point A on the top of a building that stands on a horizontal plane. The particle moves freely under gravity until it lands at point B on top of a second building that stands on the same horizontal plane.



a Find the time at which the particle is travelling parallel to the vector $2\mathbf{i} + \mathbf{j}$. **(3 marks)**

The particle strikes B at a speed of $30\,\text{m s}^{-1}$. Given that B is below A,

b find the horizontal distance, x m, between the two buildings. **(7 marks)**

Gold

For this question take $g = 10\,\text{m s}^{-2}$.

Two particles P and Q are projected simultaneously from the top of a cliff with velocities $(30\mathbf{i} + 10\mathbf{j})\,\text{m s}^{-1}$ and $(20\mathbf{i} + 20\mathbf{j})\,\text{m s}^{-1}$ respectively. Find:

a the time at which the two particles are instantaneously travelling in the same direction **(5 marks)**

b the exact distance between them at this point. **(3 marks)**

Problem solving Set B

In these questions you may assume that **i** and **j** are the unit vectors due east and north respectively.

Bronze

At time t seconds a particle P has position vector $\mathbf{r}$ m relative to a fixed origin O where

$\mathbf{r} = 4t^2\mathbf{i} + 3t(t^2 - 1)\mathbf{j}$, $t \geqslant 0$

Work out:

a the velocity of P at time t seconds **(3 marks)**

b the speed of the particle when $t = 6$, correct to 3 significant figures **(2 marks)**

c the exact time at which the particle is moving in a north-easterly direction. **(3 marks)**

Silver

At time t seconds, where $t \geqslant 0$, a particle P moves in a horizontal plane in such a way that its velocity $\mathbf{v}$ m s^{-1} is given by

$\mathbf{v} = 3\mathbf{i} + (t^{-\frac{1}{2}} + 1)\mathbf{j}$

When $t = 0$, P is at a point A. When $t = 9$, P is at a point B.

a Find the distance AB. **(5 marks)**

Given that the position vector of the point A relative to a fixed origin O is $-2\mathbf{j}$ m,

b find the exact time at which the particle is due east of O. **(3 marks)**

Gold

At time t seconds a particle P has position vector $\mathbf{r}$ m relative to a fixed origin O where

$\mathbf{r} = \cos \pi t\mathbf{i} + 3\sin^2 \pi t\mathbf{j}$, $t \geqslant 0$

Find, correct to 3 significant figures, the first time at which the particle is moving parallel to the vector $\mathbf{i} - \mathbf{j}$. **(7 marks)**

Now try this → **Exam question bank Q23, Q27, Q30, Q33, Q39, Q44**

Exam question bank

This bank of exam-style questions have not been ordered by topic. Read each question carefully to work out which skills and techniques you will need to apply.

Section A Statistics

1 A choir is made up of 18 women and 15 men. Of these, 10 of the women and 6 of the men can read music.

a Draw a two-way table to show this information. **(2 marks)**

b One member is chosen at random. Find:

i P(Reads music) **ii** P(Can't read music | Female)

iii P(Male | Reads music) **(3 marks)**

2 A chemist carried out an experiment seven times, and recorded the masses of the reactant and product each time.

Mass of reactant, m (grams)	3	5	7	10	15	20	30
Mass of product, p (grams)	2.7	4.1	5.2	6.8	9.1	11.3	15.4

The data are believed to be modelled by a relationship of the form $p = am^k$, where a and k are constants. The data are coded using $x = \log m$ and $y = \log p$.
The equation of the regression line of y on x is found to be $y = 0.081 + 0.749x$.

a Find the value of a and the value of k. **(2 marks)**

b Give a reason why this model would not be suitable for estimating the mass of the product in a similar experiment with 50 grams of reactant. **(1 mark)**

3 The random variable X is normally distributed with mean μ and variance σ^2.

a Write down the distribution of the sample mean $\overline{X}$ of a random sample of size n. **(1 mark)**

A machine in a factory makes metal sheets with width X cm. The factory supervisor wants to use a sample to estimate the mean of X.

b Determine how large a random sample is needed so that the supervisor can be 95% certain that the sample mean width will differ from the population mean width by less than 1 cm. Assume that it is known that $\sigma = 3.2$ cm. **(4 marks)**

4 Data on the daily total sunshine, s hours, and daily mean visibility, v Dm, is taken from the large data set for Leeming in July 2015.

Sunshine, s (hours)	5.9	14.5	4.6	6.8	6.5	10.7
Visibility, v (Dm)	3200	3400	1900	2200	2500	3600

a Calculate the product moment correlation coefficient for these data. **(1 mark)**

b Test, at the 5% level of significance, the claim that there is positive correlation between the daily total sunshine and visibility. State your hypotheses clearly. **(3 marks)**

5 Two events A and B are such that $P(A \mid B) = P(A) = P(B) = P(B \mid A) = P(A' \cap B')$.
Find $P(A)$. **(4 marks)**

6 **a** Write down two conditions under which the normal distribution may be used as an approximation to the binomial distribution. **(2 marks)**

A student is given that the random variable $X \sim B(250, 0.48)$ can be approximated by a normal random variable Y.

The student is asked to find an estimate for $P(X < 130)$.

The student's attempt is shown

$Y \sim N(\mu, \sigma^2)$

$\mu = np = 250 \times 0.48 = 120$

$\sigma = np(1 - p) = 250 \times 0.48 \times 0.52 = 62.4$

$Y \sim (120, 62.4^2)$

$P(X < 130) \approx P(Y < 129.5) = 0.5605$

b Identify the error made by the student. **(1 mark)**

c Use a normal approximation to find the correct estimate for $P(X < 130)$. **(2 marks)**

7 From the large data set, the daily mean air temperatures, t °C, and the daily mean windspeeds, w knots, were recorded for a location in the northern hemisphere during the last week of October 2015.

Temperature, t (°C)	23.0	23.0	22.6	24.1	22.2	20.4	19.0
Windspeed, w (kn)	6.5	7.6	6.3	6.0	6.0	3.5	3.1

a Using your knowledge of the large data set, suggest a location where the data was taken. **(1 mark)**

The corresponding data for Perth are shown in this table.

Temperature, t (°C)	19.7	21.3	21.0	20.6	21.0	20.8	18.8
Windspeed, w (kn)	10.1	12.3	9.9	7.4	6.7	7.8	8.5

b Calculate the product moment correlation coefficient for these data. **(1 mark)**

c With reference to your answer to part **b**, comment on the suitability of a linear regression model for these data. **(2 marks)**

8 In a probability experiment, two fair six-sided dice are thrown. A trial is considered a 'success' if the larger number shown on either dice is prime.

a Find the probability of success on a single trial. **(3 marks)**

The experiment is repeated 180 times, and the number of successes, X, is recorded.

b Write down the distribution of X. **(1 mark)**

c Explain why X can be approximated with a normal distribution $X \sim N(\mu, \sigma^2)$, giving the values of μ and σ. **(4 marks)**

d Use this normal distribution to estimate the probability that $X = 80$. **(1 mark)**

9 Abi and Brianna are both taking their driving tests.

A is the event that Abi passes her driving test.

B is the event that Brianna passes her driving test.

$P(A) = 0.4$, $P(B) = 0.3$ and $P(A \cup B) = 0.58$.

Find the probability that:

a both Abi and Brianna pass their driving tests **(1 mark)**

b Brianna fails her test given that Abi passes **(2 marks)**

c Abi passes her test given that Brianna passes **(2 marks)**

d State whether the events A and B are statistically independent. You must give a reason for your answer. **(1 mark)**

10 Data are collected on the amounts of water, W million gallons per day, used to generate electricity, E millions of kilowatt hours, from a sample of eight different hydroelectric generators.

Water, W	24 000	109 000	19 000	14 000	67 000	13 000	3000	46 000
Electricity, E	1090	3040	840	2190	10670	830	1600	1980

a Calculate the product moment correlation coefficient for these data, correct to 3 significant figures. **(1 mark)**

b Test, at the 10% significance level, whether these results show evidence of a positive linear relationship between W and E. State your hypotheses clearly. **(3 marks)**

Data are then collected on the amounts of water, W million gallons per day, used to generate electricity, E millions of kilowatt hours, from a sample of eight different days for one hydroelectric generator.

Water, W	24 000	32 000	14 000	22 000	37 000	43 000	27 000	19 000
Electricity, E	1090	1640	670	870	2380	2460	1950	940

c Calculate the product moment correlation coefficient for these data, to 3 s.f. **(1 mark)**

d Test, at the 10% significance level, whether these results show evidence of a positive linear relationship between W and E. State your hypotheses clearly. **(1 mark)**

e Give a possible reason for the difference in the results of the hypothesis tests in parts **b** and **d**. **(1 mark)**

11 A packing plant fills bags with gravel. The weights, X kg, of bags of gravel can be modelled by a normal distribution with mean 40 kg and standard deviation 2 kg.

a Find $P(X > 42)$. **(2 marks)**

b Find the weight that is exceeded by 1% of the bags. **(3 marks)**

Three bags are selected at random.

c Find the probability that two weigh more than 41 kg and one weighs less than 41 kg. **(4 marks)**

Bags are rejected if they weigh less than 40 kg.

d Given that a bag is not rejected, find the probability that it weighs more than 42 kg. **(3 marks)**

12 Members of a film club watch comedies (C), sci-fi (S) and foreign language films (F).

$P(C) = 0.85$, $P(S) = 0.6$ and $P(F) = 0.3$.

No one watches both sci-fi films and foreign language films and $P(C \cap S) = 0.45$.

a Draw a Venn diagram to illustrate these probabilities. **(4 marks)**

b Find:

i $P(F \cup S)$

ii $P((C \cap S) \cup (C \cap F))$ **(2 marks)**

c Are the events F' and S independent? You must justify your answer. **(2 marks)**

13 The amount of mineral water, W ml, in a bottle produced by a certain manufacturer is found to be normally distributed with mean μ and standard deviation σ.

Given that 78% of bottles contain more than 320 ml of water and 38% contain more than 334 ml of water,

a find the value of μ and the value of σ. **(6 marks)**

The bottles are packed in boxes of 8 bottles. A box meets quality standards if no more than 2 bottles contain less than 325 ml of water.

b Find the probability that a randomly chosen box meets quality standards. **(3 marks)**

A supermarket orders 600 boxes of mineral water.

c Using a suitable approximation, find the probability that fewer than 250 of these boxes meet quality standards. **(4 marks)**

14 A geologist collected these data on the diameters of solid particles in a river and the speeds of the current required to keep the particles in suspension.

Diameter of particle, d (mm)	0.2	1.3	5	11	20	45	80	160
Speed of current, v ($m s^{-1}$)	0.1	0.25	0.5	0.75	1	1.5	2.5	3.5

The data are coded using $x = d$ and $y = \log v$.

The equation of the regression line of y on x is found to be $y = 0.0074x - 0.412$.

a Find an expression for v in terms of d, giving your answer in the form $v = kb^d$, where k and b are constants to be found. **(3 marks)**

b Calculate the product moment correlation coefficient for the coded data. **(3 marks)**

The geologist believes that there is positive linear correlation between the coded data.

c Given that the data provided the geologist with sufficient evidence to reject the null hypothesis, suggest the least possible significance level for the geologist's test. **(1 mark)**

15 A call-centre collected data on how long customers had to wait before an operator was free to take their call.

The lower quartile of the data was 12.7 minutes and the interquartile range was 6.2 minutes.

It is suggested that a normal distribution could be used to model the waiting times.

a Using this model, calculate:

i the mean and standard deviation of the waiting times, correct to 3 s.f.

ii the probability that a randomly chosen customer will have to wait longer than 16 minutes. **(7 marks)**

A customer has already been waiting for 12 minutes.

b Find the probability that this customer will have to wait more than an additional 8 minutes. **(3 marks)**

On one day, two calls took more than three quarters of an hour to be answered.

c Comment on the suitability of the model in light of this information. **(1 mark)**

16 A factory produces metal components. Out of 250 components, 15 are known to be defective. A machine correctly identifies 12 of the defective components as defective. It incorrectly identifies 4 components without defects as defective.

The event D is 'component is defective' and the event M is 'machine identifies the component as defective'.

a Draw a Venn diagram to represent this situation. **(3 marks)**

b Calculate $P(M \mid D)$. **(2 marks)**

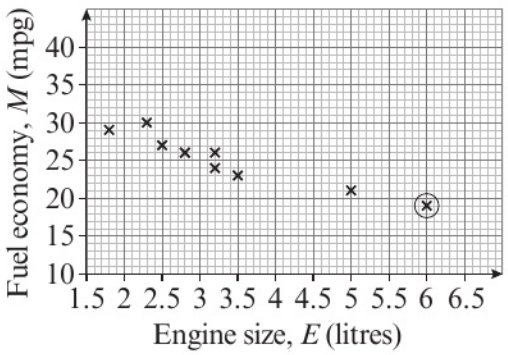
c Calculate $P(D' \mid M)$. **(2 marks)**

d With reference to your answer to part **b**, comment on the usefulness of the machine. **(1 mark)**

17 A car manufacturer records the engine sizes, E litres, and fuel economies, M miles per gallon, for a sample of ten cars.

Engine size, E (litres)	3.2	3.2	2.8	2.5	2.8	1.8	5.0	6.0	2.3	3.5
Fuel economy, M (mpg)	24	26	26	27	26	29	21	19	30	23

The data are represented on a scatter diagram.



Values greater than $Q_3 + 1.5\,(Q_3 - Q_1)$ or less than $Q_1 - 1.5\,(Q_3 - Q_1)$ are considered to be outliers.

a Show that the point circled on the diagram is an outlier. **(4 marks)**

b Suggest a reason why you might elect **not** to remove this outlier from your calculations. **(1 mark)**

c Calculate the product moment correlation coefficient for all the data. **(1 mark)**

d Suggest a possible reason for this correlation. **(1 mark)**

e With reference to your answer to part **c**, comment on the suitability of a linear regression model for these data. **(1 mark)**

The manufacturer calculated the linear regression equation as $M = 2.53E + 33.5$

f Explain why the manufacturer's equation is incorrect. **(1 mark)**

18 Serving against his regular opponent, a tennis player has a 70% chance of getting his first serve in. If his first serve is in, he then has a 60% chance of winning the point, but if his first serve is not in, he only has a 30% chance of winning the point.

a Represent this information on a tree diagram. **(2 marks)**

For a point on which this player served to his regular opponent, find the probability that:

b he won the point **(2 marks)**

c his first serve went in given that he won the point **(2 marks)**

d his first serve didn't go in given that he lost the point. **(3 marks)**

The tennis player decides to try a new tennis racket, and tests it by serving 100 times.

He finds that his first serve goes in on 62 occasions.

e Test, at the 5% level of significance, whether the new racket has changed his probability of getting his first serve in. State any assumptions you have made. **(5 marks)**

19 The reading scores, X, of a population of 7-year-olds are normally distributed with a mean of 50 and a standard deviation of 8.

a Find $P(X > 55)$. **(2 marks)**

b Find the upper quartile Q_3 of X. **(3 marks)**

c Write down the lower quartile Q_1 of X. **(1 mark)**

An outlier is defined as any value of X such that $X < h$ or $X > k$ where

$h = Q_1 - 1.5\,(Q_3 - Q_1)$ and $k = Q_3 + 1.5\,(Q_3 - Q_1)$

d Find the value of h and the value of k. **(2 marks)**

A child is selected at random.

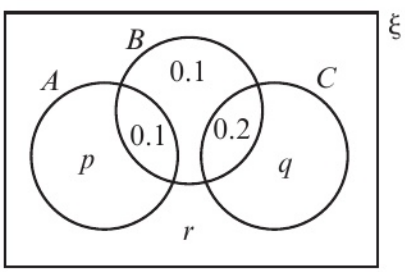
e Find the probability that their reading score is an outlier. **(3 marks)**

A teacher wants to test the theory that reading for an extra 20 minutes a day will improve reading scores. A random sample of 12 children are chosen and instructed to read for an extra 20 minutes a day for a month. They are then tested to find their reading scores. The mean score of the sample was found to be 54.

f Stating your hypotheses clearly, and using a 5% level of significance, test whether reading for 20 minutes a day has improved reading scores. **(4 marks)**

g Give one criticism of your conclusion to part **f**. **(1 mark)**

20 The Venn diagram shows the probabilities of three events A, B and C.



The events A and B are independent.

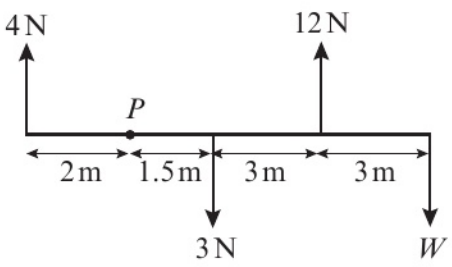
a Find the value of p. **(3 marks)**

b Given that $P(B \mid C) = \frac{4}{11}$, find the value of q and the value of r. **(4 marks)**

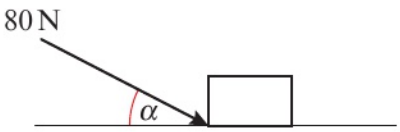
c Find $P(A \cup C \mid B)$. **(2 marks)**

Section B Mechanics

21 The diagram shows the forces acting on a light rod. Given that the resultant moment acting about P is 12 N m in the clockwise direction, work out the magnitude of force W. **(2 marks)**



22 A box is pushed along a floor. The floor is modelled as a rough horizontal plane and the box is modelled as a particle. The coefficient of friction between the box and the floor is 0.75.



The box is pushed by a force of magnitude 80 N that acts downwards at an angle of α with the floor, where $\tan\alpha = \frac{5}{12}$

Given that the box moves with constant speed, find the mass of the box. **(5 marks)**

23 At time $t = 0$, two hikers, Athena and Jeff, have position vectors relative to a fixed origin of $\begin{pmatrix}12\\-6\end{pmatrix}$ km and $\begin{pmatrix}2\\16\end{pmatrix}$ km respectively.

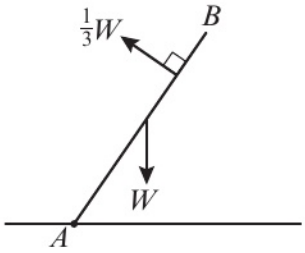
Athena walks with constant velocity $\begin{pmatrix}-1\\4\end{pmatrix}$ km h^{-1}, Jeff with constant velocity $\begin{pmatrix}2\\-2\end{pmatrix}$ km h^{-1}.

Determine whether the two hikers will meet, justifying your answer fully. **(4 marks)**

24 A uniform rod AB, of length l m and weight W N, is hinged at A.

It is held in equilibrium by a force of $\frac{1}{3}W$ acting perpendicularly to the rod $\frac{1}{4}l$ m from B.

Work out the angle that the rod makes with the horizontal. **(3 marks)**



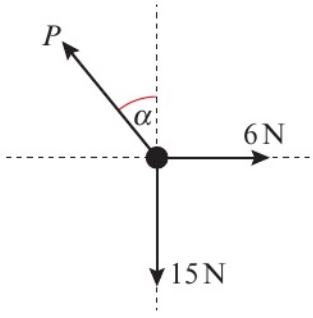
25 A box of mass 60 kg is moving along a horizontal floor under the action of a force of magnitude P N. The force acts at 60° above the floor and the box moves in a straight line at constant speed.

By modelling the box as a particle and the floor as a rough horizontal plane, with coefficient of friction 0.25 between the box and the plane,

a calculate the value of P. **(5 marks)**

b Without further calculation, briefly explain how the value of P would change if the surface is modelled as being smooth. **(1 mark)**

26 A particle of mass 3 kg is held in equilibrium by three forces as shown in the diagram.



a Find the size of angle α. **(3 marks)**

b Calculate the magnitude of P. **(1 mark)**

The force P is removed.

c Find the time taken for the particle to travel a distance of 10 m. **(4 marks)**

27 A particle P is moving in a plane. At time t seconds, it has a position vector $\mathbf{r}$ m relative to a fixed origin, where

$$\mathbf{r} = 3t(kt^2 + 1)\mathbf{i} + (5t^2 - 2)\mathbf{j},\ t \geqslant 0$$

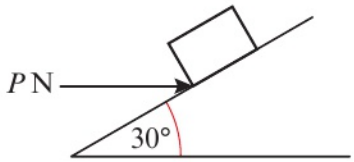
a Find an expression for the velocity of P at time t seconds. **(2 marks)**

Given that when $t = 2$, P has a speed of $5\sqrt{241}\ \text{m s}^{-1}$,

b show that $k = 2$. **(4 marks)**

28 A non-uniform rod AB, of length 16 m and weight 100 N, rests on two supports positioned at C and D. Support C lies 1 m from A and support D lies 6 m from B. A weight of 50 N is placed on the rod at B causing it to be on the point of tipping. Work out the distance of the centre of mass of the rod from A. **(3 marks)**

29 A box of weight 20 N lies on a rough plane inclined at an angle of 30° to the horizontal. A horizontal force of magnitude P N acts on the box.



The box is in equilibrium and on the point of slipping up the plane. The normal reaction of the plane on the box is 32 N, and the coefficient of friction between the box and the plane is μ. Find:

a the value of P **(3 marks)**

b the value of μ. **(3 marks)**

30 A particle P moves along the x-axis in the positive direction.

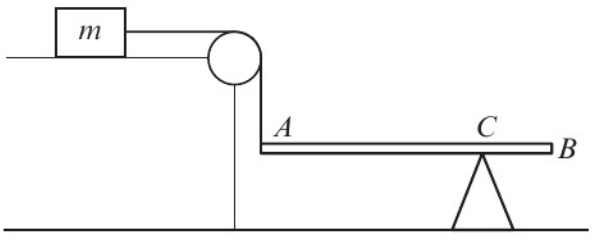
At time t seconds, the velocity of P is $v\ \text{m s}^{-1}$ and its acceleration is $\frac{2}{3}e^{-\frac{2}{5}t}\ \text{m s}^{-2}$.

When $t = 0$, the velocity of P is $1\ \text{m s}^{-1}$.

a Express v in terms of t. **(4 marks)**

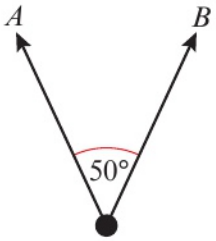
b Find the velocity of P when $t = 3$. **(2 marks)**

31 A beam AB, of length l and mass M, is attached at A to one end of a rope that passes over a smooth pulley. The other end of the rope is attached to a box of mass m that lies on a rough horizontal table. The coefficient of friction between the box and the table is μ. The beam rests in a horizontal position on a support at C, where $CB = 0.25l$.



By modelling the box as a particle, the beam as a uniform rod and the rope as light and inextensible, show that $M \leqslant k\mu m$, where k is a constant to be determined. **(4 marks)**

32 Two forces, A and B, act on a particle. Force A has magnitude 6 N and force B has magnitude 5 N. The angle between the directions of A and B is 50°.



Work out:

a the magnitude of the resultant of the two forces **(3 marks)**

b the angle between force A and the resultant. **(2 marks)**

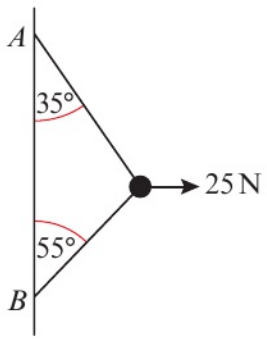
33 A particle of mass 4 kg moves under the action of a constant force $(6\mathbf{i} + 2\mathbf{j})$ N.

At time $t = 0$ seconds, the particle has position vector $(4\mathbf{i} - 2\mathbf{j})$ m and is moving with velocity $\mathbf{u}$ m s^{-1}.

At time $t = 6$ seconds, the particle has position vector $(-3\mathbf{i} + 8\mathbf{j})$ m.

Find $\mathbf{u}$. **(6 marks)**

34 A bead is threaded on a string that is attached to a vertical wall at the points A and B. The bead is held in equilibrium by a horizontal force of magnitude 25 N. The string is modelled as being light and inextensible, and the bead is modelled as being smooth.



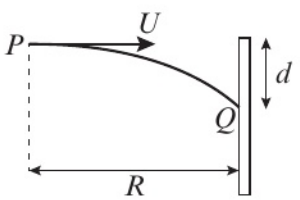
a Find the tension in the string. **(2 marks)**

b Work out the mass of the bead. **(2 marks)**

c State how you have used the fact that the bead is smooth in your calculations. **(1 mark)**

d Briefly explain how the tension in the string would be affected if the string is not modelled as being light. **(1 mark)**

35 A darts player throws a dart at a dartboard which hangs vertically. The motion of a dart is modelled as a particle moving freely under gravity. The dart is projected horizontally with speed U from point P, and moves in a vertical plane which is perpendicular to the plane of the dartboard. It hits the board at a point Q which is a distance d below P.



a Show that the horizontal distance, R, from the point where the dart was thrown to the dartboard is given by $R = U\sqrt{\frac{2d}{g}}$ **(4 marks)**

b Work out the value of d when a dart is thrown horizontally with a speed of 11.2 m s^{-1} from a distance of 2.25 m. **(3 marks)**

36 A sledge of mass 30 kg is pulled up a rough plane by means of a rope that is parallel to the line of greatest slope of the plane. The plane is inclined at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$

The coefficient of friction between the sledge and the plane is 0.2, and the sledge is accelerating up the slope with acceleration $1.25\,\text{m s}^{-2}$.

Find the tension in the rope. **(6 marks)**

37 A uniform beam AB has mass 10 kg and length 6 m. The beam rests in equilibrium in a horizontal position, resting on two smooth supports. One support is at end A, and the other is at a point C on the beam, where $AC = 4$ m.

a Find the reaction of the support on the beam at C. **(3 marks)**

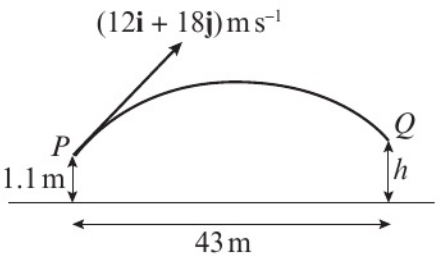
A mass of 40 kg is placed on the beam at the point D. The beam remains in equilibrium and the reactions on the beam at A and C are now equal.

b Find the distance AD. **(7 marks)**

38 A cricketer strikes a cricket ball a point P which is 1.1 m above horizontal ground.

The unit vectors **i** and **j** are horizontal and vertical respectively.

Immediately after being struck, the ball has velocity $(12\mathbf{i} + 18\mathbf{j})\,\text{m s}^{-1}$.



The ball moves freely under gravity and can be modelled as a particle.

a Find the maximum height above the ground reached by the ball. **(3 marks)**

A fielder stands in the line of flight of the ball at the point Q, a distance of 43 m from P and catches the ball.

b Work out the height, h, at which the fielder catches the ball. **(3 marks)**

c State how adjusting the model to take air resistance into account would alter your answer to part **b**. **(1 mark)**

39 A particle is projected with velocity $(23\mathbf{i} + 18\mathbf{j})\,\text{m s}^{-1}$ from a point O that lies on horizontal ground. The particle moves freely under gravity. Using $g = 9.8\,\text{m s}^{-2}$, work out:

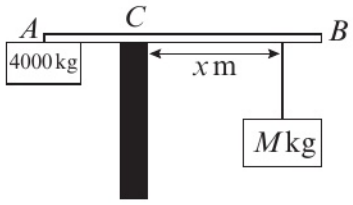
a the greatest height reached by the particle **(2 marks)**

b the horizontal distance travelled by the particle **(3 marks)**

c the time at which the particle is moving parallel to the vector $2\mathbf{i} - \mathbf{j}$. **(3 marks)**

40 A uniform ladder AB has mass m kg and length 2 m. End A rests on rough horizontal ground and end B rests against a smooth vertical wall. The ladder makes an angle of 60° with the horizontal. A man of mass $3m$ kg stands on the ladder at C where $AC = 0.5$ m. Given that the ladder is in limiting equilibrium, work out value of the coefficient of friction between the ladder and the ground. **(8 marks)**

41 The beam of a crane is modelled as a uniform rod AB, of length 25 m and mass 5000 kg, resting in horizontal equilibrium. The beam is supported by a tower at C, where $AC = 10$ m. A counterbalance mass of 4000 kg is placed at A and a load of mass M kg is hung from a wire placed at a distance of x m from the supporting tower, where $x \geqslant 6$.



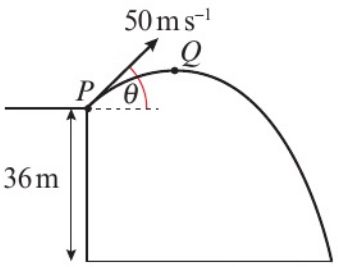
a Determine the range of loads that can be lifted by the crane. **(6 marks)**

b Criticise the model in relation to the beam of the crane. **(1 mark)**

42 A particle is projected with speed 50 m s^{-1} from a point P on a cliff that is 36 m above horizontal ground. The particle is projected at an angle θ to the horizontal.

The point Q is the highest point of the path of the ball and is 10 m above the level of P.

The particle moves freely under gravity.

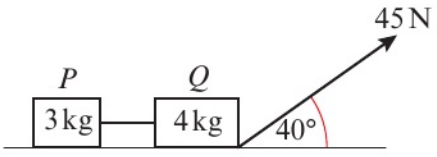


a Find the size of angle θ. **(3 marks)**

b Work out the time after projection at which the particle is moving with its original speed. **(4 marks)**

c Calculate the angle that the velocity of the particle makes with ground at the instant it lands. **(3 marks)**

43 Two boxes P and Q, of masses 3 kg and 4 kg respectively, are joined by a light inextensible string. Initially the boxes are at rest on a rough horizontal plane with the string taut. A constant force of 45 N is applied to Q at an angle of 40° as shown in the diagram.



The force is applied for 3 seconds and during this time Q travels a distance of 5 m. The coefficient of friction between P and the plane is μ and the coefficient of friction between Q and the plane is 2μ.

a Work out the acceleration of Q. **(2 marks)**

b Calculate the value of μ. **(4 marks)**

c Find the tension in the string. **(3 marks)**

d State how you have used the fact that the string is inextensible in your calculations. **(1 mark)**

44 At time t seconds a particle is moving in a plane with acceleration $\mathbf{a}\,\text{m s}^{-2}$, where

$\mathbf{a} = 3(t + 2)\mathbf{i} + 2(t - 1)\mathbf{j}$, $t \geqslant 0$

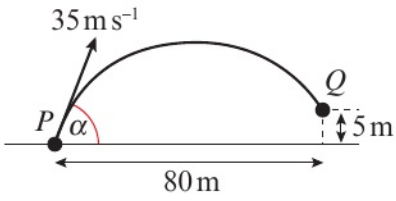
When $t = 0$, the velocity of the particle is $-2\mathbf{i} + 3\mathbf{j}\,\text{m s}^{-1}$.

Find:

a the speed of the particle when $t = 1.2$ **(6 marks)**

b the time when the particle is moving parallel to the vector $\mathbf{i} + \mathbf{j}$. **(4 marks)**

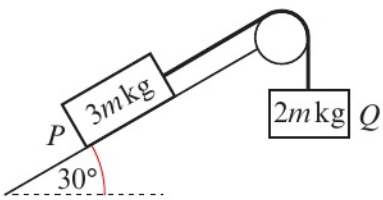
45 A golfer strikes a golf ball from a point P which lies on horizontal ground. The golfer wants the ball to land close to the hole, which lies a horizontal distance of 80 m from P and a vertical height of 5 m above P. The ball is projected with speed $35\,\text{m s}^{-1}$ at an angle α to the horizontal and can be modelled as a particle moving freely under gravity.



a Use the model to find the larger angle α that will ensure the golf ball lands on the hole. **(8 marks)**

b Comment on the likelihood of the golf ball actually landing on the hole. **(1 mark)**

46 Two boxes P and Q, of masses $3m$ kg and $2m$ kg respectively, are attached to the ends of a light inextensible string. Box P lies on a rough surface that is inclined at 30° to the horizontal. The coefficient of friction between P and the surface is 0.1.



The string passes over a small smooth pulley fixed on the edge of the surface. Box Q hangs freely below the pulley.

The system is released from rest with the strings taut.

a Work out the time taken for P to reach a speed of $10\,\text{m s}^{-1}$. **(5 marks)**

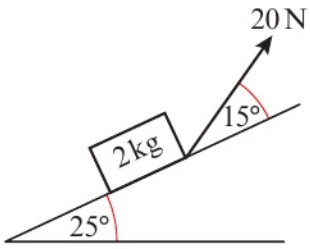
b State one assumption you have made in your calculation for part **a**. **(1 mark)**

At the instant P has velocity $10\,\text{m s}^{-1}$, the string snaps.

c Work out the distance travelled by P from the time the string snaps to the time P comes to instantaneous rest. **(5 marks)**

d Determine whether P will remain at rest or begin to slide back down the slope, fully justifying your answer. **(2 marks)**

47 A block P of mass 2 kg is moving up a rough plane at a constant speed of $8\,\text{m s}^{-1}$ under the action of a force of magnitude 20 N. The plane is inclined at 25° to the horizontal, and the force acts at 15° to the inclined plane as shown in the diagram.



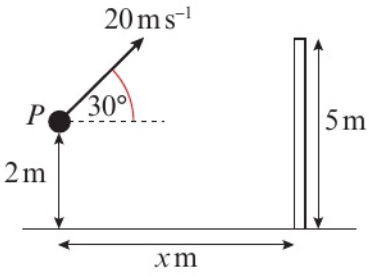
The coefficient of friction between the block and the plane is μ.

a Work out the value of μ. **(6 marks)**

The force of magnitude 20 N is removed.

b Find the distance that P travels from the instant when the force is removed to the instant when it comes to rest. **(4 marks)**

48 A ball is thrown with speed $20\,\text{m s}^{-1}$ from a point, P, that lies 2 m above horizontal ground. The angle of projection is 30° above the horizontal and the ball moves freely under gravity. A wall of height 5 m lies between P and the point where the ball lands. The horizontal distance from point P to the wall is x m.

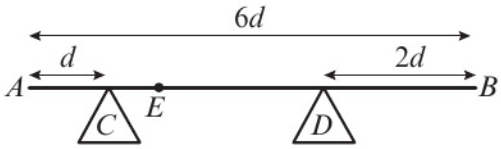


a Given that the ball clears the wall, find the range of possible values of x. **(6 marks)**

b Find the speed of the ball as it hits the ground. **(4 marks)**

49 A non-uniform ladder AB, of mass m and length l, rests in equilibrium with the end A on a rough horizontal floor and the end B against a rough vertical wall. The ladder makes an angle of 60° with the floor. The coefficient of friction between the ladder and the floor is 0.3 and the coefficient of friction between the ladder and the wall is 0.7. The centre of mass of the ladder is at the point M. Given that the ladder is on the point of slipping at both ends, determine, in terms of l, the distance AM. **(12 marks)**

50 A non-uniform rod AB, of mass m and length $6d$, rests horizontally in equilibrium on two supports at C and D, where $AC = d$ and $DB = 2d$, as shown. The centre of mass of the rod is at the point E. A particle of mass m is placed on the rod at B and the rod is on the point of tipping about D.

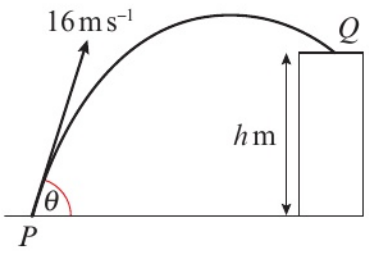


a Show that $AE = 2d$. **(4 marks)**

The particle is moved from B to the midpoint of the rod and the rod remains in equilibrium.

b Find the magnitude of the normal reaction between the support at D and the rod. **(5 marks)**

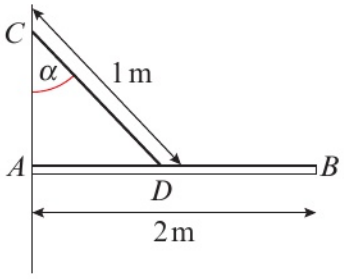
51 A ball is projected with speed $16\,\text{m s}^{-1}$ from a point P on horizontal ground. The angle of projection is θ above the horizontal, where $\sin\theta = \frac{4}{5}$. A horizontal platform is at height h metres above the ground. The ball moves freely under gravity until it hits the platform at the point Q. The speed of the ball immediately before it hits the platform at Q is $12\,\text{m s}^{-1}$.



a Find the value of h. **(4 marks)**

b Find the horizontal distance from P to Q. **(4 marks)**

52 A beam AB, of weight 20 N and length 2 m, is smoothly hinged to a vertical wall at A. The beam is held in equilibrium in a horizontal position by a rope of length 1 m. One end of the rope is fixed to the point C vertically above A and the other end is fixed at a point D such that the angle ACD is α. The beam is modelled as a uniform rod and the rope is modelled as a light inextensible string.



a Show that the tension in the string is equal to $\dfrac{40}{\sin 2\alpha}$ **(4 marks)**

b Given that the tension in the string is 80 N, find the two possible sizes of angle α. **(3 marks)**

Given further that α takes the lesser of these two values,

c find the direction and magnitude of the force exerted on the beam by the hinge at A. **(8 marks)**

d Without further calculation, state how the direction of this force would change if α took the greater of its two possible values. **(2 marks)**

Percentage points of the normal distribution

The values z in the table are those which a random variable $Z \sim N(0, 1)$ exceeds with probability p; that is, $P(Z > z) = 1 - \Phi(z) = p$.

p	z	p	z
0.5000	0.0000	0.0500	1.6449
0.4000	0.2533	0.0250	1.9600
0.3000	0.5244	0.0100	2.3263
0.2000	0.8416	0.0050	2.5758
0.1500	1.0364	0.0010	3.0902
0.1000	1.2816	0.0005	3.2905

Critical values for correlation coefficients

This table concerns tests of the hypothesis that a population correlation coefficient ρ is 0. The values in the table are the minimum values which need to be reached by a sample correlation coefficient in order to be significant at the level shown on a one-tailed test.

Product moment coefficient					
		Level			**Sample**
0.10	**0.05**	**0.025**	**0.01**	**0.005**	**Level**
0.8000	0.9000	0.9500	0.9800	0.9900	4
0.6870	0.8054	0.8783	0.9343	0.9587	5
0.6084	0.7293	0.8114	0.8822	0.9172	6
0.5509	0.6694	0.7545	0.8329	0.8745	7
0.5067	0.6215	0.7067	0.7887	0.8343	8
0.4716	0.5822	0.6664	0.7498	0.7977	9
0.4428	0.5494	0.6319	0.7155	0.7646	10
0.4187	0.5214	0.6021	0.6851	0.7348	11
0.3981	0.4973	0.5760	0.6581	0.7079	12
0.3802	0.4762	0.5529	0.6339	0.6835	13
0.3646	0.4575	0.5324	0.6120	0.6614	14
0.3507	0.4409	0.5140	0.5923	0.6411	15
0.3383	0.4259	0.4973	0.5742	0.6226	16
0.3271	0.4124	0.4821	0.5577	0.6055	17
0.3170	0.4000	0.4683	0.5425	0.5897	18
0.3077	0.3887	0.4555	0.5285	0.5751	19
0.2992	0.3783	0.4438	0.5155	0.5614	20
0.2914	0.3687	0.4329	0.5034	0.5487	21
0.2841	0.3598	0.4227	0.4921	0.5368	22
0.2774	0.3515	0.4133	0.4815	0.5256	23
0.2711	0.3438	0.4044	0.4716	0.5151	24
0.2653	0.3365	0.3961	0.4622	0.5052	25
0.2598	0.3297	0.3882	0.4534	0.4958	26
0.2546	0.3233	0.3809	0.4451	0.4869	27
0.2497	0.3172	0.3739	0.4372	0.4785	28
0.2451	0.3115	0.3673	0.4297	0.4705	29
0.2407	0.3061	0.3610	0.4226	0.4629	30
0.2070	0.2638	0.3120	0.3665	0.4026	40
0.1843	0.2353	0.2787	0.3281	0.3610	50
0.1678	0.2144	0.2542	0.2997	0.3301	60
0.1550	0.1982	0.2352	0.2776	0.3060	70
0.1448	0.1852	0.2199	0.2597	0.2864	80
0.1364	0.1745	0.2072	0.2449	0.2702	90
0.1292	0.1654	0.1966	0.2324	0.2565	100

Answers

Students' answers may differ slightly from those shown depending on whether calculators or tables have been used.

CHAPTER 1

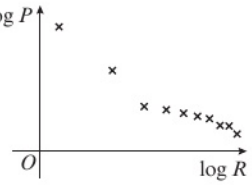
1.1 Exponential models

1 **a** $y = kb^x$ **b** $k = 5.01$, $b = 15.8$ (3 s.f.)

2 **a** $y = ax^n$ **b** $a = 31.6$ (3 s.f.), $n = 0.8$

3 $k = 126$, $b = 3.16$ (3 s.f.)

4 **a**

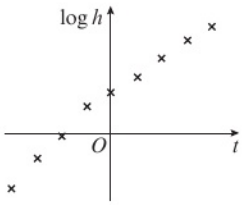


b Strong negative correlation

c Yes; the variables show a linear relationship when log P is plotted against log R.

d $P = 1000R^{-0.355}$

5 **a**



b Strong positive correlation

c The variables show a linear relationship when log h is plotted against t.

d $k = 2.99$, $b = 1.08$ (3 s.f.)

6 $E = 0.0603\,W^{1.02}$, $a = 0.0603$, $n = 1.02$ (3 s.f.)

7 $k = 31.6$, $b = 1860$ (3 s.f.)

8 **a** $a = 398$ (3 s.f.), $n = 0.67$

b 900 kg/ha is outside the range of the data (extrapolation).

1.2 Measuring correlation

1 **a** Approximately 0.7 **b** Approximately 0.3

c Approximately −0.9

2 **a** The type and strength of linear correlation between x and y.

b 0.9783

3 **a** −0.9355

b There is a strong negative correlation. The parts of the channel with larger depth have a lower current velocity.

4 −0.954 (3 s.f.)

5 **a**

P	87	9	73	62	7	44	53	98	29	35
W	95	19	58	41	14	37	47	82	24	26
log W	1.98	1.28	1.76	1.61	1.15	1.57	1.67	1.91	1.38	1.41

b 0.975 (3 s.f.)

c There is strong positive linear correlation with data in the form log W against P, which suggests an exponential model is a good fit.

d $a = 14.1$, $b = 1.02$ (3 s.f.)

6 **a**

E	3.2	3.2	2.8	2.5	2.8	1.8	5	6	2.3	3.5
M	24	26	26	27	26	29	21	19	30	23
log E	0.505	0.505	0.447	0.398	0.447	0.255	0.699	0.778	0.362	0.544
log M	1.38	1.41	1.41	1.43	1.41	1.46	1.32	1.28	1.48	1.36

b −0.962 (3 s.f.)

c There is strong negative linear correlation with data in the form log M against log E, which suggests a model of the form $y = kx^n$ is a good fit.

d $k = 38.9$ (3 s.f.), $n = -0.383$

7 **a** −0.852

b −0.852 is close to −1, so the data values show a strong to moderate negative correlation. A linear regression model is suitable for these data.

8 **a** Data on daily maximum gust is not available for this day.

b −0.0549. The n/a data entry is omitted when calculating r.

c Jamie's statement is not valid because there may be a non-linear relationship between the two variables *or* the sample size is too small to draw this conclusion.

1.3 Hypothesis testing for zero correlation

1 **a** H_0: $\rho = 0$, H_1: $\rho > 0$, critical value = 0.3061 < 0.4125.

Reject H_0: there is reason to believe at the 5% level of significance that there is a correlation between the scores.

b H_0: $\rho = 0$, H_1: $\rho > 0$, critical value = 0.4226 > 0.4125.

Accept H_0: there is no evidence of correlation between the two scores at the 1% level of significance.

2 **a** H_0: $\rho = 0$, H_1: $\rho \neq 0$, critical value = 0.4973 < 0.62.

Reject H_0: there is reason to believe at the 10% level of significance that there is a correlation between the mass and the length of the otters.

b H_0: $\rho = 0$, H_1: $\rho \neq 0$, critical value = 0.6581 > 0.62. Accept H_0: there is no evidence of correlation between the mass and the length of the otters at the 2% level of significance.

3 **a** −0.728

b H_0: $\rho = 0$, H_1: $\rho \neq 0$, critical value = 0.8114. Accept H_0: there is no evidence of correlation between the engine size and fuel economy at the 5% level of significance.

4 H_0: $\rho = 0$, H_1: $\rho > 0$, critical value = 0.7887, $r = 0.861$.

Reject H_0: there is evidence of linear correlation at the 1% level of significance. There is evidence to suggest the company is correct.

5 H_0: $\rho = 0$, H_1: $\rho < 0$, critical value = 0.4716.

Reject H_0: there is evidence of negative correlation at the 1% level of significance. There is evidence to suggest the council is correct.

6 1%

7 9

8 **a** 0.812

b $r = 0.812$, H_0: $\rho = 0$, H_1: $\rho > 0$
Critical value = 0.7887 < 0.812.
Reject H_0: there is evidence of positive correlation at the 1% level of significance.

9 **a** 0.786

b H_0: $\rho = 0$, H_1: $\rho > 0$, critical value = 0.7067.
Reject H_0: there is evidence of linear correlation at the 2.5% level of significance.

Problem solving

B $r = 0.870$ (3 s.f.). H_0: $\rho = 0$, H_1: $\rho > 0$

Critical value = 0.8745 > 0.870.

Accept H_0: there is no evidence of positive correlation between x and y at the 0.5% level of significance.

S **a** The data points do not fit a straight line.

b r = 0.992 (3 s.f.). H_0: $\rho = 0$, H_1: $\rho > 0$

Critical value = 0.7293 < 0.992.
Reject H_0: there is evidence of positive correlation at the 5% level of significance.

G **a** Using Spanish results, mean = 69.4 and s.d. = 7.61, so 86 > mean + 2 × s.d.

b All data:
$r = -0.2313$. H_0: $\rho = 0$, H_1: $\rho \neq 0$, $n = 7$, 5% in each tail gives critical value = −0.6694 < −0.2313.

Accept H_0: there is no evidence of correlation between the marks, so claim would be incorrect.
Removing point E:
$r = 0.8185$. H_0: $\rho = 0$, H_1: $\rho \neq 0$, $n = 6$, 5% in each tail gives critical value = 0.7293 < 0.8185 Reject H_0: there is evidence of correlation between the marks, so claim would be correct.

The removal of the outlier would affect the conclusion.

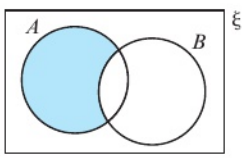
c Point E is likely to be an incorrect data entry (as the Spanish score is greater than the maximum number of marks available on the test, 75). It would be reasonable to remove this data point and conclude that there is evidence of correlation between the marks in the two tests.

CHAPTER 2

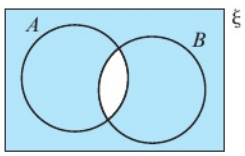
2.1 Set notation

1 **a** $A' \cap B$ **b** $(A \cup B)'$ **c** $(A \cap B') \cup (A' \cap B)$
d $(A \cup B)'$ **e** $(A \cap B) \cup (B \cap C) \cup (A \cap C)$
f $A \cap C \cap B'$

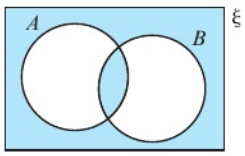
2 **a**



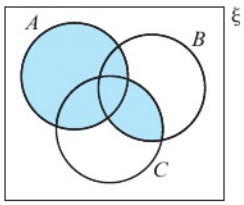
b



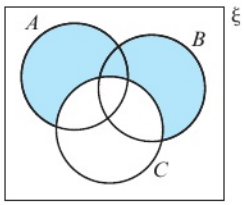
c



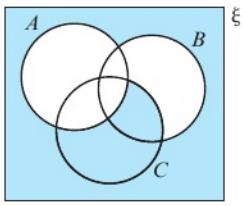
3 **a**



b



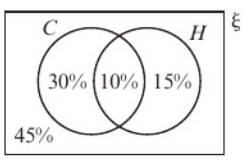
c



4 **a** $\frac{3}{13}$ **b** $\frac{1}{4}$ **c** $\frac{3}{52}$
d $\frac{11}{26}$ **e** $\frac{10}{13}$ **f** $\frac{9}{52}$

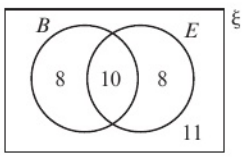
5 **a** 0.5 **b** 0.6 **c** 0.1 **d** 0.5

6 **a**



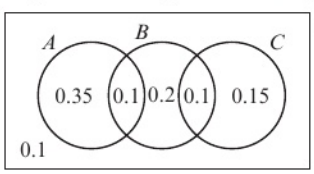
b **i** 0.1 **ii** 0.3 **iii** 0.15

7 **a**



b **i** $\frac{26}{37}$ **ii** $\frac{8}{37}$ **iii** $\frac{27}{37}$

8 **a**



b **i** $P(A \cap C) = 0$
ii $P(B) \times P(C) = 0.4 \times 0.25 = 0.1 = P(B \cap C)$

c **i** 0.3 **ii** 0.8 **iii** 0.2

2.2 Conditional probability

1 **a** $\frac{21}{40}$ **b** $\frac{4}{7}$ **c** $\frac{9}{20}$ **d** $\frac{11}{19}$

2 **a**

Green spinner \ Yellow spinner	1	2	3	4
1	1	2	3	4
2	2	4	6	8
3	3	6	9	12
4	4	8	12	16

b i $\frac{1}{8}$ ii $\frac{1}{4}$ iii $\frac{1}{3}$

3 **a**

Dice 2 \ Dice 1	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	2	3	4	5	6
3	3	3	3	4	5	6
4	4	4	4	4	5	6
5	5	5	5	5	5	6
6	6	6	6	6	6	6

b $\frac{7}{36}$ **c** $\frac{1}{3}$ **d** $\frac{1}{11}$

4 **a** i $\frac{31}{73}$ ii $\frac{12}{23}$ iii $\frac{7}{13}$

b Not mutually exclusive. The total number of people in all of the classes is 142. So there are 22 members that go to both classes.

5 $\frac{1}{3}$

6 **a**

	H	H'	Total
L	36	28	64
L'	46	30	76
Total	82	58	140

b i $\frac{19}{35}$ ii $\frac{1}{5}$ iii $\frac{9}{16}$ iv $\frac{14}{29}$

c $P(L) \times P(H) = \frac{16}{35} \times \frac{41}{70} = \frac{328}{1225} \neq \frac{9}{35} = P(L \cap H)$. Therefore L and H are not independent.

7 **a** $\frac{1}{9}$ **b** $\frac{43}{76}$ **c** $\frac{35}{58}$ **d** $\frac{4}{29}$

8 **a**

	Furina	Purskers	Whilix	Total
Adult cats	11	17	10	38
Kittens	7	7	8	22
Total	18	24	18	60

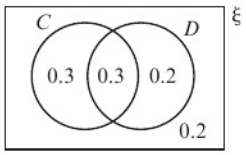
b i $\frac{11}{18}$ ii $\frac{4}{9}$ iii $\frac{7}{22}$

c $\frac{31}{60}$

2.3 Conditional probabilities in Venn diagrams

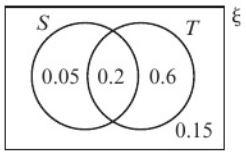
1 **a** 0.85 **b** 0.25 **c** 0.625 **d** $\frac{12}{17}$

2 **a**



b i 0.8 ii 0.6 iii 0.5 iv 0.5

3 **a**



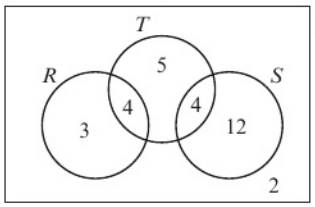
b i 0.2 ii 0.25 iii 0.8 iv 1

4 **a** $\frac{1}{3}$ **b** 0.5 **c** $\frac{1}{3}$ **d** 0.5

5 **a** 0.4 **b** $\frac{5}{9}$ **c** $\frac{5}{6}$ **d** $\frac{5}{13}$

6 **a** $\frac{5}{9}$ **b** $\frac{2}{3}$ **c** $\frac{10}{11}$ **d** $\frac{2}{3}$

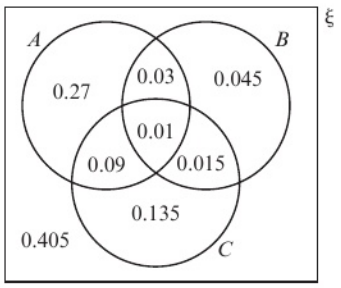
7 **a**



b $\frac{3}{20}$

c Not independent

8 **a**



b i 0.4 ii 0.9 iii 0.25

9 $x = 0.32$, $y = 0.48$

2.4 Probability formulae

1 **a** 0.2 **b** 0.5 **c** 0.7 **d** 0.9

2 **a** 0.3

b i 0.25 ii $\frac{2}{3}$ iii $\frac{6}{11}$

c No. For example, because $P(Y \mid X) \neq P(Y)$

3 **a** 0.35 **b** 0.875 **c** 0.75

4 **a** $P(E \cup F) = 2.5x - 0.3$

b $P(E \cup F) + P(E' \cap F') = 1$

$2.5x - 0.3 + 0.05 = 1$

$\Rightarrow 2.5x = 1.25$

$\Rightarrow x = 0.5$

c i 0.75 ii 0.2 iii 0.8

5 0.1

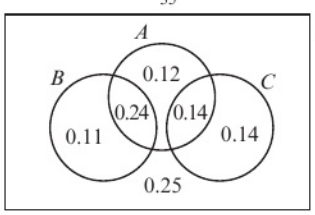
6 **a** i 0.15 ii 0.25 **b** 0.25

7 **a** 0.1 **b** 0.3 **c** 0.4

d 0.25 **e** 0.75 **f** $\frac{1}{3}$

8 **a** 0.61 **b** $\frac{11}{35}$ **c** 0.28

d



e 0.14

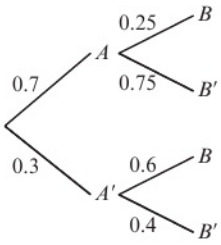
9 **a** 0.08 **b** 0.2 **c** 0.75 **d** 0.16

10 **a** 0.95 **b** 0.35 **c** $\frac{5}{12}$

d Not independent, since $P(H \mid G) \neq P(H)$

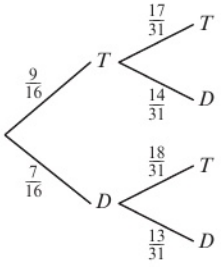
2.5 Tree diagrams

1 **a**



b **i** 0.175 **ii** 0.12 **iii** 0.75 **iv** $\frac{35}{71}$

2 **a**

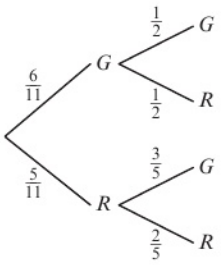


b $\frac{153}{496}$ **c** $\frac{63}{124}$ **d** $\frac{13}{49}$

3 **a** $\frac{7}{12}$ **b** 0.6

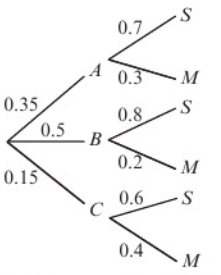
4 $\frac{44}{89}$

5 **a**



b $\frac{5}{11}$ **c** 0.4

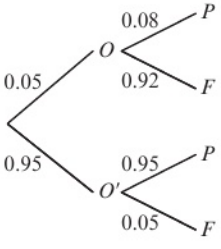
6 **a**



b **i** 0.1 **ii** 0.735

c $\frac{12}{53}$

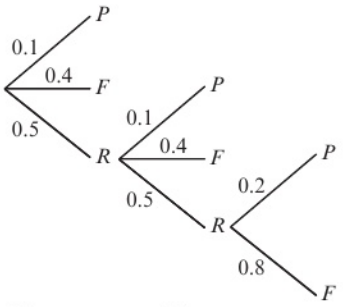
7 **a**



b 0.9065 **c** $\frac{95}{187}$

d Only 49% of the microchips that fail the test actually have defects. Therefore the test is not effective.

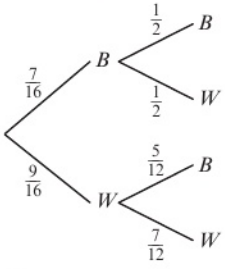
8 **a**



b 0.2 **c** 0.5

Problem solving: Set A

B **a**



b $\frac{29}{64}$ **c** $\frac{3}{5}$

S $\frac{45}{67}$

G **a** $\frac{1}{6}$ **b** $a = 0.15, b = 0.45$

Problem solving: Set B

B **a** **i** 0.25 **ii** 0.15

b $\frac{1}{3}$

c Not independent: $P(Y|Z) = \frac{1}{3} \neq P(Y) = 0.15$.

d If Zoe doesn't meet her target, Yvonne is more likely to miss hers and vice versa.

S **a** $P(B) = x$, $P(A) = 3x$, $P(A \cap B) = 3x^2$

$3x + x - 3x^2 = \frac{7}{12}$

$\Rightarrow 3x^2 - 4x + \frac{7}{12} = 0$

$\Rightarrow x = \frac{1}{6}$

b $\frac{1}{12}$ **c** $\frac{1}{6}$

G $x = 0.15$, $y = 0.15$ and $z = 0.2$

CHAPTER 3

3.1 The normal distribution

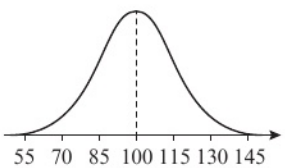
1 **a** Not suitable: number of children is discrete.

b Not suitable: the distribution is unlikely to be symmetrical.

c Suitable, e.g. likely to be similar times.

d Suitable, e.g. likely to be similar length.

2



3 The distribution shown does not have a bell-shaped curve.

4 **a** 68% **b** 95%

5 1.2

6 32.5 kg

7 $\mu = 42$ g, $\sigma = 1.5$ g

8 **a** 0.5 **b** 0.34 **c** 0.025

9 **a** **i** 0.516 (3 s.f.) **ii** 0.613 (3 s.f.) **iii** 0.968 (3 s.f.)

b Using the normal distribution the probabilities are 0.5, 0.68 and 0.95. The probabilities are close, so the model is suitable.

3.2 Finding probabilities for normal distributions

1 **a** 0.7734 **b** 0.8413 **c** 0.6368

2 **a** 0.0548 **b** 0.1587 **c** 0.7606

3 **a** 0.1587 **b** 0.5889 **c** 0.7632

4 **a** 0.7602 **b** 0.0786 **c** 0.1739

5 **a** **i** 0.2023 **ii** 0.7977

b $P(M > 11) + P(M < 11) = 1$ which is the total probability of all possible events, so $P(M = 11) = 0$.

6 **a** 0.2858 **b** 0.7142

7 **a** 0.4861 **b** 0.5640

8 **a** **i** 0.0548 **ii** 0.3446 **b** 0.1209

9 **a** **i** 0.2023 **ii** 0.1056 **b** 0.1150

10 **a** 0.1420 **b** 0.4020 **c** 0.0002

11 **a** 0.0487

b The height of an athlete is likely to have an effect on the athlete's weight, so the assumption of independence is not sensible.

3.3 The inverse normal distribution function

1 **a** 37.98 **b** 42.02 **c** 40.76

2 **a** 13.32 **b** 13.65 **c** 15.30 **d** 15.02

3 **a** **i** 23.00 **ii** 27.00 **b** 0.394

4 **a** **i** 76.0 (3 s.f.) **ii** 86.0 (3 s.f.) **b** 0.673

5 **a** E.g. $a = 94.9$, $b = 105$ (3 s.f.)

b For any value of a chosen, where $P(X < a) \leq 0.4$, there is a value of b such that $P(a < X < b) = 0.6$.

6 **a** 29.7 (3 s.f.) **b** 21.3 (3 s.f.) **c** 25

7 **a** 60.8 (3 s.f.) **b** 4.05 (3 s.f.)

8 **a** 185 (3 s.f.) **b** 16.8 (3 s.f.)

9 $k = 7$

3.4 The standard normal distribution

1 **a** 0.9699 **b** 0.1056 **c** 0.2206

d 0.4495 **e** 0.2123 **f** 0.7499

2 **a** 1.39 (3 s.f.) **b** 1.81 (3 s.f.) **c** −0.960 (3 s.f.)

d 0.440 (3 s.f.) **e** 2.75 (3 s.f.) **f** 0.842 (3 s.f.)

3 **a** −0.8 **b** −3.1 **c** −3.04 **d** 2.324

4 **a** $\Phi(1.2)$ **b** $1 - \Phi(1.6)$ **c** $\Phi(0.8) + \Phi(0.4) - 1$

5 **a** 1.645 (4 s.f.)

b 77 (nearest whole number)

6 **a** −0.8416 (3 s.f.) **b** £150

7 **a** −0.8416, 0.8416 (4 s.f.)

b 21.26 cm, 18.74 cm (4 s.f.)

3.5 Finding μ and σ

1 23.0 (3 s.f.)

2 2.50 (3 s.f.)

3 **a** 36.0 (3 s.f.) **b** 0.7674

4 **a** 25.0 (3 s.f.) **b** 0.0793

5 $\mu = 18.0$ (3 s.f.) $\sigma = 4.50$ (3 s.f.)

6 **a** $\mu = 7.00$ (3 s.f.), $\sigma = 2.20$ (3 s.f.)

b 0.5889

7 **a** The lengths tend to be symmetrical around a mean. The distribution of lengths will form a bell curve.

b Mean = 25.5 cm, standard deviation = 0.890 (3 s.f.)

c 0.2412

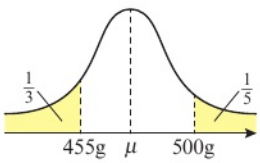
8 4.35 (3 s.f.)

9 $\sigma = 81.9$ (3 s.f.), $a = 22.2$ (3 s.f.)

10 **a** 42.0 (3 s.f.) **b** 0.1388 **c** 0.0023

11 **a** 0.5941 (4 s.f.) **b** 0.1337 **c** 0.0951

12 **a**



b $\mu = 470$ (3 s.f.), $\sigma = 35.4$ (3 s.f.) **c** 90.7 (3 s.f.)

3.6 Approximating a binomial distribution

1 **a** **i** Not suitable, p not close to 0.5

b **i** Not suitable, n not large

c **i** Suitable, n large and p close to 0.5

ii $N(62.4, 5.47^2)$ (3 s.f.)

d **i** Suitable, n large and p close to 0.5

ii $N(189, 10.2^2)$ (3 s.f.)

e **i** Suitable, n large and p close to 0.5

ii $N(324, 12.2^2)$

f **i** Not suitable, p not close to 0.5

2 **a** 0.4769 **b** 0.9815 **c** 0.0268

3 **a** 0.8854 **b** 0.9600 **c** 0.0413

4 0.6397

5 **a** 0.1502 **b** 0.0905 **c** 150

6 **a** 0.0770 **b** 0.3% (1 s.f.)

7 **a** The student did not apply the continuity correction $P(X \leqslant 110) \approx P(Y \leqslant 110.5)$.

b 0.0774 **c** 0.0990 **d** 0.4016

3.7 Hypothesis testing with the normal distribution

1 **a** The mean of the 50 distances in the sample (or $\bar{x}$, or the sample mean)

b $H_0: \mu = 200$, $H_1: \mu > 200$

c $\overline{X} \sim N\left(200, \frac{15^2}{50}\right)$

d $\overline{X} > 203.5$ (4 s.f.)

2 **a** Rejected if $P(\overline{X} \leqslant \bar{x}) < 0.1$.

b $P(\overline{X} < 16.6) = 0.0893$ (4 s.f.). Reject H_0, accept H_1.

3 **a** H_0: $\mu = 45$, H_1 : $\mu > 45$, $\overline{X} > 46.4$ hours

b $\bar{x} = 50 > 46.4$

There is evidence that the new battery is an improvement.

4 **a** A one-tailed test is used to test when it is claimed that the mean has increased or decreased. A two-tailed test is used when it is thought that the mean has changed in either direction.

b $\overline{X} < 33.4$ or $\overline{X} > 34.6$ (3 s.f.)

c 33.7 is not in the critical region, so accept H_0.

5 $H_0 : \mu = 200$, $H_1 : \mu \neq 200$.
Critical region is $\overline{X} < 197.8$, $\overline{X} > 202.2$ (4 s.f.). Accept H_0.

6 $H_0 : \mu = 5.9$, $H_1 : \mu > 5.9$. Critical region is $\overline{X} > 6.389$ (4 s.f.). Reject H_0; there is evidence for the zoologist's claim.

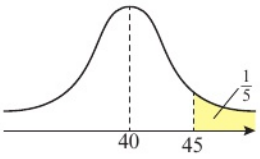
7 **a** H_0: $\mu = 12$, $H_1 : \mu < 12$

b 10.3 minutes (3 s.f.)

c $12.3 > 12$ so $P(\overline{X} < \bar{x})$ will be greater than 50%. There is insufficient evidence to reject H_0, so conclude the waiting times have not reduced.

Problem solving: Set A

B **a**



b $\sigma = 5.94$ **c** 0.3682

S **a** $\mu = 19.9$ (3 s.f.), $\sigma = 0.507$ (3 s.f.)

b 50.7% (3 s.f.)

G **a** $\mu = 104$ cm (3 s.f.), $\sigma = 28.2$ cm (3 s.f.)

b 14 prizes

c 200 cm is more than 3 standard deviations from the mean, so according to the model the probability of getting a value > 200 cm is very small (less than 0.001) so the model is still suitable.

Problem solving: Set B

B **a** **i** 0.4249 **ii** 0.0825 **iii** 0.9871

b 100% is 5 standard deviations from the mean so virtually impossible to occur as an outcome, so the interviewer's claim is not valid.

S **a** **i** 0.2173 **ii** 0.1370

b If $X > 45$ then X must be greater than 43, so for both to be true we only need $X > 45$.

c 0.6305

G **a** 0.1673 **b** 0.0316 **c** 0.3129

CHAPTER 4

4.1 Moments

1 **a** 42 N m in the anticlockwise direction

b 2.4 N m in the anticlockwise direction

c 48 N m in the clockwise direction

2 **a** 7.05 N m (3 s.f.) in the clockwise direction

b 77.3 N m (3 s.f.) in the anticlockwise direction

c 4.2 N m in the clockwise direction

3 0.3 m

4 **a** 80 N cm in the clockwise direction

b 80 N cm in the anticlockwise direction

5 **a** 1 N m in the clockwise direction

b 2 N m in the anticlockwise direction

6 30°

7 0.4 m

4.2 Resultant moments

1 5 N m in the anticlockwise direction

2 34 N m in the anticlockwise direction

3 0.161 N m (3 s.f.) in the anticlockwise direction

4 1 m

5 $\frac{19}{6}$ N

6 $\frac{23}{6}$ N

7 **a** 312 kN m (3 s.f.) anticlockwise

b Assumptions include the barge is modelled as a rectangular lamina; direction in which the forces act are constant; forces are constant; resistance due to air/water is ignored.

8 Taking moments about P, the clockwise moment is

$4F\cos 45° - 5000\cos 45°$

In order for the door to move, the clockwise moment must be greater than 0, so

$4F\cos 45° - 5000\cos 45° > 0$

So $F > 1250$ N as required

4.3 Equilibrium

1 **a** 50 N **b** 100 N

2 $R_1 = \frac{50}{3}$ N, $R_2 = \frac{250}{3}$ N

3 60°

4 $T_A = \frac{1}{3}mg$ N and $T_C = \frac{2}{3}mg$ N

5 2.75 m

6 Taking moments about D:

$m_A g \times 0.6l + Mg \times 0.1l = m_R g \times 0.2l$

$0.6m_A + 0.1M = 0.2m_R$

$6m_A + M = 2m_R$

$M = 2(m_R - 3m_A)$

7 $10g\sqrt{2}$ N

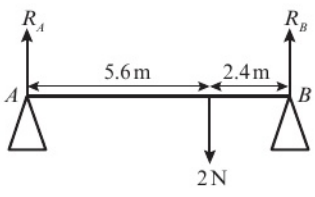
8 **a** **i** $\frac{10}{3}g$ N **ii** $\frac{20}{3}g$ N

b $\frac{5}{3}g(x + 4)$N **c** $0 < x \leqslant 2$ m

4.4 Centres of mass

1 0.9 m

2 **a**



b $R_A = 0.6$ N, $R_B = 1.4$ N

3 $7m$

4 14.7 N (3 s.f.)

5 1.5 m

6 2.11 m (3 s.f.)

7 **a** **i** $\frac{10}{3}g$ N **ii** $\frac{5}{3}g$ N

b Mass of the oar is unlikely to pass through the midpoint of the oar.

c 0.6 m

4.5 Tilting

1 1

2 9

3 **a** $\frac{40g}{7}$ N

b 0.5

4 80 kg

5 $\frac{8}{9}$

6 1200

7 **a** Take moments about D:

$2mg \times ED = mg \times 2d$

Simplify to $ED = d$.

b $1.75mg$

8 **a** 0.6 kg **b** $0 < x \leqslant 0.8$

Problem solving: Set A

B 3.25

S **a** $T_A = 1000$ N, $T_B = 500$ N

b $\frac{14}{9}$ m

G $0.6x$ m

Problem solving: Set B

B $1 \leqslant x \leqslant 13.6$

S 250

G Take moments about D: $400(3 - 2x) = Wx$

$$1200 - 800x = Wx$$

$$(W + 800)x = 1200$$

$$x = \frac{1200}{W + 800}$$

CHAPTER 5

5.1 Resolving forces

1 **a** **i** 2 m s^{-2} **ii** 133 N (3 s.f.)

b **i** 2.46 m s^{-2} **ii** 40.4 N (3 s.f.)

c **i** 0.421 m s^{-2} **ii** 65.8 N (3 s.f.)

2 **a** 21.7 N (3 s.f.) at an angle of 18.0° (3 s.f.) to the left of the **+j** vector

b 5.57 N (3 s.f.) at an angle of 28.9° (3 s.f.) below the **+i** vector

c 3.80 N (3 s.f.) at an angle of 26.8° (3 s.f.) above the **−i** vector

3 **a** **i** 44.0° (3 s.f.) **ii** 41.2 N (3 s.f.)

b **i** 82.9° (3 s.f.) **ii** 7.50 N (3 s.f.)

c **i** 6.95° (3 s.f.) **ii** 21.7 N (3 s.f.)

4 **a** 2.57 m s^{-2} (3 s.f.) **b** 129 N (3 s.f.)

5 **a** $\frac{10\sqrt{3}}{3}$

b No resistance due to surface friction.

6 8

7 **a** 30°

b For the initial acceleration of the box:

R(→): $6\cos 30° = 3a$ so $a = \sqrt{3}$ m s^{-2}

For the acceleration of the box after P has been applied:

R(→): $P\cos 60° + 6\cos 30° = 3 \times 2\sqrt{3}$

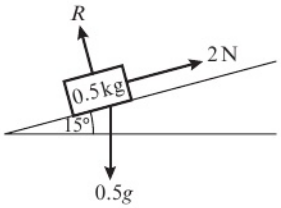
So $P = 6\sqrt{3}$ N

8 **a** 41.4° (3 s.f.) **b** 6.75 (3 s.f.)

5.2 Inclined planes

1 **a** $\frac{5\sqrt{3}}{2}g$ N **b** $\frac{1}{2}g$ m s^{-2}

2 **a**



b 1.46 m s^{-2} (3 s.f.)

3 **a** 3.35 N (3 s.f.) **b** 18.9 m s^{-1} (3 s.f.)

4 R(↙): $mg\sin\theta = ma$ so $a = g\sin\theta$

5 **a** 15.9 m s^{-1} (3 s.f.) **b** 4.23 seconds (3 s.f.)

6 6

7 $0.3g$ m s^{-2} down the slope.

8 6.85 seconds

5.3 Friction

1 **a** **i** $0.4g$ N, 2.04 m s^{-2} **ii** $0.7g$ N, 0.57 m s^{-2}

b The frictional force will only be as large as it needs to be to oppose the 8 N force, so it will have a magnitude of 8 N.

2 **a** 0.102 (3 s.f.) **b** 0.412 (3 s.f.) **c** 0.255 (3 s.f.)

3 **a** 0.673 (3 s.f.) **b** 2.06 (3 s.f.) **c** 2.12 (3 s.f.)

4 1.75 m s^{-2} (3 s.f.)

5 **a** 80.4 m s^{-1} (3 s.f.) **b** 20.5 seconds (3 s.f.)

6 R(↖): $R = mg\cos\theta$

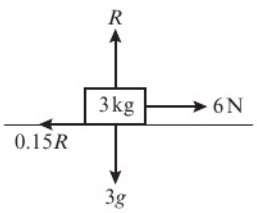
R(↙): $mg\sin\theta - \mu mg\cos\theta = ma$

So $a = g(\sin\theta - \mu\cos\theta)$

7 **a** 2300 (3 s.f.) **b** 580 (3 s.f.)

Problem solving: Set A

B **a**



b 9.54 m

S 11.3 m s^{-1}

G R(↑): $R + 2g\sin\theta = mg$

So $R = mg - 2g\sin\theta$

R(→): $2g\cos\theta - \mu(mg - 2g\sin\theta) = ma$

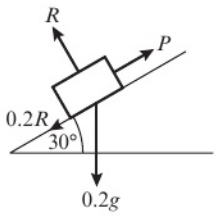
$$a = \frac{2g\cos\theta + 2g\mu\sin\theta}{m} - \mu g$$

Using $v = u + at$ and $u = 0$ gives

$$v = \left(\frac{2\cos\theta + 2\mu\sin\theta}{m} - \mu\right)gt$$

Problem solving: Set B

B **a**



b $\frac{49\sqrt{3}}{250}$ N **c** 1.36 (3 s.f.)

S **a** R(↗): $F > mg\sin\alpha + \mu R$

$F > \frac{3}{5}mg + 0.1 \times \frac{4}{5}mg$

$F > \frac{17}{25}mg$

b 6.44 (3 s.f.)

G $\alpha = 13.7°$ or $30.7°$

CHAPTER 6

6.1 Horizontal projection

1 **a** $u_y = 0$, $a_y = 9.8\,\text{m s}^{-2}$, $s_y = 50\,\text{m}$

Substitute into $s = ut + \frac{1}{2}at^2$ to give $50 = \frac{9.8}{2}t^2$

Rearrange to give $t = \frac{10\sqrt{5}}{7}$ seconds

b $\frac{150\sqrt{5}}{7}$ m

2 **a** $30t$ m **b** $4.9t^2$ m

3 **a** 2.5 seconds **b** $\frac{245}{8}$

4 **a** 14.0 (3 s.f.)

b Air resistance negligible

5 **a** 40 m **b** 0.4 m

c Q will be closer to the horizontal surface.

6 $\frac{14 - \sqrt{10}}{10}$ m

7 **a** 2.08 m s^{-1} **b** $\frac{441}{640}$ m

c The table height would be less than previously calculated.

6.2 Horizontal and vertical components

1 **a** $5\sqrt{3}$ m s^{-1} **b** 5 m s^{-1}

2 $(10\sqrt{2}\,\mathbf{i} + 10\sqrt{2}\,\mathbf{j})$ m s^{-1}

3 13 m s^{-1} at an angle of 22.6° (3 s.f.) above the horizontal

4 **a** 13.6 m s^{-1} (3 s.f.) and 6.34 m s^{-1} (3 s.f.)

b $(13.6\mathbf{i} + 6.34\mathbf{j})$ m s^{-1}

5 **a** 7.88 m s^{-1} (3 s.f.) and −1.39 m s^{-1} (3 s.f.)

b $(7.88\mathbf{i} - 1.39\mathbf{j})$ m s^{-1}

6 **a** 60 m s^{-1} and 144 m s^{-1}

b $(60\mathbf{i} + 144\mathbf{j})$ m s^{-1}

7 5 m s^{-1} at an angle of 36.9° (3 s.f.) above the horizontal

6.3 Projection at any angle

1 10 seconds

2 **a** 2.45 seconds (3 s.f.) **b** 12.2 m (3 s.f.)

3 34.6 m (3 s.f.)

4 88.1 (3 s.f.)

5 188 m (3 s.f.)

6 1.10 seconds (3 s.f.)

7 **a** **i** 42.3° (3 s.f.) **ii** 46.5 (3 s.f.)

b 38.7 m s^{-1} (3 s.f.)

8 44.5 m (3 s.f.)

9 57.6 m (3 s.f.)

6.4 Projectile motion formulae

1 At maximum height, $v_y = 0$.

Substitute $u_y = 20\sin\alpha$, $a = -g$, $v_y = 0$ into

$v^2 = u^2 + 2as$ to give $0 = (20\sin\alpha)^2 - 2gh$

Rearrange to give $h = \frac{200\sin^2\alpha}{g}$

2 **a** $R = \frac{U^2\sin 2\alpha}{g}$ **b** 45°

3 **a** Substitute $u_x = U\cos\alpha$, $a_x = 0$, $s_x = R$ into

$s_x = u_x t + \frac{1}{2}a_x t^2$ to give $t = \frac{R}{U\cos\alpha}$

This gives the time when the particle lands, so the time when the particle is at maximum height is $\frac{R}{2U\cos\alpha}$

Substitute $u_y = U\sin\alpha$, $a_y = -g$, $t = \frac{R}{2U\cos\alpha}$ into

$s_y = u_y t + \frac{1}{2}a_y t^2$ to give $h = \frac{R\tan\alpha}{2} - \frac{gR^2}{8U^2\cos^2\alpha}$

b Use $\frac{1}{\cos^2\alpha} \equiv \sec^2\alpha \equiv 1 + \tan^2\alpha$ to give

$h = \frac{R\tan\alpha}{2} - \frac{gR^2}{8U^2}(1 + \tan^2\alpha)$

4 $\frac{1}{g}\left|u_A^2\sin 2\alpha_A - u_B^2\sin 2\alpha_B\right|$

5 $d = 3u\sqrt{\frac{2h}{g}}$

6 **a** Let d be the range of the projectile and t_1 be the time at which it lands.

From horizontal motion: $t_1 = \frac{d}{a}$ (1)

For the vertical motion at time of greatest height:

$u = b$, $v = 0$, $t = \frac{t_1}{2}$, $a = -g$

Substitute into $v = u + at$ to give $0 = b - \frac{gt_1}{2}$ (2)

Substitute (1) into (2) to give $0 = b - \frac{gd}{2a}$

Rearrange to give $d = \frac{2ab}{g}$

b 52.5 m s^{-1} (3 s.f.)

7 Use $u_y = U\sin\alpha$, $a = -g$, $s_y = 0$ to find $t = \frac{2U\sin\alpha}{g}$. Then use $u_x = U\cos\alpha$, $a = 0$, $t = \frac{2U\sin\alpha}{g}$ to find $s = \frac{2U^2\sin\alpha\cos\alpha}{g}$

Finally, using $\sin 2\alpha = 2\sin\alpha\cos\alpha$ gives $s = \frac{U^2\sin 2\alpha}{g}$

8 **a** When projectile is at the top of the wall, from horizontal motion: $t = \frac{40}{U\cos\alpha}$ (1)

For the vertical motion: $10 = Ut\sin\alpha - \frac{g}{2}t^2$ (2)

Substitute (1) into (2) to give

$$10 = U\sin\alpha\left(\frac{40}{U\cos\alpha}\right) - \frac{g}{2}\left(\frac{40}{U\cos\alpha}\right)^2$$

$$\Rightarrow \frac{800g}{U^2}\left(\frac{1}{\cos\alpha}\right)^2 - 40\tan\alpha + 10 = 0$$

$$\Rightarrow \frac{800g}{U^2}(1 + \tan^2\alpha) - 40\tan\alpha + 10 = 0$$

b 21.4°, 82.7° (3 s.f.)

Problem solving: Set A

B **a** R(↑): $s = ut + \frac{1}{2}at^2$

$0 = (2U\sin\alpha)t - \frac{1}{2}gt^2$

$0 = t(2U\sin\alpha - \frac{1}{2}gt)$

$t = 0$ (point of projection) or $2U\sin\alpha = \frac{1}{2}gt$

$\Rightarrow t = \frac{4U\sin\alpha}{g}$

b R(→): $s = ut$

$= 2U\cos\alpha\left(\frac{4U\sin\alpha}{g}\right)$

$= \frac{8U^2\sin\alpha\cos\alpha}{g} = \frac{4U^2\sin 2\alpha}{g}$

S **a** R(→): $u_x = U\cos\alpha = \frac{4}{5}U$

R(↑): $u_h = U\sin\alpha = \frac{3}{5}U$

R(→): $s = ut \Rightarrow x = \frac{4}{5}Ut$ (1)

R(↑): $s = ut + \frac{1}{2}at^2 \Rightarrow h = \frac{3}{5}Ut - \frac{1}{2}gt^2$ (2)

Rearrange (1) and substitute into (2)

$h = \frac{3}{5}U\left(\frac{5x}{4U}\right) - \frac{1}{2}g\left(\frac{5x}{4U}\right)^2$

$h = \frac{3}{4}x - \frac{25}{32}\left(\frac{gx^2}{U^2}\right)$

b $7\sqrt{17}\,\text{m s}^{-1}$

G **a** 40

b The arrows will reach the point of intersection at different times.

c $(80 - 4\sqrt{400 - 10h})$ m

Problem solving: Set B

B **a** $21.1\,\text{m s}^{-1}$ **b** 2.24 seconds (3 s.f.)

S **a** 39.4 m above ground **b** 57.7° (3 s.f.)

G **a** $56\,\text{m s}^{-1}$ **b** 152 m (3 s.f.)

c 25.6 m above ground

d Using $v^2 = u^2 + 2as$

$(-40)^2 = (-6)^2 - 20s \Rightarrow s = 78.2 > 75$ so the particle hits the ground before achieving a velocity of $(40\mathbf{i} - 40\mathbf{j})\,\text{m s}^{-1}$.

CHAPTER 7

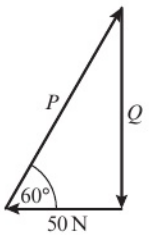
7.1 Static particles

1 **a** $P\cos 60° + Q\cos 45° = 40$

b $P\sin 60° = Q\sin 45°$

c $P = 29.3$ N (3 s.f.), $Q = 35.9$ N (3 s.f.)

2 **a**



b $P = 100$ N, $Q = 50\sqrt{3}$ N

3 $P = 5$ N, $Q = 5\sqrt{3}$ N

4 $P = 10$ N, $Q = 10\sqrt{3}$ N

5 $P = 22.2$ N (3 s.f.), $Q = 44.1$ N (3 s.f.)

6 **a** $\alpha = 41.8°$ (3 s.f.) **b** 22.4 N (3 s.f.)

7.2 Modelling with statics

1 $2\sqrt{3}$ N

2 **a** $15\sqrt{2}$ N **b** $15(1 + \sqrt{2})$ N

c Tension is the same on both sides of the bead.

3 **a** $\frac{60g}{13}$ N **b** $\frac{25g}{13}$

4 $T_P = 14.3$ N (3 s.f.), $T_Q = 10.1$ N (3 s.f.)

5 **a** 11.4 N (3 s.f.) **b** 0.143 kg (3 s.f.)

c The tension in the string would be greater at the top of the string as it would have to support the additional weight of the string below it.

6 **a** 45.6° (3 s.f.)

b The angle would be smaller.

7 R(↗): $2g - mg\sin\alpha = 0$, rearrange to find $\alpha = \arcsin\left(\frac{2}{m}\right)$

8 R(↗): $4P\cos 30° - P\cos 60° - 20\sin 30° = 0$ so

$P = \frac{20\sin 30°}{4\cos 30° - \cos 60°} = 3.3737\ldots$

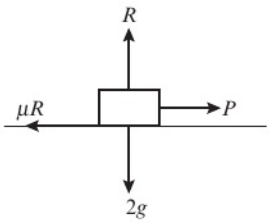
R(↖): $R + 4P\sin 30° + P\sin 60° - 20\cos 30° = 0$ so

$R = 20\cos 30° - P(4\sin 30° + \sin 60°)$

Substituting gives $R = 7.65$ N as required.

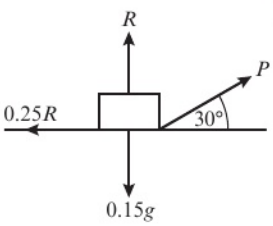
7.3 Friction and static particles

1 **a**



b 3.92 N

2 **a**



b 0.371 N (3 s.f.)

3 **a** 37.9 N (3 s.f.) **b** 0.268 (3 s.f.)

4 **a** 23.3 N (3 s.f.) **b** 14.6 N (3 s.f.)

5 0.603 or 3.32 (3 s.f.)

6 R(↖): $R = mg\cos\alpha$, R(↗): $\mu R = g + mg\sin\alpha$

Substitute and rearrange to find μ.

7 **a** 702 (3 s.f.)

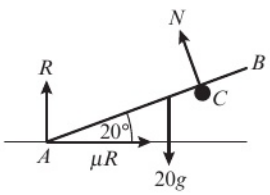
b Valid because air resistance not a factor in static situation; rotational forces unlikely to affect situation.

8 $0.116 \leqslant F \leqslant 0.549$ (3 s.f.)

7.4 Static rigid bodies

1 **a** $N = F$ **b** $R = 25g$ N **c** 70.7 N (3 s.f.)

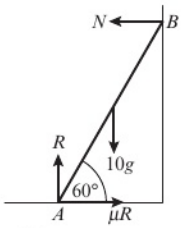
2 **a**



b 132 N (3 s.f.) **c** 0.622 (3 s.f.)

d Reaction at the peg acts perpendicular to the rod.

3 **a**



b $\frac{5\sqrt{3}g}{3}$ N **c** $\frac{\sqrt{3}}{6}$

4 **a** 63.4° (3 s.f.)

b Assumption means no friction at points of contact with the ladder. Wall and floor are unlikely to be smooth in real life.

c α would be smaller.

5 **a** 282 N (3 s.f.) **b** 64.4° (3 s.f.)

6 **a** 107 N (3 s.f.) **b** 165 N (3 s.f.)

7 **a** 539 N

b 483 N (3 s.f.) at an angle of 14.7° (3 s.f.) above the horizontal.

8 **a** Take moments about A to find $T = \frac{2Mg}{3\sin\alpha}$

b R($\rightarrow$):

$R_x = T\cos\alpha$

$= \frac{2Mg}{3\sin\alpha}\cos\alpha$

$= \frac{2Mg}{3\tan\alpha}$

R($\uparrow$):

$R_y + T\sin\alpha = Mg$

$R_y + \frac{2Mg}{3\sin\alpha}\sin\alpha = Mg$

$R_y + \frac{2}{3}Mg = Mg$

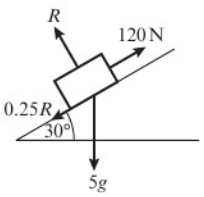
$R_y = \frac{1}{3}Mg$

c 38.7°

d The angle would be larger (because $\tan\alpha = 2\mu$).

7.5 Dynamics and inclined planes

1 **a**



b 42.4 N **c** 17.0 m s^{-2} (3 s.f.)

2 **a** 1.96 N up the slope

b R($\swarrow$) to find the resultant force acting on the block, which is 6.53 N (3 s.f.) down the slope, therefore the block will begin to slide.

c 6.53 m s^{-2} (3 s.f.)

3 1.94 (3 s.f.)

4 1.66 seconds (3 s.f.)

5 **a** 6.42 m s^{-2} (3 s.f.) **b** 3.83 seconds (3 s.f.)

c R($\swarrow$) to find the resultant force acting on the particle to be 13.0 N (3 s.f.) (acting down the slope), therefore the particle will begin to slide.

6 0.589 (3 s.f.)

7 0.0149 (3 s.f.)

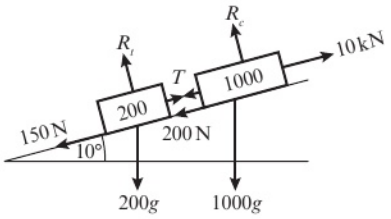
8 5.03 seconds

9 When the rope snaps the sled has velocity 29.2 m s^{-1} (3 s.f.) and acceleration of −7.45 m s^{-2} (3 s.f.) up the slope.

Use $v^2 = u^2 + 2as$ to find that the sled will travel 57.4 m (3 s.f.) before coming to rest. Therefore the sled will not reach B.

7.6 Connected particles

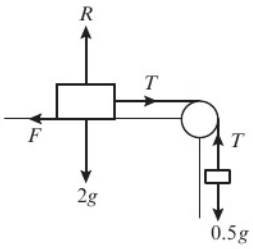
1 **a**



b **i** 6.34 m s^{-2} (3 s.f.) **ii** 1760 N (3 s.f.)

c Acceleration of car and trailer are the same.

2 **a**



b $\frac{1}{25}g$ m s^{-2} **c** $\frac{12}{25}g$ N

d Tension in string is the same on both sides of the pulley.

3 **a** $1.8g$ N **b** $0.1g$ m s^{-2} **c** 2.47 seconds

4 **a** 1800 N (3 s.f.) **b** 2210 N (3 s.f.)

c 2.09 seconds (3 s.f.)

5 **a** $\mu < \frac{1}{3}$ **b** 6.38 seconds

c For example, box A does not reach ground; box B does not reach pulley; boxes can be modelled as particles.

6 **a** $2mg\sin 30° = mg$

$mg\sin 45° = \frac{1}{\sqrt{2}}mg < mg$

So block B slides up the slope.

b Resolve parallel to the plane for each block to find

$T = mg - 2ma$ and $T = ma + \frac{\sqrt{2}}{2}mg$

Then solve to find $a = \frac{2-\sqrt{2}}{6}g$

Use $s = ut + \frac{1}{2}at^2$ with $u = 0$, $s = d$, $a = \frac{2-\sqrt{2}}{6}g$ to find t.

Problem solving: Set A

B **a** 19.9 N (3 s.f.) **b** 1.71 seconds (3 s.f.)

c It would take longer to come to rest.

S **a** 11.3° (3 s.f.) **b** 5.20 seconds (3 s.f.)

c The size of α would be greater, as a greater component of the weight of the particle would need to act down the slope to maintain limiting equilibrium.

G **a** $\alpha > 8.53°$ (3 s.f.) **b** 8.08 seconds (3 s.f.)

c The component of the weight that acts down the slope and the frictional force both have a factor of m, so the conditions for limiting equilibrium and the acceleration of the particle would be unchanged.

Problem solving: Set B

B **a** 19.5 N **b** 2.27 (3 s.f.)

c Tension equal on both sides of the pulley.

S **a** 8.51° (3 s.f.) or 20.6° (3 s.f.)

b 6.82 N (3 s.f.)

c The force exerted on the pulley would be greater.

G **a** $\mu < \frac{7}{48}$

b $a = \frac{11}{325}g$

c **i** a is independent of m, so the acceleration would be unchanged.

ii Tension initially is $T = mg + ma$; when m is doubled $T = 2mg + 2ma = 2(mg + ma)$, so the tension in the string would double.

CHAPTER 8

8.1 Vectors in kinematics

1 $(9\mathbf{i} - 13\mathbf{j})$ m

2 **a** $(-2\mathbf{i} + 3\mathbf{j})$ m **b** 3 seconds

3 **a** $(80\mathbf{i} + 80\mathbf{j})$ m **b** 113 m (3 s.f.)

4 $\sqrt{145}$ m s^{-1} on a bearing of 085.2° (3 s.f.)

5 $(1.5\mathbf{i} - \mathbf{j})$ m s^{-2}

6 **a** $a = -11$, $b = -18$ **b** 79.5 m (3 s.f.)

7 **a** $\mathbf{r} = \begin{pmatrix} 1540 - 220t \\ 2500 - 400t \end{pmatrix}$ m

b 2020 m (3 s.f.) **c** 7 seconds

8 **a** $\sqrt{29}$ m s^{-1} **b** $\begin{pmatrix} 2.4 \\ 2 \end{pmatrix}$ m s^{-2} **c** $\begin{pmatrix} 54.2 \\ 14 \end{pmatrix}$ m

9 **a** Equate $\mathbf{r} = \mathbf{r}_0 + t\mathbf{v}$ for A and B to find the walkers have the same position vector at $t = 4$, so they meet.

b $\begin{pmatrix} 10 \\ 2 \end{pmatrix}$ km

10 **a** $6\mathbf{i} - 8\mathbf{j}$ **b** 6 seconds

8.2 Vector methods with projectiles

1 **a** $\begin{pmatrix} 15 \\ 5.4 \end{pmatrix}$ m s^{-1} **b** 15.9 m s^{-1} (3 s.f.)

2 **a** $\begin{pmatrix} 12t \\ 19t - 4.9t^2 \end{pmatrix}$ m **b** $\begin{pmatrix} 6 + 12t \\ 6 + 19t - 4.9t^2 \end{pmatrix}$ m

3 **a** $\begin{pmatrix} 23t \\ 15 + 13t - 4.9t^2 \end{pmatrix}$ m

b **i** 3.52 seconds (3 s.f.) **ii** 81.0 m (3 s.f.)

4 2.5 m

5 **a** $a = 24$, $b = 24.5$ **b** 30.6 m (3 s.f.)

6 **a** 13.9 seconds (3 s.f.) **b** 464 m (3 s.f.)

7 **a** 30.3 m (3 s.f.) **b** 13.3 m s^{-1} (3 s.f.)

8 $p = 12.25$, $q = 29.4$

8.3 Variable acceleration in one dimension

1 **a** $2(\pi + 2)$ m s^{-1} **b** 2 m s^{-2}

2 **a** $\left(e^t + \frac{1}{3}t^3 - 1\right)$ m s^{-1}

b $\left(e^3 + \frac{11}{4}\right)$ m

3 **a** $2(1 + 4\ln 4)$ m s^{-1} **b** 2 m s^{-2}

4 $\left(-2\ln t + \frac{\cos(2\pi t)}{4\pi^2} + 2t - \frac{1}{4\pi^2} - 2\right)$ m

5 **a** $-\frac{1}{2}$ **b** $\left(-\frac{1}{2}t\sin t - \frac{1}{2}\cos t + \frac{1}{2}\right)$ m

c 1 m

6 **a** 2 **b** $-\frac{8}{e}$

7 **a** $v = \frac{t}{\sqrt{t^2 + 15}}$ m s^{-1}

b $a = \frac{15}{(t^2 + 15)^{\frac{3}{2}}}$ which is always positive, so v is increasing for all values of t. Max speed $= \frac{7}{8}$ m s^{-1}

8.4 Differentiating vectors

1 **a** $\left(54\mathbf{i} - \frac{5}{6}\mathbf{j}\right)$ m s^{-1}

b 6.5 m s^{-1} at an angle of 22.6° (3 s.f.) below the positive $\mathbf{i}$-direction.

2 **a** $-5\mathbf{j}$ m s^{-2}

b 101° (3 s.f.) below the positive $\mathbf{i}$-direction.

3 **a** $\sqrt{85}$ m **b** $(2\mathbf{i} + 24\mathbf{j})$ m s^{-2}

4 $9e^6\mathbf{i}$ m s^{-2}

5 **a** 0.5 seconds **b** 4.22 m s^{-1}

6 **a** 4 m s^{-1} **b** 1 second **c** $(-\mathbf{i} + 3\mathbf{j})$ m

7 **a** 13.6 m s^{-1} (3 s.f.)

b $((48t^2 + 48t + 24)e^{2t(t+1)}\mathbf{i} - e^t\mathbf{j})$ m s^{-2}

8 **a** $\mathbf{r} = (5t^2 + 5t)\mathbf{i} + (3 - t^2)\mathbf{j}$

$\frac{d\mathbf{r}}{dt} = (10t + 5)\mathbf{i} - 2t\mathbf{j}$

$\frac{d^2\mathbf{r}}{dt^2} = 10\mathbf{i} - 2\mathbf{j}$

So P moves with constant acceleration.

b $10\sqrt{26}$

8.5 Integrating vectors

1 **a** $\left(\left(\frac{5}{2}t^2 - 2\right)\mathbf{i} + 2(t - 1)\mathbf{j}\right)$ m s^{-1}

b 20.9 m s^{-1}

2 $\left((3t - 1)\mathbf{i} + \left(3 - \frac{2}{3}t^3\right)\mathbf{j}\right)$ m

3 **a** $\left(\left(\frac{1}{2}t^2 - t^3 + t + 4\right)\mathbf{i} + \left(\frac{3}{2}t^2 + 2t + 1\right)\mathbf{j}\right)$ m

b 25.7 m

4 **a** $\frac{4}{\pi}\mathbf{j}$ m s^{-1} **b** $\frac{8}{\pi}\mathbf{j}$ m

5 **a** $\left(\left(\frac{3}{2}t^2 + 2\right)\mathbf{i} - 2t\mathbf{j}\right)$ m s^{-1}

b 11.7 m s^{-1} (3 s.f.)

c 14.7° (3 s.f.) below the positive $\mathbf{i}$-direction.

6 **a** $(t(2t + 1)\mathbf{i} + (1 - 2kt)\mathbf{j})$ m

b $\frac{5}{4}$

c $(10\mathbf{i} - 4\mathbf{j})$ m

7 $\mathbf{a} = (3t - 1)\mathbf{i} + e^t\mathbf{j}$

$\mathbf{v} = \left(\frac{3}{2}t^2 - t\right)\mathbf{i} + e^t\mathbf{j} + \mathbf{c}$

At $t = 0$, $\mathbf{v} = 7\mathbf{i} - 4\mathbf{j}$

$\mathbf{v} = \left(\frac{3}{2}t^2 - t + 7\right)\mathbf{i} + (e^t - 5)\mathbf{j}$

In order for $\mathbf{v}$ to be parallel to $\mathbf{j}$, $\frac{3}{2}t^2 - t + 7 = 0$ must have a real solution. Since $b^2 - 4ac < 0$ there are no real values of t that satisfy $\frac{3}{2}t^2 - t + 7 = 0$.

Problem solving: Set A

B **a** $(15\mathbf{i} - 13\mathbf{j})$ m s^{-1} **b** 6 m

S **a** 2.5 seconds **b** 115 m (3 s.f.)

G **a** 4 seconds **b** $40\sqrt{2}$ m

Problem solving: Set B

B **a** $(8t\mathbf{i} + (9t^2 - 3)\mathbf{j})\,\text{m s}^{-1}$

b $325\,\text{m s}^{-1}$ **c** $\dfrac{4+\sqrt{43}}{9}$ seconds

S **a** 30.9 m (3 s.f.) **b** $(4 - 2\sqrt{3})$ seconds

G 0.447 seconds (3 s.f.)

Exam Question Bank

Section A: Statistics

1 **a**

	Reads music	Can't read music	Total
Female	10	8	18
Male	6	9	15
Total	16	17	33

b **i** $\frac{16}{33}$ **ii** $\frac{4}{9}$ **iii** $\frac{3}{8}$

2 **a** $a = 1.21$ (3 s.f.), $k = 0.749$

b 50 grams is outside the range of the data (extrapolation).

3 **a** $\overline{X} \sim \text{N}\left(\mu, \frac{\sigma^2}{n}\right)$ **b** 40

4 **a** 0.731 (3 s.f.)

b H_0: $\rho = 0$, H_1: $\rho > 0$, critical value = 0.7293 < 0.731. Reject H_0: there is evidence of positive correlation at the 5% level of significance, so claim is correct.

5 $\dfrac{3-\sqrt{5}}{2}$

6 **a** p is close to 0.5; n is large

b Incorrect calculation of standard deviation, $\sigma = \sqrt{62.4}$

c 0.8854

7 **a** Jacksonville or Beijing **b** 0.133 (3 s.f.)

c The data shows a very weak positive correlation so a linear model may not be best. There may be other variables affecting the relationship or a different model might be a better fit.

8 **a** $\frac{17}{36}$ **b** $X \sim \text{B}\left(180, \frac{17}{36}\right)$

c n is large and p (0.4722) is close to 0.5, $X \sim \text{N}(85, 6.7^2)$.

d 0.0451 (3 s.f.)

9 **a** 0.12 **b** 0.7 **c** 0.4

d $\text{P}(A) = \text{P}(A \mid B)$, therefore independent.

10 **a** 0.496

b H_0: $\rho = 0$, H_1: $\rho > 0$, critical value = 0.5067. Accept H_0: there is no evidence of linear correlation at the 10% level of significance.

c 0.937

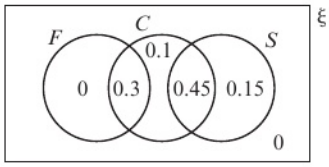
d H_0: $\rho = 0$, H_1: $\rho > 0$, critical value = 0.5067. Reject H_0: there is evidence of linear correlation at the 10% level of significance.

e Weaker positive correlation in part **b** due to different efficiencies of the different generators.

11 **a** 0.1587 **b** 44.65 kg (4 s.f.)

c 0.197 **d** 0.3173

12 **a**



b **i** 0.9 **ii** 0.75

c No, $\text{P}(F' \cap S) = 0.6 \neq \text{P}(F') \times \text{P}(S)$
$= 0.7 \times 0.6 = 0.42$

13 **a** $\mu = 330$ ml (3 s.f.), $\sigma = 13.0$ (3 s.f.)

b 0.430 (3 s.f.) **c** 0.248 (3 s.f.)

14 **a** $k = 0.387$, $b = 1.02$ (3 s.f.)

b $r = 0.799$ (3 s.f.)

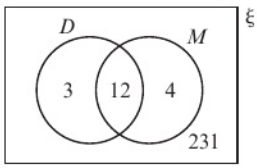
c 1% (critical value = 0.7887)

15 **a** **i** $\mu = 15.8$, $\sigma = 4.60$ (3 s.f.) **ii** 0.483 (3 s.f.)

b 0.227 (3 s.f.)

c 45 mins is more than 6 standard deviations away from the mean, so virtually impossible according to the normal distribution model. The model is unlikely to be suitable.

16 **a**



b 0.8 **c** 0.25

d 20% of the defective components are not identified by the machine and 25% of the components identified as defective are not so test is not very effective.

17 **a** Using data for engine size,
$Q_3 + 1.5\,(Q_3 - Q_1) = 3.5 + 1.5 = 5 < 6$

b It follows the trend of the data, or it is a reasonable engine capacity for a large car.

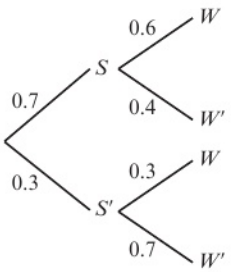
c −0.945

d Cars with larger engines are larger and therefore less fuel efficient.

e Strong negative correlation, therefore a linear model is likely to be suitable.

f Negative correlation, so expect gradient to be negative.

18 **a**



b 0.51 **c** $\frac{14}{17}$ **d** $\frac{3}{7}$

e Assuming that each serve is independent and the probability of success is constant.
H_0: $p = 0.7$, H_1: $p \neq 0.7$. $\text{P}(X \leqslant 62) = 0.053 > 0.025$.
There is insufficient evidence to reject H_0. There is no evidence that proportion of successful first serves has changed.

19 **a** 0.2660 **b** 55.4 (3 s.f.)
c 44.6 (3 s.f.) **d** $h = 28.4$, $k = 71.6$ (3 s.f.)
e 0.0070 (4 d.p.)
f H_0: $\mu = 50$, H_1: $\mu > 50$
Critical region is $\overline{X} > 53.79...$ Reject H_0. There is evidence that the reading scores have improved.
g Increase could have been due to other factors, e.g. second time taking test increases familiarity.

20 **a** 0.15 **b** $q = 0.35$, $r = 0.1$ **c** 0.75

Section B: Mechanics

21 7.13 N (3 s.f.)

22 6.91 kg

23 Equating **i**-direction position:
$12 - t = 2 + 2t \Rightarrow t = \frac{10}{3}$
Equation **j**-direction position:
$-6 + 4t = 16 - 2t \Rightarrow t = \frac{11}{3}$
So the hikers will not meet.

24 60°

25 **a** 205 (3 s.f.)
b P would be 0 as there would be no frictional force acting on the particle.

26 **a** 21.8° (3 s.f.) **b** 16.2 N (3 s.f.) **c** 1.93 s (3 s.f.)

27 **a** $\mathbf{v} = ((9kt^2 + 3)\mathbf{i} + 10t\mathbf{j})\,\text{m s}^{-1}$
b At $t = 2$ s, $\mathbf{v} = (36k + 3)\mathbf{i} + 20\mathbf{j}$ then use $|\mathbf{v}| = 5\sqrt{241}$ to find k.

28 7 m

29 **a** 29.4 **b** 0.482 (3 s.f.)

30 **a** $\left(\frac{8 - 5e^{-\frac{2t}{5}}}{3}\right)\text{m s}^{-1}$ **b** 2.16 m s^{-1} (3 s.f.)

31 Take moments about C to find $T = \frac{1}{3}Mg$.
At point of slipping, $T = \mu R = \mu mg$.
Equate values of T to find $M \leqslant 3\mu m$.

32 **a** 9.98 N (3 s.f.) **b** 22.6° (3 s.f.)

33 $\mathbf{u} = \left(-\frac{17}{3}\mathbf{i} + \frac{1}{6}\mathbf{j}\right)\text{m s}^{-1}$

34 **a** 18.0 N (3 s.f.) **b** 0.450 kg (3 s.f.)
c The tension in the string is the same on both sides of the bead.
d The tension in the string would be greater at the top as it would have to support the mass of the string below it.

35 **a** Use $s = ut + \frac{1}{2}at^2$ horizontally to find $R = Ut$.
Use $s = ut + \frac{1}{2}at^2$ vertically to find $t = \sqrt{\frac{2d}{g}}$
Substitute to find $R = U\sqrt{\frac{2d}{g}}$
b 19.8 cm (3 s.f.)

36 261 N (3 s.f.)

37 **a** $7.5g$ N **b** 1.75 m

38 **a** 17.6 m (3 s.f.) **b** 2.68 m (3 s.f.)
c Fielder catches the ball nearer the ground, or the ball might not reach the fielder before hitting the ground.

39 **a** $\frac{810}{49}$ m **b** $\frac{4140}{49}$ m **c** 3.01 s (3 s.f.)

40 0.180 (3 s.f.)

41 **a** $1830 \leqslant M \leqslant 4580$ kg (3 s.f.)
b The beam is unlikely to be uniform, or the beam does not rest freely on the tower.

42 **a** 16.3° **b** 2.86 seconds (3 s.f.)
c 32.0° (3 s.f.)

43 **a** $\frac{10}{9}\text{m s}^{-2}$ **b** 0.534 (3 s.f.) **c** 19.0 N (3 s.f.)
d Both particles have the same acceleration.

44 **a** 7.64 m s^{-1} (3 s.f.) **b** 0.602 seconds (3 s.f.)

45 **a** 69.5°
b The ball is unlikely to land on the hole exactly. Valid reasons include the model does not consider air resistance and/or rotational motion of the ball, the golfer is unlikely to hit the ball at the exact angle or speed, and the measurements may not be exact.

46 **a** 21.2 seconds (3 s.f.)
b P does not reach the pulley, Q does not reach the floor.
c 8.70 m (3 s.f.)
d R(↙): Force $= 3mg \sin 30° = 1.5mg$
$F_{max} = \mu R = 0.1 \times 3mg \cos 30° = \frac{3\sqrt{3}}{20}mg < 1.5mg$
So P slides back down the slope.

47 **a** 0.877 (3 s.f.) **b** 2.68 m (3 s.f.)

48 **a** $6.33 < x < 29.0$ (3 s.f.) **b** 21.0 m s^{-1} (3 s.f.)

49 $0.603l$ (3 s.f.)

50 **a** Let distance CE be x
Taking moments about D:
$(3d - x)mg = 2dmg$
$\Rightarrow 3d - x = 2d$
$\Rightarrow x = d$ and so $AE = 2d$
b mg N

51 **a** 5.71 (3 s.f.) **b** 19.6 m (3 s.f.)

52 **a** $AD = \sin\alpha$
Moments about A: $T \sin\alpha \cos\alpha = 20$
So $T = \frac{20}{\frac{1}{2}\sin 2\alpha} = \frac{40}{\sin 2\alpha}$
b 15° and 75°
c 60.9 N at an angle of 70.1° to the horizontal below the beam.
d The angle would be closer to the horizontal (or above the beam).