Complex numbers 1A

1 a $\sqrt{9}\sqrt{(-1)} = 3\text{i}$

b $\sqrt{49}\sqrt{(-1)} = 7\text{i}$

c $\sqrt{121}\sqrt{(-1)} = 11\text{i}$

d $\sqrt{1000}\sqrt{(-1)} = 100\text{i}$

e $\sqrt{225}\sqrt{(-1)} = 15\text{i}$

f $\sqrt{5}\sqrt{(-1)} = \text{i}\sqrt{5}$

g $\sqrt{12}\sqrt{(-1)} = \sqrt{4}\sqrt{3}\sqrt{(-1)} = 2\text{i}\sqrt{3}$

h $\sqrt{45}\sqrt{(-1)} = \sqrt{9}\sqrt{5}\sqrt{(-1)} = 3\text{i}\sqrt{5}$

i $\sqrt{200}\sqrt{(-1)} = \sqrt{100}\sqrt{2}\sqrt{(-1)} = 10\text{i}\sqrt{2}$

j $\sqrt{147}\sqrt{(-1)} = \sqrt{49}\sqrt{3}\sqrt{(-1)} = 7\text{i}\sqrt{3}$

2 a $(5+8)+\text{i}(2+9) = 13+11\text{i}$

b $(4+1)+\text{i}(10-8) = 5+2\text{i}$

c $(7-3)+\text{i}(6-5) = 4+\text{i}$

d $\left(\frac{1}{2}+\frac{5}{2}\right)+\text{i}\left(\frac{1}{3}+\frac{5}{3}\right) = 3+2\text{i}$

e $(20-11)+\text{i}(12-3) = 9+9\text{i}$

f $(2--5)+\text{i}(-1-3) = 7-4\text{i}$

g $(-4--8)+\text{i}(-6--8) = 4+2\text{i}$

h $(3\sqrt{2}-\sqrt{2})+\text{i}(1--1) = 2\sqrt{2}+2\text{i}$

i $(-2+1--12)+\text{i}(-7+3-1) = 11-5\text{i}$

j $(18-15-3)+\text{i}(5--2-7) = 0$

3 a $14+4\text{i}$

b $24-12\text{i}$

c $(6+2\text{i})+(6+3\text{i}) = (6+6)+\text{i}(2+3)$
$= 12+5\text{i}$

d $(20+15\text{i})+(4-8\text{i}) = (20+4)+\text{i}(15-8)$
$= 24+7\text{i}$

e $\frac{6-4\text{i}}{2} = \frac{6}{2}-\frac{4}{2}\text{i}$
$= 3-2\text{i}$

f $\frac{15+25\text{i}}{5} = \frac{15}{5}-\frac{25}{5}\text{i}$
$= 3+5\text{i}$

g $\frac{9+11\text{i}}{3} = \frac{9}{3}+\frac{11}{3}\text{i}$
$= 3+\frac{11}{3}\text{i}$

h $\frac{-8+3\text{i}}{4}-\frac{7-2\text{i}}{2} = -2+\frac{3}{4}\text{i}-\frac{7}{2}+\text{i}$
$= \left(-\frac{4}{2}-\frac{7}{2}\right)+\text{i}\left(\frac{3}{4}+\frac{4}{4}\right)$
$= -\frac{11}{2}+\frac{7}{4}\text{i}$

4 a $\frac{4-2\text{i}}{\sqrt{2}} = \frac{4-2\text{i}}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$
$= \frac{4\sqrt{2}-2\sqrt{2}\text{i}}{2}$
$= 2\sqrt{2}-\text{i}\sqrt{2}$

b $\frac{2-6\text{i}}{1+\sqrt{3}} = \frac{2-6\text{i}}{1+\sqrt{3}}\times\frac{\left(1-\sqrt{3}\right)}{\left(1-\sqrt{3}\right)}$
$= \frac{2-2\sqrt{3}-6\text{i}+6\sqrt{3}\text{i}}{1-3}$
$= \frac{2-2\sqrt{3}}{-2}+\frac{6\sqrt{3}-6}{-2}\text{i}$
$= \left(-1+\sqrt{3}\right)+\left(3-3\sqrt{3}\right)\text{i}$

5 a $z-w=(7-6\mathrm{i})-(7+6\mathrm{i})$
$=(7-7)+\mathrm{i}(-6-6)$
$=-12\mathrm{i}$

b $w+z=(7+6\mathrm{i})+(7-6\mathrm{i})$
$=(7+7)+\mathrm{i}(6-6)$
$=14$

6 $7+2\mathrm{i}=z_2-z_1$
$=(-3+b\mathrm{i})-(a+9\mathrm{i})$
$=(-3-a)+(b-9)\mathrm{i}$
Equate real parts:
$7=-3-a\Rightarrow a=-10$
Equate imaginary parts:
$2=b-9\Rightarrow b=11$

7 a $z_1-z_2=(4+\mathrm{i})-(7-3\mathrm{i})$
$=(4-7)+\mathrm{i}(1--3)$
$=-3+4\mathrm{i}$

b $4z_2=4(7-3\mathrm{i})$
$=28-12\mathrm{i}$

c $2z_1+5z_2=2(4+\mathrm{i})+5(7-3\mathrm{i})$
$=8+2\mathrm{i}+35-15\mathrm{i}$
$=43-13\mathrm{i}$

8 a $z+w=(a+b\mathrm{i})+a-b\mathrm{i}$
$=2a$
So $z+w$ is always real.

b $z-w=(a+b\mathrm{i})-(a-b\mathrm{i})$
$=2b\mathrm{i}$
So $z-w$ is always imaginary.