

Argand diagrams 2C

$$1 \text{ a} \quad r = \sqrt{(2^2 + 2^2)} = \sqrt{8} = 2\sqrt{2}$$

$$\tan \alpha = \frac{2}{2} = 1. \quad \alpha = \frac{\pi}{4}$$

$$\theta = \frac{\pi}{4}$$

$$2 + 2i = 2\sqrt{2} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)$$

$$b \quad r = \sqrt{(0^2 + 3^2)} = \sqrt{9} = 3$$

$$\tan \alpha = \infty \quad \alpha = \frac{\pi}{2}$$

$$\theta = \frac{\pi}{2}$$

$$3i = 3 \left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right)$$

$$c \quad r = \sqrt{((-3)^2 + 4^2)} = \sqrt{25} = 5$$

$$\tan \alpha = \frac{4}{-3}. \quad \alpha = 0.927 \text{ rad.}$$

$$\theta = \pi - \alpha = 2.21$$

$$-3 + 4i = 5(\cos 2.21 + i \sin 2.21)$$

$$d \quad r = \sqrt{(1^2 + (-\sqrt{3})^2)} = \sqrt{4} = 2$$

$$\tan \alpha = \frac{\sqrt{3}}{1}. \quad \alpha = \frac{\pi}{3}$$

$$\theta = -\frac{\pi}{3}$$

$$1 - \sqrt{3}i = 2 \left(\cos \left(\frac{-\pi}{3} \right) + i \sin \left(\frac{-\pi}{3} \right) \right)$$

$$e \quad r = \sqrt{((-2)^2 + (-5)^2)} = \sqrt{29}$$

$$\tan \alpha = \frac{5}{-2}. \quad \alpha = 1.190 \text{ rad}$$

$$\theta = -(\pi - \alpha) = -1.95$$

$$-2 - 5i = \sqrt{29}(\cos(-1.95) + i \sin(-1.95)).$$

$$f \quad r = \sqrt{((-20)^2 + 0^2)} = \sqrt{400} = 20$$

$$\tan \alpha = 0$$

$$\theta = \pi$$

$$-20 = 20(\cos \pi + i \sin \pi)$$

$$g \quad r = \sqrt{(7^2 + (-24)^2)} = \sqrt{625} = 25$$

$$\tan \alpha = \frac{24}{7}. \quad \alpha = 1.287 \text{ rad}$$

$$\theta = -1.29$$

$$7 - 24i = 25(\cos(-1.29) + i \sin(-1.29))$$

$$h \quad r = \sqrt{((-5)^2 + 5^2)} = \sqrt{50} = 5\sqrt{2}$$

$$\tan \alpha = \frac{5}{-5} = -1. \quad \alpha = \frac{3\pi}{4}$$

$$\theta = \pi - \alpha = \frac{3\pi}{4}$$

$$-5 + 5i = 5\sqrt{2} \left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right).$$

$$2 \text{ a} \quad \frac{3}{1+i\sqrt{3}} = \frac{3(1-i\sqrt{3})}{(1+i\sqrt{3})(1-i\sqrt{3})}$$

$$= \frac{3-3i\sqrt{3}}{1(1-i\sqrt{3})+i\sqrt{3}(1-i\sqrt{3})}$$

$$= \frac{3-3i\sqrt{3}}{1-i\sqrt{3}+i\sqrt{3}-3i^2}$$

$$= \frac{3-3i\sqrt{3}}{4}$$

$$= \frac{3}{4} - \frac{3\sqrt{3}}{4}i$$

$$r = \sqrt{\left(\frac{3}{4}\right)^2 + \left(-\frac{3\sqrt{3}}{4}\right)^2} = \sqrt{\frac{9}{16} + \frac{27}{16}}$$

$$= \sqrt{\frac{36}{16}} = \frac{3}{2}$$

$$\tan \alpha = \frac{3\sqrt{3}}{4} \div \frac{3}{4} = \sqrt{3} \quad \alpha = \frac{\pi}{3}$$

$$\theta = -\frac{\pi}{3}$$

$$\frac{3}{1+i\sqrt{3}} = \frac{3}{2} \left(\cos \left(\frac{-\pi}{3} \right) + i \sin \left(\frac{-\pi}{3} \right) \right)$$

$$\begin{aligned}
 2 \quad b \quad \frac{1}{2-i} &= \frac{2+i}{(2-i)(2+i)} \\
 &= \frac{2+i}{2(2+i)-i(2+i)} \\
 &= \frac{2+i}{4+2i-2i-i^2} \\
 &= \frac{2+i}{5} = \frac{2}{5} + \frac{1}{5}i \\
 r &= \sqrt{\left(\frac{2}{5}\right)^2 + \left(\frac{1}{5}\right)^2} = \sqrt{\frac{4}{25} + \frac{1}{25}} \\
 &= \sqrt{\frac{5}{25}} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5} \\
 \tan \alpha &= \frac{1}{5} \div \frac{2}{5} = \frac{1}{2}. \quad \alpha = 0.4636 \text{ rad.} \\
 \theta &= 0.46 \\
 \frac{1}{2-i} &= \frac{\sqrt{5}}{5} (\cos 0.46 + i \sin 0.46)
 \end{aligned}$$

$$\begin{aligned}
 c \quad \frac{1+i}{1-i} &= \frac{(1+i)(1+i)}{(1-i)(1+i)} \\
 &= \frac{1(1+i) + i(1+i)}{1(1+i) - i(1+i)} = \frac{1+i+i+i^2}{1+i-i-i^2} \\
 &= \frac{2i}{2} = i \\
 r &= \sqrt{(0^2 + 1^2)} = 1 \\
 \tan \alpha &= \infty, \quad \alpha = \frac{\pi}{2} \\
 \theta &= \frac{\pi}{2} \\
 \frac{1+i}{1-i} &= 1 \left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right)
 \end{aligned}$$

$$\begin{aligned}
 3 \quad a \quad 5 \left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right) \\
 &= 5(0+i) \\
 &= 5i
 \end{aligned}$$

$$\begin{aligned}
 b \quad \frac{1}{2} \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right) \\
 &= \frac{1}{2} \left(\frac{\sqrt{3}}{2} + \frac{1}{2}i \right) \\
 &= \frac{\sqrt{3}}{4} + \frac{1}{4}i
 \end{aligned}$$

$$\begin{aligned}
 c \quad 6 \left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \right) \\
 &= 6 \left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i \right) \\
 &= -3\sqrt{3} + 3i
 \end{aligned}$$

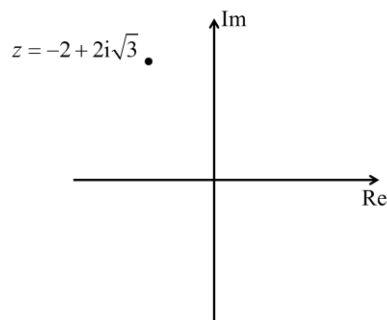
$$\begin{aligned}
 d \quad 3 \left(\cos \left(-\frac{2\pi}{3} \right) + i \sin \left(-\frac{2\pi}{3} \right) \right) \\
 &= 3 \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i \right) \\
 &= -\frac{3}{2} - \frac{3\sqrt{3}}{2}i
 \end{aligned}$$

$$\begin{aligned}
 e \quad 2\sqrt{2} \left(\cos \left(-\frac{\pi}{4} \right) + i \sin \left(-\frac{\pi}{4} \right) \right) \\
 &= 2\sqrt{2} \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i \right) \\
 &= 2 - 2i
 \end{aligned}$$

$$\begin{aligned}
 f \quad -4 \left(\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6} \right) \\
 &= -4 \left(-\frac{\sqrt{3}}{2} - \frac{1}{2}i \right) \\
 &= 2\sqrt{3} + 2i
 \end{aligned}$$

$$\begin{aligned}
 4 \quad a \quad z &= 4 \left(\cos \left(\frac{2\pi}{3} \right) + i \sin \left(\frac{2\pi}{3} \right) \right) \\
 &= 4 \left(-\frac{1}{2} + i \frac{\sqrt{3}}{2} \right) \\
 &= -2 + 2i\sqrt{3}
 \end{aligned}$$

4 b



$$5 \quad |z| = 7 \text{ and } \arg z = \frac{11\pi}{6}$$

$$\begin{aligned} z &= 7 \left(\cos \left(\frac{11\pi}{6} \right) + i \sin \left(\frac{11\pi}{6} \right) \right) \\ &= 7 \cos \left(\frac{11\pi}{6} \right) + 7i \sin \left(\frac{11\pi}{6} \right) \\ &= \frac{7\sqrt{3}}{2} - \frac{7}{2}i \end{aligned}$$

$$\text{So } p = \frac{7\sqrt{3}}{2} \text{ and } q = -\frac{7}{2}$$

$$6 \quad |z| = 5 \text{ and } \arg z = -\frac{4\pi}{3}$$

$$\begin{aligned} z &= 5 \left(\cos \left(-\frac{4\pi}{3} \right) + i \sin \left(-\frac{4\pi}{3} \right) \right) \\ &= 5 \cos \left(-\frac{4\pi}{3} \right) + 5i \sin \left(-\frac{4\pi}{3} \right) \\ &= -\frac{5}{2} + \frac{5i\sqrt{3}}{2} \end{aligned}$$

$$\text{So } a = -\frac{5}{2} \text{ and } b = \frac{5\sqrt{3}}{2}.$$