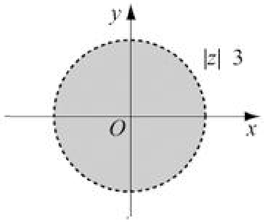
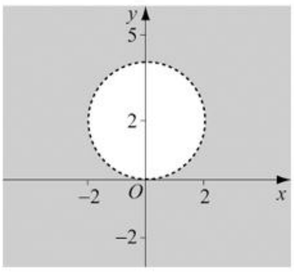
Argand diagrams 2F

1 a

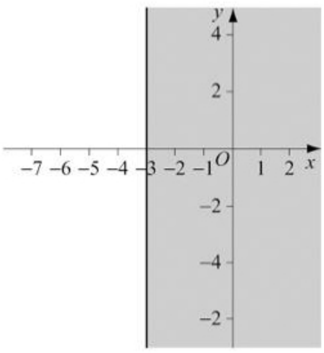


$|z| = 3$ represent a circle centre $(0, 0)$, radius 3

b $|z-(2\text{i})| > 2$

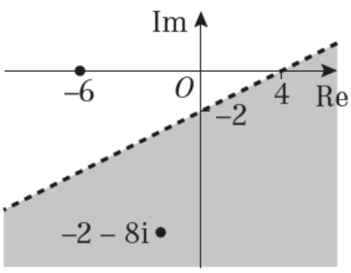


c $|z+7| \geq |z-1|$



$|z+7| = |z-1|$ represents the perpendicular bisector of the line joining $(-7, 0)$ to $(1, 0)$ which has equation $x = -3$.

d $|z+6| > |z+2+8\text{i}|$



$|z+6| = |z+2+8\text{i}|$ represents a perpendicular bisector of the line joining $(-6, 0)$ to $(-2, -8)$.

$|x+\text{i}y+6| = |x+\text{i}y+2+8\text{i}|$

$\Rightarrow |x+6+\text{i}y| = |(x+2)+\text{i}(y+8)|$

$\Rightarrow |(x+6)+\text{i}y|^2 = |(x+2)+\text{i}(y+8)|^2$

$\Rightarrow (x+6)^2 + y^2 = (x+2)^2 + (y+8)^2$

$\Rightarrow x^2 + 12x + 36 + y^2 = x^2 + 4x + 4 + y^2 + 16y + 64$

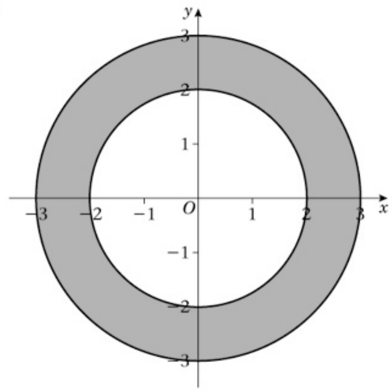
$\Rightarrow 12x + 36 = 4x + 16y + 68$

$\Rightarrow 8x + 36 - 68 = 16y$

$\Rightarrow 8x - 32 = 16y$

$\Rightarrow y = \dfrac{1}{2}x - 2$

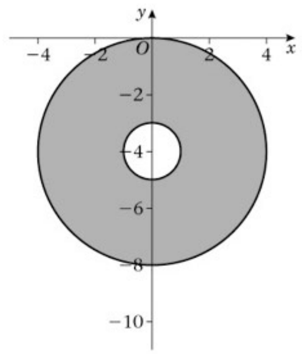
1 e $2 \leqslant |z| \leqslant 3$



$|z| = 2$ represents a circle centre (0, 0), radius 2

$|z| = 3$ represents a circle centre (0, 0), radius 3

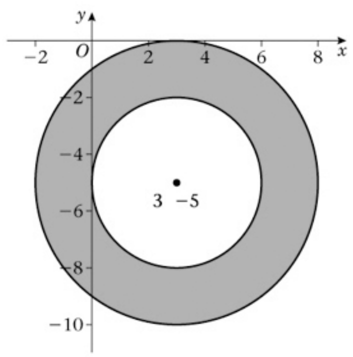
f $1 \leqslant |z+4\text{i}| \leqslant 4$



$|z+4\text{i}| = 1$ represents a circle centre (0, 4), radius 1.

$|z+4\text{i}| = 4$ represents a circle centre (0, –4), radius 4.

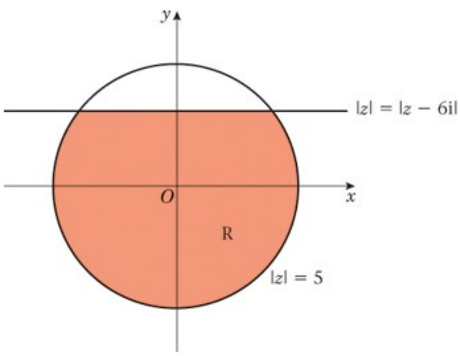
g $3 \leqslant |z-3+5\text{i}| \leqslant 5$



$|z-(3-5\text{i})| = 3$ represents a circle centre (3, –5), radius 3.

$|z-(3-5\text{i})| = 3$ represents a circle centre (3, –5), radius 5.

2 $|z| \leqslant 5$ and $|z| \leqslant |z-6\text{i}|$



$|z| = 5$ represents a circle centre (0, 0), radius 5

$|z| = |z-6\text{i}|$ represents a perpendicular bisector of the line joining (0, 0), to (0, 6) and has the equation $y = 3$.

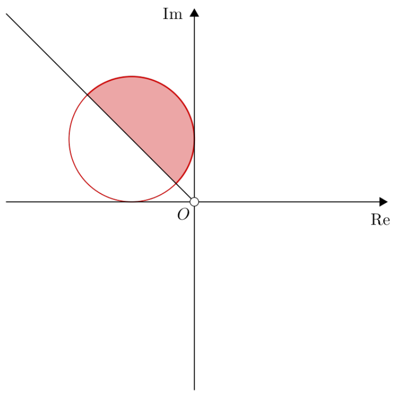
3 $|z+1-\mathrm{i}|\leqslant 1$

$|z-(-1+\mathrm{i})|\leqslant 1$

This represents the region on the boundary and inside the circle with centre $(-1,1)$ and radius 1.

$0\leqslant \arg z\leqslant \frac{3\pi}{4}$ This represents the half-line from $(0,0)$ with angle θ, where $0\leqslant\theta\leqslant\frac{3\pi}{4}$.

The shaded area shown is where these two regions overlap.



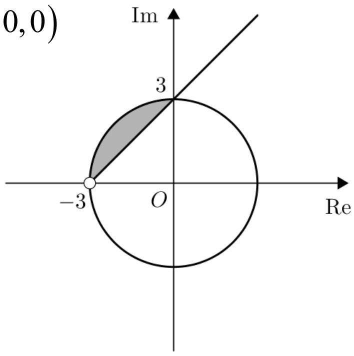
4 $z\in\mathbb{C}:|z|\leqslant 3$

This represents the region on the boundary and inside the circle with centre $(0,0)$ and radius 3.

$z\in\mathbb{C}:\frac{\pi}{4}\leqslant\arg(z+3)\leqslant\pi$

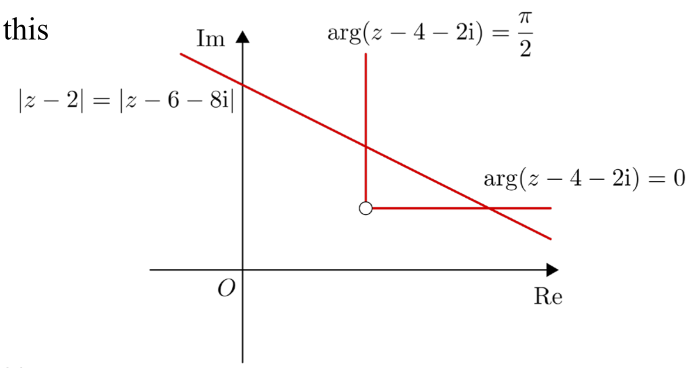
This represents the half-line from $(-3,0)$ with angle θ, where $\frac{\pi}{4}\leqslant\theta\leqslant\pi$.

The shaded area shown is where these two regions overlap.



5 a i $|z-2|=|z-6-8\mathrm{i}|$ represents the perpendicular bisector of the line joining (2, 0) to (6, 8).

Write $z=x+\mathrm{i}y$ to find a Cartesian form for this perpendicular bisector.



$|x+\mathrm{i}y-2|=|x+\mathrm{i}y-6-8\mathrm{i}|$

$\Rightarrow|(x-2)+\mathrm{i}y|=|(x-6)+\mathrm{i}(y-8)|$

$\Rightarrow|(x-2)+\mathrm{i}y|^2=|(x-6)+\mathrm{i}(y-8)|^2$

$\Rightarrow(x-2)^2+y^2=(x-6)^2+(y-8)^2$

$\Rightarrow x^2-4x+4+y^2=x^2-12x+36+y^2-16y+64$

$\Rightarrow -4x+4=-12x-16y+100$

$\Rightarrow 8x+16y-96=0\quad(\div 8)$

$\Rightarrow x+2y-12=0$

$\Rightarrow 2y=-x+12$

$\Rightarrow y=-\frac{1}{2}x+6$

ii $\arg(z-4-2\mathrm{i})=0$

$\arg(z-(4+2\mathrm{i}))=0$

This is a half line from (4, 2) making an angle of 0 with the positive Re axis.

5 a iii $\arg(z-4-2\mathrm{i})=\dfrac{\pi}{2}$

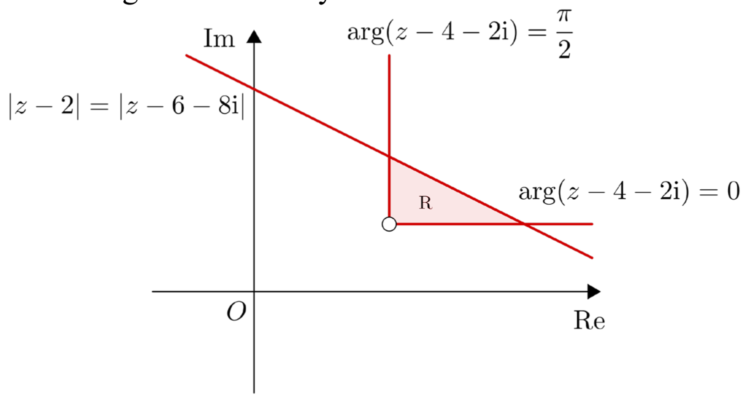
$\arg(z-(4+2\mathrm{i}))=\dfrac{\pi}{2}$

This is a half line from (4, 2) making an angle of $\dfrac{\pi}{2}$ with the positive Re axis.

b $\{z\in\mathbb{C}:|z-2|\leqslant|z-6-8\mathrm{i}|\}$ represents the region $y\leqslant-\dfrac{1}{2}x+6$

$\left\{z\in\mathbb{C}:0\leqslant\arg(z-4-2\mathrm{i})\leqslant\dfrac{\pi}{2}\right\}$ represented by the region in between and including the two half-lines $\arg(z-(4+2\mathrm{i}))=0$ and $\arg(z-(4+2\mathrm{i}))=\dfrac{\pi}{2}$

The region satisfied by both of these is shaded.



6 a i $|x+\mathrm{i}y+10|=|x+\mathrm{i}y-6-4\sqrt{2}\mathrm{i}|$

so $(x+10)^2+y^2=(x-6)^2+(y-4\sqrt{2})^2$

$\cancel{x^2}+20x+100+\cancel{y^2}=\cancel{x^2}-12x+36+\cancel{y^2}-8\sqrt{2}y+32$

$32x=-8\sqrt{2}\,y-32$

$8\sqrt{2}y+32(x+1)=0$

$y+2\sqrt{2}(x+1)=0$

$y=-2\sqrt{2}(x+1)$

ii $(x+1)^2+y^2=9$

$(x^2+2x+y^2=8)$

b Substitute $y=-2\sqrt{2}(x+1)$ into $(x+1)^2+y^2=9$

$(x+1)^2+8(x+1)^2=9$

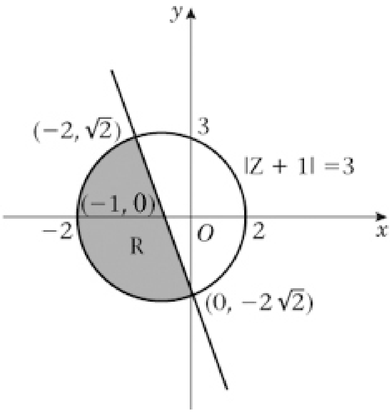
$9(x+1)^2=9$

$x+1=\pm1$

$x=0,-2$ $\quad$ $(0,\ -2\sqrt{2})$ and $(-2,\ 2\sqrt{2})$

$z=-2\sqrt{2}\mathrm{i}$ and $z=-2+2\sqrt{2}\mathrm{i}$

6 c



The region $|z+10| \leqslant |z-6-4\mathrm{i}\sqrt{2}|$ is given by $y \leqslant -2\sqrt{2}(x+1)$

$|z+1| \leqslant \sqrt{3}$ is the area inside the circle $(x+1)^2 + y^2 = 9$

The shaded area satisfies both of these inequalities.

Challenge

$A = \{z \in \mathbb{C} : |z + 5 + 8\mathrm{i}| \leqslant 5\}$

$A = \{z \in \mathbb{C} : |z - (-5 - 8\mathrm{i})| \leqslant 5\}$

This represents the region on the boundary and inside the circle with centre $(-5, -8)$ and radius 5.

$B = \{z \in \mathbb{C} : |z + 8 + 4\mathrm{i}| \leqslant |z + 2 + 12\mathrm{i}|\}$

$B = \{z \in \mathbb{C} : |z - (-8 - 4\mathrm{i})| \leqslant |z - (-2 - 12\mathrm{i})|\}$

This represents the region of all points closer to $(-8, -4)$ than $(-2, -12)$.

To find this region, first find the perpendicular bisector of $(-8, -4)$ than $(-2, -12)$.

The gradient of the line joining $(-8, -4)$ than $(-2, -12)$ is $-\frac{8}{6} = -\frac{4}{3}$

The gradient of the perpendicular bisector is $\frac{3}{4}$

The midpoint of $(-8, -4)$ than $(-2, -12)$ is $(-5, -8)$

The equation of the perpendicular bisector is found by using $y - y_1 = m(x - x_1)$ with $m = \frac{3}{4}$ and $(x_1, y_1) = (-5, -8)$

$y + 8 = \frac{3}{4}(x + 5)$

$y + 8 = \frac{3}{4}x + \frac{15}{4}$

$y = \frac{3}{4}x - \frac{17}{4}$

As $(-8, -4)$ is above this line, the shaded region will also be above the line.

$C = \left\{z \in \mathbb{C} : 0 \leqslant \arg(z + 10 + 8\mathrm{i}) \leqslant \frac{\pi}{4}\right\}$

$C = \left\{z \in \mathbb{C} : 0 \leqslant \arg(z - (-10 - 8\mathrm{i})) \leqslant \frac{\pi}{4}\right\}$

The region is the half-line from $(-10, -8)$ which forms an angle θ with the positive real axis, where $0 \leqslant \theta \leqslant \frac{\pi}{4}$

Remember, we require the region C', so shade the area outside of C.

The diagram shows the correct shaded region together with labels of each locus.