Roots of polynomials Mixed exercise 4

1 $ax^4+bx^3+cx^2+dx+e=0$

Roots are $\alpha=\frac{1}{5},\beta=-\frac{2}{5},\gamma=-\frac{3}{5}$ and $\delta=-\frac{1}{2}$.

$\alpha+\beta+\gamma+\delta=\frac{1}{5}-\frac{2}{5}-\frac{3}{5}-\frac{1}{2}=-\frac{13}{10}$

$\alpha\beta+\alpha\gamma+\alpha\delta+\beta\gamma+\beta\delta+\gamma\delta$

$=\left(\frac{1}{5}\right)\left(-\frac{2}{5}\right)+\left(\frac{1}{5}\right)\left(-\frac{3}{5}\right)+\left(\frac{1}{5}\right)\left(-\frac{1}{2}\right)$

$+\left(-\frac{2}{5}\right)\left(-\frac{3}{5}\right)+\left(-\frac{2}{5}\right)\left(-\frac{1}{2}\right)+\left(-\frac{3}{5}\right)\left(-\frac{1}{2}\right)$

$=-\frac{2}{25}-\frac{3}{25}-\frac{1}{10}+\frac{6}{25}+\frac{2}{10}+\frac{3}{10}$

$=\frac{11}{25}$

$\alpha\beta\gamma+\alpha\beta\delta+\alpha\gamma\delta+\beta\gamma\delta$

$=\left(\frac{1}{5}\right)\left(-\frac{2}{5}\right)\left(-\frac{3}{5}\right)+\left(\frac{1}{5}\right)\left(-\frac{2}{5}\right)\left(-\frac{1}{2}\right)$

$+\left(\frac{1}{5}\right)\left(-\frac{3}{5}\right)\left(-\frac{1}{2}\right)+\left(-\frac{2}{5}\right)\left(-\frac{3}{5}\right)\left(-\frac{1}{2}\right)$

$=\frac{6}{125}+\frac{2}{50}+\frac{3}{50}-\frac{6}{50}$

$=\frac{7}{250}$

$\alpha\beta\gamma\delta=\left(\frac{1}{5}\right)\left(-\frac{2}{5}\right)\left(-\frac{3}{5}\right)\left(-\frac{1}{2}\right)=-\frac{6}{250}$

So $-\frac{13}{10}=-\frac{b}{a}\Rightarrow\frac{b}{a}=\frac{13}{10}$

$\frac{11}{25}=\frac{c}{a}\Rightarrow\frac{c}{a}=\frac{11}{25}$

$\frac{7}{250}=-\frac{d}{a}\Rightarrow\frac{d}{a}=-\frac{7}{250}$

$-\frac{6}{250}=\frac{e}{a}\Rightarrow\frac{e}{a}=-\frac{6}{250}$

$ax^4+bx^3+cx^2+dx+e=0$

$\Rightarrow x^4+\frac{b}{a}x^3+\frac{c}{a}x^2+\frac{d}{a}x+\frac{e}{a}=0$

So the equation is:

$x^4+\frac{13}{10}x^3+\frac{11}{25}x^2-\frac{7}{250}x-\frac{6}{250}=0$

Multiply by 250 for integer coefficients:

$250x^4+325x^3+110x^2-7x-6=0$

2 a $x^3+px^2+37x-52=0$

So $a=1,b=p,c=37$ and $d=-52$.

$\alpha\beta+\beta\gamma+\gamma\alpha=\frac{c}{a}=37$

$\alpha\beta\gamma=-\frac{d}{a}=52$

$\alpha+\beta+\gamma=-\frac{b}{a}=-p$, so

$p=-(\alpha+\beta+\gamma)$

b If $\alpha=3-2\text{i}$, then $\beta=3+2\text{i}$

Substitute $\alpha=3-2\text{i}$ and $\beta=3+2\text{i}$ into

$\alpha\beta\gamma=52$

$(3-2\text{i})(3+2\text{i})\gamma=52$

$(9+6\text{i}-6\text{i}-4\text{i}^2)\gamma=52$

$13\gamma=52$

$\gamma=4$

So the roots are $3-2\text{i},3+2\text{i}$ and 4.

$p=-(\alpha+\beta+\gamma)$

$=-((3-2\text{i})+(3+2\text{i})+4)=-10$

3 a $2x^3+5x^2-2x+q=0$

So $a=2,b=5,c=-2$ and $d=q$.

If $x=-2+\text{i}$ is a root then $x=-2-\text{i}$ is also a root.

$\alpha+\beta+\gamma=-\frac{b}{a}=-\frac{5}{2}$

$\alpha\beta+\beta\gamma+\gamma\alpha=\frac{c}{a}=-\frac{2}{2}=-1$

$\alpha\beta\gamma=-\frac{d}{a}=-\frac{q}{2}$

Substitute $\alpha=-2+\text{i}$ and $\beta=-2-\text{i}$ into

$\alpha+\beta+\gamma=-\frac{5}{2}$.

$(-2+\text{i})+(-2-\text{i})+\gamma=-\frac{5}{2}$

$-4+\gamma=-\frac{5}{2}$

$\gamma=\frac{3}{2}$

The other two roots are $-2-\text{i}$ and $\frac{3}{2}$.

3 b $\alpha\beta\gamma = -\frac{q}{2}$, so

$q = -2\alpha\beta\gamma$

$= -2(-2+\mathrm{i})(-2-\mathrm{i})\left(\frac{3}{2}\right)$

$= (-3)(4+2\mathrm{i}-2\mathrm{i}-\mathrm{i}^2)$

$= (-3)(5)$

$= -15$

So $q = -15$.

4 $x^4 - 40x^3 + 510x^2 - 220x + 1729 = 0$

So $a = 1, b = -40, c = 510, d = -2200$ and $e = 1729$.

Let $\alpha = \alpha$, $\beta = \alpha + 2k$, $\gamma = \alpha + 4k$ and $\delta = \alpha + 6k$.

$\alpha + \beta + \gamma + \delta$

$= \alpha + (\alpha + 2k) + (\alpha + 4k) + (\alpha + 6k)$

$= 4\alpha + 12k$

But $\alpha + \beta + \gamma + \delta = -\frac{b}{a} = 40$

So $4\alpha + 12k = 40$ or $\alpha + 3k = 10$

$\alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta$

$= \alpha(\alpha + 2k) + \alpha(\alpha + 4k) + \alpha(\alpha + 6k)$

$\quad + (\alpha + 2k)(\alpha + 4k) + (\alpha + 2k)(\alpha + 6k)$

$\quad + (\alpha + 4k)(\alpha + 6k)$

$= \alpha^2 + 2\alpha k + \alpha^2 + 4\alpha k + \alpha^2 + 6\alpha k$

$\quad + \alpha^2 + 6\alpha k + 8k^2 + \alpha^2 + 8\alpha k + 12k^2$

$\quad + \alpha^2 + 10\alpha k + 24k^2$

$= 6\alpha^2 + 36\alpha k + 44k^2$

But $\alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta = \frac{c}{a} = 510$

So $6\alpha^2 + 36\alpha k + 44k^2 = 510$

or $3\alpha^2 + 18\alpha k + 22k^2 = 255$

If $\alpha + 3k = 10$, then $\alpha = 10 - 3k$

Substitute $\alpha = 10 - 3k$ into

$3\alpha^2 + 18\alpha k + 22k^2 = 255$.

$3(10-3k)^2 + 18(10-3k)k + 22k^2 = 255$

$3(100 - 60k + 9k^2) + 18k(10-3k) + 22k^2 = 255$

$300 - 180k + 27k^2 + 180k - 54k^2 + 22k^2 = 255$

$-5k^2 = -45$

$k^2 = 9$

$k = \pm 3$

If $k = 3$, $\alpha = 10 - 3(3) = 1$

If $k = -3$, $\alpha = 10 - 3(-3) = 19$

Either way the roots are $1, 7, 13$ and 19.

5 a $24x^4 - 58x^3 + 17x^2 + dx + e = 0$

So $a = 24, b = -58, c = 17, d = d$ and $e = e$

$\alpha = \frac{1}{2}, \beta = -\frac{1}{3}, \gamma = 2$ and $\delta = \delta$.

$\alpha + \beta + \gamma + \delta = \frac{1}{2} + \left(-\frac{1}{3}\right) + 2 + \delta$

$= \frac{13}{6} + \delta$

But $\alpha + \beta + \gamma + \delta = -\frac{b}{a} = \frac{58}{24} = \frac{29}{12}$

So $\frac{13}{6} + \delta = \frac{29}{12}$, hence $\delta = \frac{3}{12} = \frac{1}{4}$

The fourth root is $\frac{1}{4}$.

b $\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta$

$= \left(\frac{1}{2}\right)\left(-\frac{1}{3}\right)(2) + \left(\frac{1}{2}\right)\left(-\frac{1}{3}\right)\left(\frac{1}{4}\right)$

$+ \left(\frac{1}{2}\right)(2)\left(\frac{1}{4}\right) + \left(-\frac{1}{3}\right)(2)\left(\frac{1}{4}\right)$

$= -\frac{1}{3} - \frac{1}{24} + \frac{1}{4} - \frac{1}{6}$

$= -\frac{8}{24} - \frac{1}{24} + \frac{6}{24} - \frac{4}{24}$

$= -\frac{7}{24}$

But $\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta = -\frac{d}{a} = -\frac{d}{24}$

So $-\frac{d}{24} = -\frac{7}{24}$, hence $d = 7$

$\alpha\beta\gamma\delta = \left(\frac{1}{2}\right)\left(-\frac{1}{3}\right)(2)\left(\frac{1}{4}\right) = -\frac{1}{12}$

But $\alpha\beta\gamma\delta = \frac{e}{a} = \frac{e}{24}$

So $\frac{e}{24} = -\frac{1}{12}$, hence $e = -2$.

6 a $x^4 + 2x^3 + mx^2 + nx + 85 = 0$

So $a = 1, b = 2, c = m, d = n$ and $e = 85$.

Given $\alpha = -2 + \mathrm{i}$ and $\beta = \alpha^* = -2 - \mathrm{i}$

$\alpha + \beta + \gamma + \delta = (-2+\mathrm{i}) + (-2+\mathrm{i}) + \gamma + \delta$

$= -4 + \gamma + \delta$

But $\alpha + \beta + \gamma + \delta = -\frac{b}{a} = -2$

So $-4 + \gamma + \delta = -2$ hence $\gamma + \delta - 2 = 0$

$\alpha\beta\gamma\delta = (-2+\mathrm{i})(-2+\mathrm{i})\gamma\delta$

$= 5\gamma\delta$

But $\alpha\beta\gamma\delta = \frac{e}{a} = 85$

So $5\gamma\delta = 85$ hence $\gamma\delta - 17 = 0$

6 b If $\gamma\delta - 17 = 0$, then $\gamma = \dfrac{17}{\delta}$

Substitute $\gamma = \dfrac{17}{\delta}$ into $\gamma + \delta - 2 = 0$

$\dfrac{17}{\delta} + \delta - 2 = 0$

Multiply by δ :

$17 + \delta^2 - 2\delta = 0$

$\delta^2 - 2\delta + 17 = 0$

Solve by completing the square:

$(\delta - 1)^2 - 1 + 17 = 0$

$(\delta - 1)^2 = -16$

$\delta - 1 = \pm 4\mathrm{i}$

$\delta = 1 \pm 4\mathrm{i}$

So the roots are $-2 \pm \mathrm{i}$ and $1 \pm 4\mathrm{i}$

But $\alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta = \dfrac{c}{a} = m$

$= (-2+i)(-2-i) + (-2+i)(1+4i) + (-2+i)(1-4i)$
$+ (-2-i)(1+4i) + (-2-i)(1-4i) + (1+4i)(1-4i)$

and cancelling the imaginary terms gives us

$= 5 - 2 - 2 - 2 + 17 = 14 = m$

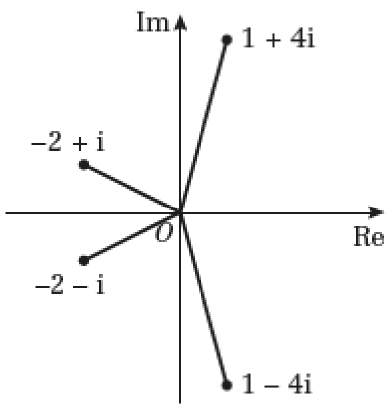
Now $\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta = \dfrac{-d}{a} = n$

$= 5(\gamma + \delta) + 17(\alpha + \beta) = (5)(2) + (17)(-4) = -58 = -n$

So

$n = 58$

c



7 a $4x^4 - 16x^3 + 115x^2 + 4x - 29 = 0$

So $a = 4, b = -16, c = 115, d = 4$ and $e = -29$

Show $\alpha = 2 - 5\mathrm{i}$ is a root:

$4(2-5\mathrm{i})^4 - 16(2-5\mathrm{i})^3 + 115(2-5\mathrm{i})^2$
$+4(2-5\mathrm{i}) - 29 = 0$

7 a Use

$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

with $a = 2$ and $b = -5\mathrm{i}$

$(2-5\mathrm{i})^4 = (2)^4 + 4(2)^3(-5\mathrm{i}) + 6(2)^2(-5\mathrm{i})^2$
$\quad + 4(2)(-5\mathrm{i})^3 + (-5\mathrm{i})^4$

$(2-5\mathrm{i})^4 = 16 + (32)(-5\mathrm{i}) + (24)(-5\mathrm{i})^2$
$\quad + (8)(-5\mathrm{i})^3 + (-5\mathrm{i})^4$

$(2-5\mathrm{i})^4 = 16 - 160\mathrm{i} + 600\mathrm{i}^2 - 1000\mathrm{i}^3 + 625\mathrm{i}^4$

Use the fact that $\mathrm{i}^2 = -1$, $\mathrm{i}^3 = -\mathrm{i}$ and $\mathrm{i}^4 = 1$ to simplify:

$(2-5\mathrm{i})^4 = 16 - 160\mathrm{i} - 600 + 1000\mathrm{i} + 625$
$\quad = 41 + 840\mathrm{i}$

Use $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ with $a = 2$ and $b = -5\mathrm{i}$

$(2-5\mathrm{i})^3 = (2)^3 + 3(2)^2(-5\mathrm{i})$
$\quad + 3(2)(-5\mathrm{i})^2 + (-5\mathrm{i})^3$
$\quad = 8 + (12)(-5\mathrm{i})$
$\quad + (6)(-5\mathrm{i})^2 + (-5\mathrm{i})^3$
$\quad = 8 + -60\mathrm{i} + 150\mathrm{i}^2 - 125\mathrm{i}^3$

Use the fact that $\mathrm{i}^2 = -1$ and $\mathrm{i}^3 = -\mathrm{i}$ to simplify:

$(2-5\mathrm{i})^3 = 8 + -60\mathrm{i} - 150 + 125\mathrm{i}$
$\quad = -142 + 65\mathrm{i}$

$(2-5\mathrm{i})^2 = 4 - 10\mathrm{i} - 10\mathrm{i} + 25\mathrm{i}^2$
$\quad = -21 - 20\mathrm{i}$

So $4(2-5\mathrm{i})^4 - 16(2-5\mathrm{i})^3 + 115(2-5\mathrm{i})^2$
$+4(2-5\mathrm{i}) - 29 = 0$ becomes:

$4(41+840\mathrm{i}) - 16(-142+65\mathrm{i})$
$+115(-21-20\mathrm{i}) + 4(2-5\mathrm{i}) - 29 = 0$

$164 + 3360\mathrm{i} + 2272 - 1040\mathrm{i}$
$-2415 - 2300\mathrm{i} + 8 - 20\mathrm{i} - 29 = 0$

$(164 + 2272 - 2514 + 8 - 29)$
$+(3360 - 1040 - 2300 - 20)\mathrm{i} = 0$

So $\alpha = 2 - 5\mathrm{i}$ is a root.

7 b If $\alpha = 2-5\text{i}$ is a root, then $\beta = 2+5\text{i}$ is also a root. Let the other two roots be γ and δ.

$\alpha+\beta+\gamma+\delta = (2-5\text{i})+(2+5\text{i})+\gamma+\delta$
$= 4+\gamma+\delta$

But $\alpha+\beta+\gamma+\delta = -\frac{b}{a} = 4$

So $4+\gamma+\delta = 4$ hence $\gamma+\delta = 0$ and $\gamma = -\delta$.

$\alpha\beta\gamma\delta = (2-5\text{i})(2+5\text{i})\gamma\delta$
$= 29\gamma\delta$

But $\alpha\beta\gamma\delta = \frac{e}{a} = -\frac{29}{4}$

So $29\gamma\delta = -\frac{29}{4}$, hence $\gamma\delta = -\frac{1}{4}$.

Substitute $\gamma = -\delta$ into $\gamma\delta = -\frac{1}{4}$.

$-\delta^2 = -\frac{1}{4}$

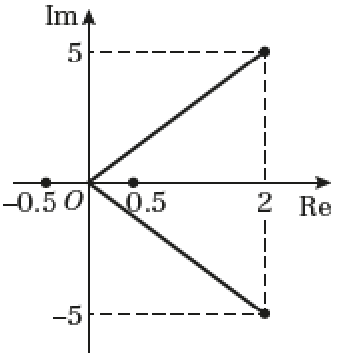
$\delta^2 = \frac{1}{4}$

$\delta = \pm\frac{1}{2}$

So $\gamma = \mp\frac{1}{2}$

So the roots are $2-5\text{i}, 2+5\text{i}, \frac{1}{2}$ and $-\frac{1}{2}$.

c



8 a $2x^3 - 5x^2 + 11x - 9 = 0$

So $a = 2,\ b = -5,\ c = 11$ and $d = -9$

$\alpha+\beta+\gamma = -\frac{b}{a} = \frac{5}{2}$

$\alpha\beta+\beta\gamma+\gamma\alpha = \frac{c}{a} = \frac{11}{2}$

$\alpha\beta\gamma = -\frac{d}{a} = \frac{9}{2}$

b i $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma} = \frac{\alpha\beta+\beta\gamma+\gamma\alpha}{\alpha\beta\gamma} = \frac{\left(\frac{11}{2}\right)}{\left(\frac{9}{2}\right)} = \frac{11}{9}$

ii $\alpha^2+\beta^2+\gamma^2$

$= (\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\beta\gamma+\gamma\alpha)$

$= \left(\frac{5}{2}\right)^2 - 2\left(\frac{11}{2}\right)$

$= \frac{25}{4} - 11$

$= -\frac{19}{4}$

iii $(\alpha-1)(\beta-1)(\gamma-1)$

$= \alpha\beta\gamma - (\alpha\beta+\beta\gamma+\gamma\alpha)$
$+ (\alpha+\beta+\gamma) - 1$

$= \frac{9}{2} - \frac{11}{2} + \frac{5}{2} - 1$

$= \frac{1}{2}$

9 a $px^4 + 12x^3 + 6x^2 + 5x - 7 = 0$

So $a = p, b = 12, c = 6, d = 5$ and $e = -7$

$\alpha\beta\gamma\delta = \frac{e}{a} = -\frac{7}{p}$

But $\alpha\beta\gamma\delta = -1$

So $-\frac{7}{p} = -1$, hence $p = 7$.

b $\sum\alpha = -\frac{b}{a} = -\frac{12}{7}$

$\sum\alpha\beta = \frac{c}{a} = \frac{6}{7}$

$\sum\alpha\beta\gamma = -\frac{d}{a} = -\frac{5}{7}$

c $\alpha^2+\beta^2+\gamma^2+\delta^2$

$= (\alpha+\beta+\gamma+\delta)^2$
$-2(\alpha\beta+\alpha\gamma+\alpha\delta+\beta\gamma+\beta\delta+\gamma\delta)$

$= \left(-\frac{12}{7}\right)^2 - 2\left(\frac{6}{7}\right)$

$= \frac{144}{49} - \frac{12}{7}$

$= \frac{144}{49} - \frac{84}{49}$

$= \frac{60}{49}$

10 a $5x^3 + cx + 21 = 0$

So $a = 5,\ b = 0,\ c = c$, and $d = 21$.

$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = \frac{c}{5}$

But $\alpha\beta + \beta\gamma + \gamma\alpha = -6$

So $\frac{c}{5} = -6$, hence $c = -30$

b $\alpha + \beta + \gamma = -\frac{b}{a} = 0$

$\alpha\beta\gamma = -\frac{d}{a} = -\frac{21}{5}$

c $(1-\alpha)(1-\beta)(1-\gamma)$

$= 1 - (\alpha + \beta + \gamma) + (\alpha\beta + \beta\gamma + \gamma\alpha) - \alpha\beta\gamma$

$= 1 - 0 + (-6) - \left(-\frac{21}{5}\right)$

$= -5 + \frac{21}{5}$

$= -\frac{4}{5}$

11 $2x^3 + 5x^2 + 7x - 2 = 0$

$w = 3x + 1$, so $x = \frac{w-1}{3}$

Substitute $x = \frac{w-1}{3}$ into

$2x^3 + 5x^2 + 7x - 2 = 0$

$2\left(\frac{w-1}{3}\right)^3 + 5\left(\frac{w-1}{3}\right)^2 + 7\left(\frac{w-1}{3}\right) - 2 = 0$

Multiply by 27 to remove fractions:

$2(w-1)^3 + 15(w-1)^2 + 63(w-1) - 54 = 0$

$2(w^3 - 3w^2 + 3w - 1) + 15(w^2 - 2w + 1)$
$+63(w-1) - 54 = 0$

$2w^3 - 6w^2 + 6w - 2 + 15w^2 - 30w + 15$
$+63w - 63 - 54 = 0$

$2w^3 + 9w^2 + 39w - 104 = 0$

12 a $6x^4 - 2x^3 - 5x^2 + 7x + 8 = 0$

$w = 2x$, so $x = \frac{w}{2}$

Substitute $x = \frac{w}{2}$ into

$6x^4 - 2x^3 - 5x^2 + 7x + 8 = 0$

$6\left(\frac{w}{2}\right)^4 - 2\left(\frac{w}{2}\right)^3 - 5\left(\frac{w}{2}\right)^2 + 7\left(\frac{w}{2}\right) + 8 = 0$

12 a Multiply by 16 to eliminate fractions:

$6w^4 - 4w^3 - 20w^2 + 56w + 128 = 0$

All coefficients are divisible by 2:

$3w^4 - 2w^3 - 10w^2 + 28w + 64 = 0$

b $w = 3x - 2$, so $x = \frac{w+2}{3}$

Substitute $x = \frac{w+2}{3}$ into

$6x^4 - 2x^3 - 5x^2 + 7x + 8 = 0$

$6\left(\frac{w+2}{3}\right)^4 - 2\left(\frac{w+2}{3}\right)^3 - 5\left(\frac{w+2}{3}\right)^2$
$+7\left(\frac{w+2}{3}\right) + 8 = 0$

Multiply by 81 to eliminate fractions:

$6(w+2)^4 - 6(w+2)^3 - 45(w+2)^2$
$+189(w+2) + 648 = 0$

All coefficients are divisible by 3:

$2(w+2)^4 - 2(w+2)^3 - 15(w+2)^2$
$+63(w+2) + 216 = 0$

$2(w^4 + 8w^3 + 24w^2 + 32w + 16)$
$-2(w^3 + 6w^2 + 12w + 8)$
$-15(w^2 + 4w + 4) + 63(w+2) + 216 = 0$

$2w^4 + 16w^3 + 48w^2 + 64w + 32$
$-2w^3 - 12w^2 - 24w - 16$
$-15w^2 - 60w - 60 + 63w + 126 + 216 = 0$

$2w^4 + 14w^3 + 21w^2 + 43w + 298 = 0$

Challenge

1 Multiply by w^3 :

$1-3w+3w^2-w^3+4w(1-2w+w^2)$
$-5w^2(1-w)-7w^3=0$

$1-3w+3w^2-w^3+4w-8w^2+4w^3$
$-5w^2+5w^3-7w^3=0$

$w^3-10w^2+w+1=0$

2 $x^3+2x^2-3x-5=0$

So $a=1,\ b=2,\ c=-3$ and $d=-5$.

$\alpha+\beta+\gamma=-\dfrac{b}{a}=-2$

Hence $\alpha+\beta+\gamma=-2$

So, $\alpha+\beta=-2-\gamma$, $\beta+\gamma=-2-\alpha$ and
$\gamma+\alpha=-2-\beta$

$w=-2-x$, so $x=-2-w$

Substitute $x=-2-w$ into
$x^3+2x^2-3x-5=0$

$(-2-w)^3+2(-2-w)^2-3(-2-w)-5=0$

$-8-12w-6w^2-w^3+2(4+4w+w^2)$
$+6+3w-5=0$

$-8-12w-6w^2-w^3+8+8w+2w^2$
$+6+3w-5=0$

$-w^3-4w^2-w+1=0$

Divide by -1 :

$w^3+4w^2+w-1=0$

3 $x^4+2x^2-5x+2=0$

$w=x^2+1$, so $x^2=w-1$.

Therefore $x=\sqrt{w-1}$.

Substitute $x=\sqrt{w-1}$ into
$x^4+2x^2-5x+2=0$.

$\left(\sqrt{w-1}\right)^4+2\left(\sqrt{w-1}\right)^2-5\left(\sqrt{w-1}\right)+2=0$

$(w-1)^2+2(w-1)-5\sqrt{w-1}+2=0$

$w^2-2w+1+2w-2-5\sqrt{w-1}+2=0$

$w^2-5\sqrt{w-1}+1=0$

Multiply by $w^2+5\sqrt{w-1}+1$.

$w^4+2w^2-25(w-1)+1=0$

$w^4+2w^2-25w+26=0$