Matrices 6A

1 a $\begin{pmatrix} 1 & 0 \\ -1 & 3 \end{pmatrix}$

The size is 2×2.

b $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

The size is 2×1.

c $\begin{pmatrix} 1 & 2 & 1 \\ 3 & 0 & -1 \end{pmatrix}$

The size is 2×3.

d $\begin{pmatrix} 1 & 2 & 3 \end{pmatrix}$

The size is 1×3.

e $\begin{pmatrix} 3 & -1 \end{pmatrix}$

The size is 1×2.

f $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

The size is 3×3.

2 $\mathbf{I}_4 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

3 Compare corresponding elements:

$a = 6$

$b = 2$

4 a $\begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix} + \begin{pmatrix} 6 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 8 & -1 \\ 1 & 4 \end{pmatrix}$

b $\begin{pmatrix} 4 & 1 \\ -1 & -2 \end{pmatrix} - \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 2 & 2 \\ -2 & -5 \end{pmatrix}$

c $\begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix} + \begin{pmatrix} 4 & 1 \\ -1 & -2 \end{pmatrix} - \begin{pmatrix} 6 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

5 a $A + B$ is $(2 \times 1) + (1 \times 2)$ Not possible

b $A - E = \begin{pmatrix} 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ -1 \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$

c $F - D + C = \begin{pmatrix} 2 & 1 & 3 \end{pmatrix} - \begin{pmatrix} 0 & 1 & -1 \end{pmatrix} + \begin{pmatrix} -1 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 4 \end{pmatrix}$

d $B + C$ is $(1 \times 2) + (1 \times 3)$ Not possible

e $F - (D + C) = \begin{pmatrix} 2 & 1 & 3 \end{pmatrix} - \left[\begin{pmatrix} 0 & 1 & -1 \end{pmatrix} + \begin{pmatrix} -1 & 1 & 0 \end{pmatrix} \right] = \begin{pmatrix} 2 & 1 & 3 \end{pmatrix} - \begin{pmatrix} -1 & 2 & -1 \end{pmatrix} = \begin{pmatrix} 3 & -1 & 4 \end{pmatrix}$

f $A - F = (2 \times 1) - (1 \times 3)$ Not possible.

g $C - (F - D) = \begin{pmatrix} -1 & 1 & 0 \end{pmatrix} - \left[\begin{pmatrix} 2 & 1 & 3 \end{pmatrix} - \begin{pmatrix} 0 & 1 & -1 \end{pmatrix} \right] = \begin{pmatrix} -1 & 1 & 0 \end{pmatrix} - \begin{pmatrix} 2 & 0 & 4 \end{pmatrix} = \begin{pmatrix} -3 & 1 & -4 \end{pmatrix}$

6

$a - 1 = 5 \Rightarrow a = 6$

$2 - c = 0 \Rightarrow c = 2$

$-1 - d = 0 \Rightarrow d = -1$

$b - (-2) = 5 \Rightarrow b = 3$

7

$1 + a = c \quad (1)$

$2 + b = 5 \Rightarrow b = 3$

$0 + c = c$

$a + 1 = c$

$b + 2 = c \quad (2)$

$c + 0 = c$

Use $b = 3$ in **(2)** $\Rightarrow c = 5$

Use $c = 5$ in **(1)** $\Rightarrow a = 4$

8 $5+a=7 \Rightarrow a=2$

$3+b=1 \Rightarrow b=-2$

$0+c=2 \Rightarrow c=2$

$-1+d=0 \Rightarrow d=1$

$2+e=1 \Rightarrow e=-1$

$1+f=4 \Rightarrow f=3$

9 a $\mathbf{A}+\mathbf{B}=\begin{pmatrix}1&-1&3\\2&0&2\\3&4&0\end{pmatrix}+\begin{pmatrix}6&3&-4\\1&1&2\\-2&0&-3\end{pmatrix}$

$=\begin{pmatrix}7&2&-1\\3&1&4\\1&4&-3\end{pmatrix}$

b $\mathbf{B}-\mathbf{C}=\begin{pmatrix}6&3&-4\\1&1&2\\-2&0&-3\end{pmatrix}-\begin{pmatrix}-2&0&3\\2&8&-6\\-1&1&1\end{pmatrix}$

$=\begin{pmatrix}8&3&-7\\-1&-7&8\\-1&-1&-4\end{pmatrix}$

c $\mathbf{C}+\mathbf{A}=\begin{pmatrix}-2&0&3\\2&8&-6\\-1&1&1\end{pmatrix}+\begin{pmatrix}1&-1&3\\2&0&2\\3&4&0\end{pmatrix}$

$=\begin{pmatrix}-1&-1&6\\4&8&-4\\2&5&1\end{pmatrix}$

d i $\mathbf{A}+\mathbf{M}=\begin{pmatrix}1&-1&3\\2&0&2\\3&4&0\end{pmatrix}+\begin{pmatrix}5&-6&b\\4&a&6\\2&0&c\end{pmatrix}$

$=\begin{pmatrix}6&-7&3+b\\6&a&8\\5&4&c\end{pmatrix}$

Equate with given $\mathbf{A}+\mathbf{M}$:

$\begin{pmatrix}6&-7&3+b\\6&a&8\\5&4&c\end{pmatrix}=\begin{pmatrix}6&-7&-2\\6&3&8\\5&4&6\end{pmatrix}$

Compare corresponding elements:

$a=3$

$3+b=-2 \Rightarrow b=-5$

$c=6$

d ii $\mathbf{M}-\mathbf{B}=\begin{pmatrix}5&-6&b\\4&a&6\\2&0&c\end{pmatrix}-\begin{pmatrix}6&3&-4\\1&1&2\\-2&0&-3\end{pmatrix}$

$=\begin{pmatrix}-1&-9&b+4\\3&a-1&4\\4&0&c+3\end{pmatrix}$

Equate with given $\mathbf{M}-\mathbf{B}$:

$\begin{pmatrix}-1&-9&b+4\\3&a-1&4\\4&0&c+3\end{pmatrix}=\begin{pmatrix}-1&-9&-5\\-2&7&-1\\-3&2&2\end{pmatrix}$

Compare corresponding elements:

$a-1=7 \Rightarrow a=8$

$b+4=-5 \Rightarrow b=-9$

$c+3=2 \Rightarrow c=-1$

10 a $3\begin{pmatrix}2&0\\4&-6\end{pmatrix}=\begin{pmatrix}6&0\\12&-18\end{pmatrix}$

b $\frac{1}{2}\begin{pmatrix}2&0\\4&-6\end{pmatrix}=\begin{pmatrix}1&0\\2&-3\end{pmatrix}$

c $2\begin{pmatrix}1\\-1\end{pmatrix}=\begin{pmatrix}2\\-2\end{pmatrix}$

d **A** and **B** are not the same size, so you can't subtract them.

11 a $3\mathbf{A}+2\mathbf{B}=3\begin{pmatrix}3&-2\\1&0\end{pmatrix}+2\begin{pmatrix}2&1\\-2&3\end{pmatrix}$

$=\begin{pmatrix}9&-6\\3&0\end{pmatrix}+\begin{pmatrix}4&2\\-4&6\end{pmatrix}$

$=\begin{pmatrix}13&-4\\-1&6\end{pmatrix}$

b $2\mathbf{A}-4\mathbf{B}=2\begin{pmatrix}3&-2\\1&0\end{pmatrix}-4\begin{pmatrix}2&1\\-2&3\end{pmatrix}$

$=\begin{pmatrix}6&-4\\2&0\end{pmatrix}-\begin{pmatrix}8&4\\-8&12\end{pmatrix}$

$=\begin{pmatrix}-2&-8\\10&-12\end{pmatrix}$

c $5\mathbf{A}-2\mathbf{B}=5\begin{pmatrix}3&-2\\1&0\end{pmatrix}-2\begin{pmatrix}2&1\\-2&3\end{pmatrix}$

$=\begin{pmatrix}15&-10\\5&0\end{pmatrix}-\begin{pmatrix}4&2\\-4&6\end{pmatrix}$

$=\begin{pmatrix}11&-12\\9&-6\end{pmatrix}$

d $\frac{1}{2}\mathbf{A}+\frac{3}{2}\mathbf{B}=\frac{1}{2}\begin{pmatrix}3&-2\\1&0\end{pmatrix}+\frac{3}{2}\begin{pmatrix}2&1\\-2&3\end{pmatrix}$

$=\begin{pmatrix}\frac{3}{2}&-1\\\frac{1}{2}&0\end{pmatrix}+\begin{pmatrix}3&\frac{3}{2}\\-3&\frac{9}{2}\end{pmatrix}$

$=\begin{pmatrix}\frac{9}{2}&\frac{1}{2}\\-\frac{5}{2}&\frac{9}{2}\end{pmatrix}$

12 a $\mathbf{M}+2\mathbf{N}=\begin{pmatrix}2&4&-1\\1&-3&-1\\0&2&2\end{pmatrix}+2\begin{pmatrix}6&-2&5\\3&-3&1\\1&-1&0\end{pmatrix}$

$=\begin{pmatrix}2&4&-1\\1&-3&-1\\0&2&2\end{pmatrix}+\begin{pmatrix}12&-4&10\\6&-6&2\\2&-2&0\end{pmatrix}$

$=\begin{pmatrix}14&0&9\\7&-9&1\\2&0&2\end{pmatrix}$

12 b $3\mathbf{M}-\mathbf{N}=3\begin{pmatrix}2&4&-1\\1&-3&-1\\0&2&2\end{pmatrix}-\begin{pmatrix}6&-2&5\\3&-3&1\\1&-1&0\end{pmatrix}$

$=\begin{pmatrix}6&12&-3\\3&-9&-3\\0&6&6\end{pmatrix}-\begin{pmatrix}6&-2&5\\3&-3&1\\1&-1&0\end{pmatrix}$

$=\begin{pmatrix}0&14&-8\\0&-6&-4\\-1&7&6\end{pmatrix}$

c $4\mathbf{M}+5\mathbf{N}=4\begin{pmatrix}2&4&-1\\1&-3&-1\\0&2&2\end{pmatrix}+5\begin{pmatrix}6&-2&5\\3&-3&1\\1&-1&0\end{pmatrix}$

$=\begin{pmatrix}8&16&-4\\4&-12&-4\\0&8&8\end{pmatrix}+\begin{pmatrix}30&-10&25\\15&-15&5\\5&-5&0\end{pmatrix}$

$=\begin{pmatrix}38&6&21\\19&-27&1\\5&3&8\end{pmatrix}$

d $\frac{2}{3}\mathbf{M}-\frac{1}{2}\mathbf{N}=\frac{2}{3}\begin{pmatrix}2&4&-1\\1&-3&-1\\0&2&2\end{pmatrix}-\frac{1}{2}\begin{pmatrix}6&-2&5\\3&-3&1\\1&-1&0\end{pmatrix}$

$=\begin{pmatrix}\frac{4}{3}&\frac{8}{3}&-\frac{2}{3}\\\frac{2}{3}&-2&-\frac{2}{3}\\0&\frac{4}{3}&\frac{4}{3}\end{pmatrix}-\begin{pmatrix}3&-1&\frac{5}{2}\\\frac{3}{2}&-\frac{3}{2}&\frac{1}{2}\\\frac{1}{2}&-\frac{1}{2}&0\end{pmatrix}$

$=\begin{pmatrix}-\frac{5}{3}&\frac{11}{3}&-\frac{19}{6}\\-\frac{5}{6}&-\frac{1}{2}&-\frac{7}{6}\\-\frac{1}{2}&\frac{11}{6}&\frac{4}{3}\end{pmatrix}$

13 $1+2k=7$

$\Rightarrow \quad 2k=6$

$\Rightarrow \quad k=3.$

$2-k=x$

$\Rightarrow \quad 2-3=x$

$\therefore \quad x=-1$

14 $2a-3=3 \Rightarrow 2a=6$

$\Rightarrow a=3$

$0-3c=3 \Rightarrow c=-1$

$2-3d=-4 \Rightarrow -3d=-6$

$\Rightarrow d=2$

$2b+3=-4 \Rightarrow 2b=-7$

$\Rightarrow b=-3.5$

15 $5-2c=9$

$\Rightarrow -4=2c$

$\Rightarrow c=-2.$

$a-4=1$

$\Rightarrow a=5.$

$b-2=3$

$\Rightarrow b=5.$

$0+2=d$

$\Rightarrow d=2$

16 $-3+2k^2=k$

$\Rightarrow 2k^2-k-3=0$

$(2k-3)(k+1)=0$

$\therefore k=\frac{3}{2}$ or -1

AND $k+2k^2=6$

$\Rightarrow 2k^2+k-6=0$

$(2k-3)(k+2)=0$

$\therefore k=\frac{3}{2}$ or -2

So common value is $k=\frac{3}{2}$

17 a $\mathbf{A}-k\mathbf{B}=\mathbf{I}_3$

$$\begin{pmatrix} p & 0 & 0 \\ 0 & q^2 & r \\ 0 & 0 & 5 \end{pmatrix} - k\begin{pmatrix} 2q & 0 & 0 \\ 0 & 4 & 6 \\ 0 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} p-2kq & 0 & 0 \\ 0 & q^2-4k & r-6k \\ 0 & 0 & 5-2k \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Compare corresponding elements:

$5-2k=1$

$2k=4$

$k=2$

b Using $k=2$,

$r-6k=0$

$r-6(2)=0 \Rightarrow r=12$

$q^2-4k=1$

$q^2-4(2)=1$

$q^2=9 \Rightarrow q=3$

$p-2kq=1$

$p-2(2)(3)=1 \Rightarrow p=13$

18 $\mathbf{P}-k\mathbf{Q}=\begin{pmatrix}0&2&c\\a&0&0\\0&b&-1\end{pmatrix}-k\begin{pmatrix}0&-1&-1\\3&d&0\\0&2&e\end{pmatrix}=\begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix}$

$=\begin{pmatrix}0&2&c\\a&0&0\\0&b&-1\end{pmatrix}-\begin{pmatrix}0&-k&-k\\3k&dk&0\\0&2k&ek\end{pmatrix}=\begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix}$

$=\begin{pmatrix}0&2+k&c+k\\a-3k&-dk&0\\0&b-2k&-1-ek\end{pmatrix}=\begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix}$

Compare corresponding elements:

$2+k=0\Rightarrow k=-2$

$a-3k=0$

$a-3(-2)=0$

$a+6=0\Rightarrow a=-6$

$b-2k=0$

$b-2(-2)=0$

$b+4=0\Rightarrow b=-4$

$c+k=0\Rightarrow c=2$

$-dk=0\Rightarrow d=0$

$-1-ek=0$

$-1-e(-2)=0$

$-1+2e=0$

$2e=1\Rightarrow e=\frac{1}{2}$