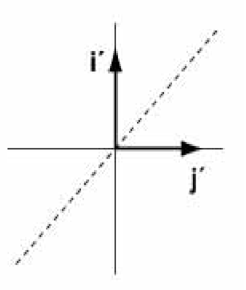
Exercise 7D

1 a $\mathbf{AB}=\begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}0 & -1\\ -1 & 0\end{pmatrix}=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$

$\begin{pmatrix}1\\0\end{pmatrix}\to\begin{pmatrix}0\\1\end{pmatrix}$

$\begin{pmatrix}0\\1\end{pmatrix}\to\begin{pmatrix}1\\0\end{pmatrix}$



Reflection in $y=x$

b $\mathbf{BA}=\begin{pmatrix}0 & -1\\ -1 & 0\end{pmatrix}\begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix}=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$ Reflection in $y=x$

c $\mathbf{AC}=\begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}2 & 0\\ 0 & 2\end{pmatrix}=\begin{pmatrix}-2 & 0\\ 0 & -2\end{pmatrix}$ Enlargement scale factor -2 centre (0, 0)

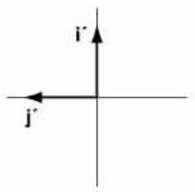
d $\mathbf{A}^2=\begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ Identity (No transformation)

[This can be thought of as a rotation of 180° +180°=360°]

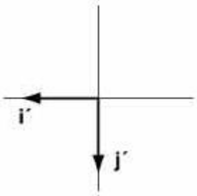
e $\mathbf{C}^2=\begin{pmatrix}2 & 0\\ 0 & 2\end{pmatrix}\begin{pmatrix}2 & 0\\ 0 & 2\end{pmatrix}=\begin{pmatrix}4 & 0\\ 0 & 4\end{pmatrix}$

Enlargement scale factor 4 centre (0, 0**)**

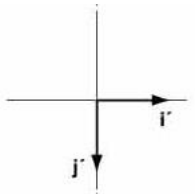
2 a Rotation of 90° anticlockwise $A=\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$



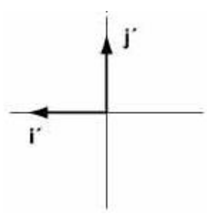
Rotation of 180° about (0, 0) $B=\begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix}$



Reflection in x-axis $C=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$



Reflection in y-axis $D=\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}$



© Pearson Education Ltd 2017. Copying permitted for purchasing institution only. This material is not copyright free.

2 b i $BC = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \quad (= D)$

Reflection in y-axis

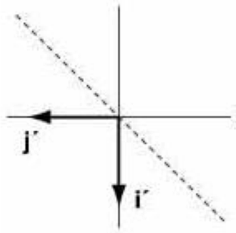
ii $CB = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \quad (= D)$

Reflection in y-axis

iii $CD = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \quad (= B)$

Rotation of 180° about (0, 0)

iv $AD = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$



Reflection in line $y = -x$.

v $BB = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Rotation of 360° about (0, 0) or Identity

vi $DAC = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$= \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad (= A)$

Rotation of 90° anticlockwise about (0, 0)

vii $\text{CBD} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

$= \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Identify –no transformation

© Pearson Education Ltd 2017. Copying permitted for purchasing institution only. This material is not copyright free.

3 a $\mathbf{RS}=\begin{pmatrix}3&0\\0&-2\end{pmatrix}\begin{pmatrix}0&1\\-1&0\end{pmatrix}=\begin{pmatrix}0&3\\2&0\end{pmatrix}$

Reflection in $y=x$ with a stretch by scale factor 3 parallel to the x-axis and by scale factor 2 parallel to the y-axis.

b $\mathbf{RT}=\begin{pmatrix}3&0\\0&-2\end{pmatrix}\begin{pmatrix}5&0\\0&5\end{pmatrix}=\begin{pmatrix}15&0\\0&-10\end{pmatrix}$

Stretch by scale factor 15 parallel to the x-axis and by scale factor -10 parallel to the y-axis

c $\mathbf{TS}=\begin{pmatrix}5&0\\0&5\end{pmatrix}\begin{pmatrix}0&1\\-1&0\end{pmatrix}=\begin{pmatrix}0&5\\-5&0\end{pmatrix}$

Enlargement by scale factor 5 about (0,0) and rotation through 270° anti-clockwise.

d $\mathbf{TR}=\begin{pmatrix}5&0\\0&5\end{pmatrix}\begin{pmatrix}3&0\\0&-2\end{pmatrix}=\begin{pmatrix}15&0\\0&-10\end{pmatrix}$

Stretch by scale factor 15 parallel to the x-axis and by scale factor -10 parallel to the y-axis.

e $\mathbf{ST}=\begin{pmatrix}0&1\\-1&0\end{pmatrix}\begin{pmatrix}5&0\\0&5\end{pmatrix}=\begin{pmatrix}0&5\\-5&0\end{pmatrix}$

Enlargement by scale factor 5 about (0,0) and rotation through 270° anti-clockwise.

f $\mathbf{RST}=\begin{pmatrix}3&0\\0&-2\end{pmatrix}\begin{pmatrix}0&5\\-5&0\end{pmatrix}=\begin{pmatrix}0&15\\10&0\end{pmatrix}$

Reflection in $y=x$ with a stretch by scale factor 15 parallel to the x-axis and by scale factor 10 parallel to the y-axis.

4 a $\mathbf{A}:\begin{pmatrix}2&0\\0&3\end{pmatrix},\mathbf{B}:\begin{pmatrix}-2&0\\0&-2\end{pmatrix},\mathbf{C}:\begin{pmatrix}4&0\\0&4\end{pmatrix}$

b i $\mathbf{AB}=\begin{pmatrix}2&0\\0&3\end{pmatrix}\begin{pmatrix}-2&0\\0&-2\end{pmatrix}=\begin{pmatrix}-4&0\\0&-6\end{pmatrix}$

ii $\mathbf{AC}=\begin{pmatrix}2&0\\0&3\end{pmatrix}\begin{pmatrix}4&0\\0&4\end{pmatrix}=\begin{pmatrix}8&0\\0&12\end{pmatrix}$

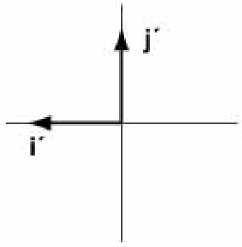
iii $\mathbf{CB}=\begin{pmatrix}4&0\\0&4\end{pmatrix}\begin{pmatrix}-2&0\\0&-2\end{pmatrix}=\begin{pmatrix}-8&0\\0&-8\end{pmatrix}$

iv $\mathbf{C}^2=\begin{pmatrix}4&0\\0&4\end{pmatrix}\begin{pmatrix}4&0\\0&4\end{pmatrix}=\begin{pmatrix}16&0\\0&16\end{pmatrix}$

v $\mathbf{ABC}=\begin{pmatrix}-4&0\\0&-6\end{pmatrix}\begin{pmatrix}4&0\\0&4\end{pmatrix}=\begin{pmatrix}-16&0\\0&-24\end{pmatrix}$

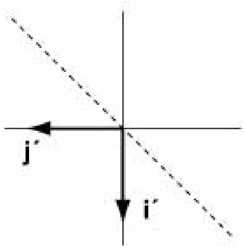
© Pearson Education Ltd 2017. Copying permitted for purchasing institution only. This material is not copyright free.

5 Reflection in y-axis



$\therefore$ Matrix is $\mathbf{Y} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

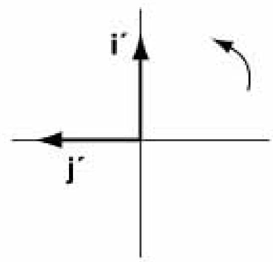
Reflection in $y = -x$



$\therefore$ Matrix $\mathbf{R} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

$$\mathbf{RY} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \to \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$
$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \to \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$



i.e Rotation of 90° anticlockwise about (0, 0).

6 $T = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ and $U = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

$$UT = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$TU = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \neq UT$$

7 a $PQ = \begin{pmatrix} -4 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix} = \begin{pmatrix} -4k & 0 \\ 0 & 2k \end{pmatrix}$

b Stretch by scale factor $-4k$ parallel to the x-axis and by scale factor $2k$ parallel to the y-axis.

c $\mathbf{QP} = \begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}\begin{pmatrix} -4 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} -4k & 0 \\ 0 & 2k \end{pmatrix} = \mathbf{PQ}$ (from part a)

8 a $\mathbf{C}^2 = \begin{pmatrix} 3 & 0 \\ 0 & 4 \end{pmatrix}\begin{pmatrix} 3 & 0 \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} 9 & 0 \\ 0 & 16 \end{pmatrix}$

b Stretch by scale factor 9 parallel to the x-axis and by scale factor 16 parallel to the y-axis.

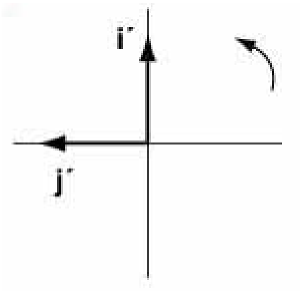
© Pearson Education Ltd 2017. Copying permitted for purchasing institution only. This material is not copyright free.

8 c $\mathbf{B}^2 = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} = \begin{pmatrix} a^2 & 0 \\ 0 & b^2 \end{pmatrix}$

Stretch by scale factor a^2 parallel to the x-axis and by scale factor b^2 parallel to the y-axis.

9 a $\mathbf{R}^2 = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

b



i.e. $\mathbf{R}^2$ represents rotation of 90° anticlockwise about (0, 0)

c **R** represents a rotation of 45° anticlockwise about (0, 0)

d $\mathbf{R}^8$ will represent rotation of $8 \times 45^\circ = 360^\circ$

This is equivalent to no transformation

$\therefore \quad \mathbf{R}^8 = \mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

10 a $\det \mathbf{M} = \frac{9}{2} + \frac{9}{2} = 9$

$\Rightarrow k = -3$ since $k < 0$

b $\mathbf{M} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix}$

$\Rightarrow \theta = 45^\circ$

11 $\mathbf{AB} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix} = \begin{pmatrix} 0 & 5 \\ -5 & 0 \end{pmatrix}$

$\det \mathbf{M} = 25$

$\Rightarrow$ Area of $T = 75 \div 25 = 3$

12 a $\begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$

b $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

© Pearson Education Ltd 2017. Copying permitted for purchasing institution only. This material is not copyright free.

12 c $TU = \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$

13 a $\mathbf{A}^2 = \begin{pmatrix} k & \sqrt{3} \\ \sqrt{3} & -k \end{pmatrix}\begin{pmatrix} k & \sqrt{3} \\ \sqrt{3} & -k \end{pmatrix} = \begin{pmatrix} k^2+3 & 0 \\ 0 & k^2+3 \end{pmatrix}$

b Enlargement centre (0,0) with scale factor k^2+3

14 $\mathbf{P}^2 = \begin{pmatrix} a & b \\ b & -a \end{pmatrix}\begin{pmatrix} a & b \\ b & -a \end{pmatrix} = \begin{pmatrix} a^2+b^2 & ab-ba \\ ab-ba & b^2+a^2 \end{pmatrix}$

$= \begin{pmatrix} a^2+b^2 & 0 \\ 0 & a^2+b^2 \end{pmatrix}$

Enlargement centre (0,0) with scale factor $\mathbf{a}^2+\mathbf{b}^2$

Challenge

a $\mathbf{P}^2 = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

$= \begin{pmatrix} \cos^2\theta-\sin^2\theta & -2\sin\theta\cos\theta \\ 2\sin\theta\cos\theta & \cos^2\theta-\sin^2\theta \end{pmatrix} = \begin{pmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}$

b Two successive anticlockwise rotations about the origin by an angle θ are equivalent to a single anticlockwise rotation by an angle 2θ.

© Pearson Education Ltd 2017. Copying permitted for purchasing institution only. This material is not copyright free.