Exercise 9C

1 $\mathbf{a}.\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta = 3\times 3\times \cos 60^\circ$

$= 3\times 3\times \frac{1}{2} = \frac{9}{2}$

2 a $\mathbf{a}.\mathbf{b} = \begin{pmatrix}5\\2\\3\end{pmatrix}.\begin{pmatrix}2\\-1\\-2\end{pmatrix} = 10-2-6 = 2$

b $\mathbf{a}.\mathbf{b} = \begin{pmatrix}10\\-7\\4\end{pmatrix}.\begin{pmatrix}3\\-5\\-12\end{pmatrix} = 30+35-48 = 17$

c $\mathbf{a}.\mathbf{b} = \begin{pmatrix}1\\1\\-1\end{pmatrix}.\begin{pmatrix}-1\\-1\\4\end{pmatrix} = -1-1-4 = -6$

d $\mathbf{a}.\mathbf{b} = \begin{pmatrix}2\\0\\-1\end{pmatrix}.\begin{pmatrix}6\\-5\\-8\end{pmatrix} = 12+0+8 = 20$

e $\mathbf{a}.\mathbf{b} = \begin{pmatrix}0\\3\\9\end{pmatrix}.\begin{pmatrix}1\\-12\\-4\end{pmatrix} = 0+36-36 = 0$

3 a $\mathbf{a}.\mathbf{b} = \begin{pmatrix}3\\7\end{pmatrix}.\begin{pmatrix}5\\1\end{pmatrix} = 15+7 = 22$

$|\mathbf{a}| = \sqrt{3^2+7^2} = \sqrt{58}$

$|\mathbf{b}| = \sqrt{5^2+1^2} = \sqrt{26}$

$\sqrt{58}\sqrt{26}\cos\theta = 22$

$\cos\theta = \frac{22}{\sqrt{58}\sqrt{26}}$

$\theta = 55.5^\circ$ (1 d.p.)

b $\mathbf{a}.\mathbf{b} = \begin{pmatrix}2\\-5\end{pmatrix}.\begin{pmatrix}6\\3\end{pmatrix} = 12-15 = -3$

$|a| = \sqrt{2^2+(-5)^2} = \sqrt{29}$

$|b| = \sqrt{6^2+3^2} = \sqrt{45}$

$\sqrt{29}\sqrt{45}\cos\theta = -3$

$\cos\theta = -\frac{-3}{\sqrt{29}\sqrt{45}}$

$\theta = 94.8^\circ$ (1 d.p.)

3 c $\mathbf{a}.\mathbf{b} = \begin{pmatrix}1\\-7\\8\end{pmatrix}.\begin{pmatrix}12\\2\\1\end{pmatrix} = 12-14+8+6$

$|\mathbf{a}| = \sqrt{1^2+(-7)^2+8^2} = \sqrt{114}$

$|\mathbf{b}| = \sqrt{12^2+2^2+1^2} = \sqrt{149}$

$\sqrt{114}\sqrt{149}\cos\theta = 6$

$\cos\theta = \frac{6}{\sqrt{114}\sqrt{149}}$

$\theta = 87.4^\circ$ (1 d.p.)

d $\mathbf{a}.\mathbf{b} = \begin{pmatrix}-1\\-1\\5\end{pmatrix}.\begin{pmatrix}11\\-3\\4\end{pmatrix} = -11+3+20 = 12$

$|\mathbf{a}| = \sqrt{(-1)^2+(-1)^2+5^2} = \sqrt{27}$

$|\mathbf{b}| = \sqrt{11^2+(-3)^2+4^2} = \sqrt{146}$

$\sqrt{27}\sqrt{146}\cos\theta = 12$

$\cos\theta = \frac{12}{\sqrt{27}\sqrt{146}}$

$\theta = 79.0^\circ$ 1 (d.p.)

e $\mathbf{a}.\mathbf{b} = \begin{pmatrix}6\\-7\\12\end{pmatrix}.\begin{pmatrix}-2\\1\\1\end{pmatrix} = -12-7+12 = -7$

$|\mathbf{a}| = \sqrt{6^2+(-7)^2+12^2} = \sqrt{229}$

$|\mathbf{b}| = \sqrt{(-2)^2+1^2+1^2} = \sqrt{6}$

$\sqrt{229}\sqrt{6}\cos\theta = -7$

$\cos\theta = \frac{-7}{\sqrt{229}\sqrt{6}}$

$\theta = 100.9^\circ$ (1 d.p.)

3 f $\mathbf{a}.\mathbf{b}=\begin{pmatrix}4\\0\\5\end{pmatrix}.\begin{pmatrix}6\\-2\\0\end{pmatrix}=24+0+0+24$

$|\mathbf{a}|=\sqrt{4^2+5^2}=\sqrt{41}$

$|\mathbf{b}|=\sqrt{6^2+(-2)^2}=\sqrt{40}$

$\sqrt{41}\sqrt{40}\cos\theta=24$

$\cos\theta=\dfrac{24}{\sqrt{41}\sqrt{40}}$

$\theta=53.7°$ (1 d.p.)

g $\mathbf{a}.\mathbf{b}=\begin{pmatrix}-5\\2\\-3\end{pmatrix}.\begin{pmatrix}2\\-2\\-11\end{pmatrix}=-10-4+33=19$

$|\mathbf{a}|=\sqrt{(-5)^2+2^2+(-3)^2}=\sqrt{38}$

$|\mathbf{b}|=\sqrt{2^2-(-2)^2+11^2}=\sqrt{129}$

$\sqrt{38}\sqrt{129}\cos\theta=19$

$\cos\theta=\dfrac{19}{\sqrt{38}\sqrt{129}}$

$\theta=74.3°$ (1 d.p.)

h $\mathbf{a}.\mathbf{b}=\begin{pmatrix}1\\1\\1\end{pmatrix}.\begin{pmatrix}1\\-1\\1\end{pmatrix}=1-1+1=1$

$|\mathbf{a}|=\sqrt{1^2+1^2+1^2}=\sqrt{3}$

$|\mathbf{b}|=\sqrt{1^2+(-1)^2+1^2}=\sqrt{3}$

$\sqrt{3}\sqrt{3}\cos\theta=1$

$\cos\theta=\dfrac{1}{\sqrt{3}\sqrt{3}}=\dfrac{1}{3}$

$\theta=70.5°$ (1 d.p.)

4 a $\begin{pmatrix}3\\5\end{pmatrix}.\begin{pmatrix}\lambda\\6\end{pmatrix}=3\lambda+30=0$

$\Rightarrow 3\lambda=-30$

$\Rightarrow \lambda=-10$

b $\begin{pmatrix}2\\6\\-1\end{pmatrix}\begin{pmatrix}\lambda\\-4\\-14\end{pmatrix}=2\lambda-24+14=0$

$\Rightarrow 2\lambda=10$

$\Rightarrow \lambda=5$

c $\begin{pmatrix}3\\\lambda\\-8\end{pmatrix}.\begin{pmatrix}7\\-5\\1\end{pmatrix}=21-5\lambda-8=0$

$\Rightarrow 5\lambda=13$

$\Rightarrow \lambda=2\dfrac{3}{5}$

d $\begin{pmatrix}9\\-3\\5\end{pmatrix}.\begin{pmatrix}\lambda\\\lambda\\3\end{pmatrix}=9\lambda-3\lambda+15=0$

$\Rightarrow 6\lambda=-15$

$\Rightarrow \lambda=-2\dfrac{1}{2}$

e $\begin{pmatrix}\lambda\\3\\-2\end{pmatrix}.\begin{pmatrix}\lambda\\\lambda\\5\end{pmatrix}=\lambda^2+3\lambda-10=0$

$\Rightarrow (\lambda+5)(\lambda-2)=0$

$\Rightarrow \lambda=-5$ or $\lambda=2$

5 a Using $\mathbf{a}=9\mathbf{i}-5\mathbf{j}+3\mathbf{k}$ and $\mathbf{b}=\mathbf{i}$,

$\mathbf{a}.\mathbf{b}=\begin{pmatrix}9\\-5\\3\end{pmatrix}.\begin{pmatrix}1\\0\\0\end{pmatrix}=9$

$|\mathbf{a}|=\sqrt{9^2+(-5)^2+3^2}=\sqrt{115}$

$|\mathbf{b}|=1$

$\sqrt{115}\cos\theta=9$

$\cos\theta=\dfrac{9}{\sqrt{115}}$

$\theta=32.9°$

b Using $\mathbf{a}=9\mathbf{i}-5\mathbf{j}+3\mathbf{k}$ and $\mathbf{b}=\mathbf{j}$,

$\mathbf{a}.\mathbf{b}=\begin{pmatrix}9\\-5\\3\end{pmatrix}.\begin{pmatrix}0\\1\\0\end{pmatrix}=-5$

$|\mathrm{a}|=\sqrt{115}, |\mathrm{b}|=1$

$\sqrt{115}\cos\theta=-5$

$\cos\theta=\dfrac{-5}{\sqrt{115}}$

$\theta=117.8°$

6 a Using $\mathbf{a} = \mathbf{i} + 11\mathbf{j} - 4\mathbf{k}$ and $\mathbf{b} = \mathbf{j}$,

$$\mathbf{a}.\mathbf{b} = \begin{pmatrix} 1 \\ 11 \\ -4 \end{pmatrix} . \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 11$$

$$|\mathbf{a}| = \sqrt{1^2 + 11^2 + (-4)^2} = \sqrt{138}$$

$$|\mathbf{b}| = 1$$

$$\sqrt{138}\cos\theta = 11$$

$$\cos\theta = \frac{11}{\sqrt{138}}$$

$$\theta = 20.5°$$

b Using $\mathbf{a} = \mathbf{i} + 11\mathbf{j} - 4\mathbf{k}$ and $\mathbf{b} = \mathbf{k}$,

$$\mathbf{a}.\mathbf{b} = \begin{pmatrix} 1 \\ 11 \\ -4 \end{pmatrix} . \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = -4$$

$$|\mathbf{a}| = \sqrt{138},\ |\mathrm{b}| = 1$$

$$\sqrt{138}\cos\theta = -4$$

$$\cos\theta = \frac{-4}{\sqrt{138}}$$

$$\theta = 109.9°$$

7 Using $\mathbf{a} = \mathbf{i} + \mathbf{j} + \mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} + \mathbf{j} + \mathbf{k}$,

$$\mathbf{a}.\mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} . \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = 2 + 1 + 1 = 4$$

$$|\mathbf{a}| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$$

$$|\mathbf{b}| = \sqrt{2^2 + 1^2 + 1^2} = \sqrt{6}$$

$$\sqrt{3}\sqrt{6}\cos\theta = 4$$

$$\cos\theta = \frac{4}{\sqrt{3}\sqrt{6}} = \frac{4}{\sqrt{3}\sqrt{3}\sqrt{2}} = \frac{4}{3\sqrt{2}}$$

$$= \frac{4}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{4\sqrt{2}}{6} = \frac{2\sqrt{2}}{3}$$

8 Using $\mathbf{a} = \mathbf{i} + 3\mathbf{j}$ and $\mathbf{b} = \mathbf{j} + \lambda\mathbf{k}$,

$$\mathbf{a}.\mathbf{b} = \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix} . \begin{pmatrix} 0 \\ 1 \\ \lambda \end{pmatrix} = 0 + 3 + 0 = 3$$

$$|\mathbf{a}| = \sqrt{1^2 + 3^2} = \sqrt{10}$$

$$|\mathbf{b}| = \sqrt{1^2 + \lambda^2} = \sqrt{1 + \lambda^2}$$

$$\sqrt{10}\sqrt{1 + \lambda^2}\cos 60° = 3$$

$$\sqrt{1 + \lambda^2} = \frac{3}{\sqrt{10}\cos 60°} = \frac{6}{\sqrt{10}}$$

Squaring both sides:

$$1 + \lambda^2 = \frac{36}{10}$$

$$\lambda^2 = \frac{26}{10} = \frac{13}{5}$$

$$\lambda = \pm\sqrt{\frac{13}{5}}$$

9 a Let the required vector be $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$.
Then

$$\begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix} . \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0 \text{ and } \begin{pmatrix} 5 \\ -2 \\ -1 \end{pmatrix} . \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$$

$$x + y - 3z = 0$$

$$5x - 2y - z = 0$$

Let $z = 1$:

$x + y = 3 \quad (\times 2)$

$5x - 2y = 1$

$2x + 2y = 6$

$5x - 2y = 1$

Adding, $7x = 7 \Rightarrow x = 1$

$1 + y = 3$, so $y = 2$

So $x = 1, y = 2$ and $z = 1$

A possible vector is $\mathbf{i} + 2\mathbf{j} + \mathbf{k}$

9 b Let the required vector be $x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$

Then $\begin{pmatrix}2\\3\\-4\end{pmatrix}.\begin{pmatrix}x\\y\\z\end{pmatrix}=0$ and $\begin{pmatrix}1\\-6\\3\end{pmatrix}.\begin{pmatrix}x\\y\\z\end{pmatrix}=0$

$2x+3y-4z=0$

$x-6y+3z=0$

Let $z=1$:

$2x+3y=4$

$x-6y=-3\quad(\times 2)$

$2x+3y=4$

$2x-12y=-6$

Subtracting, $15y=10 \quad\Rightarrow y=\frac{2}{3}$

$2x+2=4, x$ so $x=1$

So $x=1, y=\frac{2}{3}$ and $z=1$

A possible vector is $\mathbf{i}+\frac{2}{3}\mathbf{j}+\mathbf{k}$

Another possible vector is

$3\left(\mathbf{i}+\frac{2}{3}\mathbf{j}+\mathbf{k}\right)=3\mathbf{i}+2\mathbf{j}+3\mathbf{k}$

c Let the required vector be $x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$

Then

$\begin{pmatrix}4\\-4\\-1\end{pmatrix}.\begin{pmatrix}x\\y\\z\end{pmatrix}=0$ and $\begin{pmatrix}-2\\-9\\6\end{pmatrix}.\begin{pmatrix}x\\y\\z\end{pmatrix}=0$

$4x-4y-z=0$

$-2x-9y+6z=0$

Let $z=1$:

$4x-4y=1$

$-2x-9y=-6\quad(\times 2)$

$4x-4y=1$

$-4x-18y=-12$

Adding, $-22y=-11 \quad\Rightarrow\quad y=\frac{1}{2}$

$4x-2=1, \text{so } x=\frac{3}{4}$

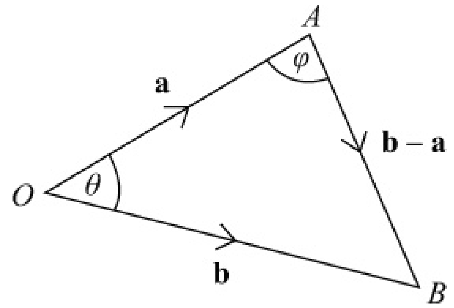
So $x=\frac{3}{4}, y=\frac{1}{2}$ and $z=1$

A possible vector is $\frac{3}{4}\mathbf{i}+\frac{1}{2}\mathbf{j}+\mathbf{k}$

Another possible vector is

$4\left(\frac{3}{4}\mathbf{i}+\frac{1}{2}\mathbf{j}+\mathbf{k}\right)=3\mathbf{i}+2\mathbf{j}+4\mathbf{k}$

10



Using **a** and **b** to find θ:

$\mathbf{a}.\mathbf{b}=\begin{pmatrix}2\\5\\1\end{pmatrix}.\begin{pmatrix}6\\1\\-2\end{pmatrix}=12+5-2=15$

$|\mathbf{a}|=\sqrt{2^2+5^2+1^2}=\sqrt{30}$

$|\mathbf{b}|=\sqrt{6^2+1^2+(-2)^2}=\sqrt{41}$

$\sqrt{30}\sqrt{41}\cos\theta=15$

$\cos\theta=\frac{15}{\sqrt{30}\sqrt{41}}$

$\theta=64.7°$

Using **AO** and **AB** to find ϕ:

$\mathbf{AO}=-\mathbf{a}=\begin{pmatrix}-2\\-5\\-1\end{pmatrix}$

$\mathbf{AB}=\mathbf{b}-\mathbf{a}=\begin{pmatrix}6\\1\\-2\end{pmatrix}-\begin{pmatrix}2\\5\\1\end{pmatrix}=\begin{pmatrix}4\\-4\\-3\end{pmatrix}$

$(-\mathbf{a}).(\mathbf{b}-\mathbf{a})=\begin{pmatrix}-2\\-5\\-1\end{pmatrix}.\begin{pmatrix}4\\-4\\-3\end{pmatrix}$

$=-8+20+3=15$

$|-a|=\sqrt{(-2)^2+(-5)^2+(-1)^2}=\sqrt{30}$

$|b-a|=\sqrt{4^2+(-4)^2+(-3)^2}=\sqrt{41}$

$\cos\phi=\frac{15}{\sqrt{30}\sqrt{41}}$

$\phi=64.7°\quad$ (1 d.p.)

(Since $|\mathrm{b}-\mathrm{a}|=|\mathrm{b}|$, AB = OB, so the triangle is isosceles).

$\angle\mathbf{OBA}=180°-64.7°-64.7°$

$=50.6°$ (1 d.p.)

Angles are

64.7°, 64.7° and 50.6° (all 1 d.p.)

11 a $|\overrightarrow{AB}| = \sqrt{(2-1)^2+(7-3)^2+(-3-1)^2}$
$= \sqrt{33}$
$|\overrightarrow{BC}| = \sqrt{(4-2)^2+(-5-7)^2+(2-(-3))^2}$
$= \sqrt{173}$

b $\overrightarrow{AB}\cdot\overrightarrow{BC} = \begin{pmatrix}1\\4\\-4\end{pmatrix}\cdot\begin{pmatrix}2\\-12\\5\end{pmatrix} = 1(2)+4(-12)-4(5)$
$= -66$
$\therefore\ -66 = \sqrt{33}\sqrt{173}\cos\theta$
$\Rightarrow \cos\theta = -\dfrac{66}{\sqrt{33}\sqrt{173}}$
$\therefore \theta = 180^o - \cos^{-1}\left(-\dfrac{66}{\sqrt{33}\sqrt{173}}\right)$
$= 29.1^o$ (1d.p.) $=\overrightarrow{BA}\cdot\overrightarrow{BC}$

12 a $\overrightarrow{OA} = \begin{pmatrix}7\\4\\4\end{pmatrix},\ \overrightarrow{OB} = \begin{pmatrix}2\\2\\1\end{pmatrix}$

$\overrightarrow{OA}\cdot\overrightarrow{OB} = \begin{pmatrix}7\\4\\4\end{pmatrix}\cdot\begin{pmatrix}2\\2\\1\end{pmatrix} = 7(2)+4(2)+4(1)$
$= 26$
$= \sqrt{7^2+4^2+4^2}\sqrt{2^2+2^2+1^2}\cos\theta$
$\Rightarrow \cos\theta = \dfrac{26}{27}$

b $\cos\theta = \dfrac{26}{27}$
$\Rightarrow \cos^2\theta = 1-\sin^2\theta = \left(\dfrac{26}{27}\right)^2 = \dfrac{676}{729}$
$\Rightarrow \sin^2\theta = \dfrac{53}{729} \Rightarrow \sin\theta = \dfrac{\sqrt{53}}{27}$
since $0^o < \theta < 180^o$.
$\therefore$ The area of the triangle ΔAOB is
$\dfrac{1}{2}|\overrightarrow{OA}||\overrightarrow{OB}|\sin\theta = \dfrac{1}{2}(9)(3)\sin\theta = \dfrac{\sqrt{53}}{2}$

13 Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OP} = \mathbf{p}$
Then $\overrightarrow{OB} = -\mathbf{a}$
$\therefore \overrightarrow{AP}\cdot\overrightarrow{PB} = \left(\overrightarrow{AO}+\overrightarrow{OP}\right)\cdot\left(\overrightarrow{PO}+\overrightarrow{OB}\right)$
$= (-\mathbf{a}+\mathbf{p})\cdot(-\mathbf{p}-\mathbf{a})$
$= \mathbf{a}\cdot\mathbf{p}+\mathbf{a}\cdot\mathbf{a}-\mathbf{p}\cdot\mathbf{p}-\mathbf{p}\cdot\mathbf{a}$
$= 0$
since $\mathbf{a}\cdot\mathbf{p} = \mathbf{p}\cdot\mathbf{a}$ and $\mathbf{a}\cdot\mathbf{a} = \mathbf{p}\cdot\mathbf{p}$ because both A and P lie on the circle

14 a $\overrightarrow{CA} = -\overrightarrow{OC}+\overrightarrow{OA} = \begin{pmatrix}-6+5\\1-1\\-4+0\end{pmatrix} = \begin{pmatrix}-1\\0\\-4\end{pmatrix}$

$\overrightarrow{CB} = -\overrightarrow{OC}+\overrightarrow{OB} = \begin{pmatrix}-6+2\\1+4\\-4+10\end{pmatrix} = \begin{pmatrix}-4\\5\\6\end{pmatrix}$

b $\overrightarrow{CA}\cdot\overrightarrow{CB} = \begin{pmatrix}-1\\0\\-4\end{pmatrix}\cdot\begin{pmatrix}-4\\5\\6\end{pmatrix} = 4-24 = -20$
$|\overrightarrow{CA}| = \sqrt{(-1)^2+(-4)^2} = \sqrt{17}$
$|\overrightarrow{CB}| = \sqrt{(-4)^2+5^2+6^2} = \sqrt{77}$

$\therefore \cos\theta = -\dfrac{20}{\sqrt{77}\sqrt{17}}$
$\Rightarrow \cos^2\theta = 1-\sin^2\theta = \dfrac{400}{1309}$
$\Rightarrow \sin\theta = \sqrt{\dfrac{909}{1309}}$
$\therefore$ Area is $\dfrac{1}{2}|\overrightarrow{CA}||\overrightarrow{CB}|\sin\theta$
$= \dfrac{1}{2}\sqrt{1309}\sqrt{\dfrac{909}{1309}} = \dfrac{3\sqrt{101}}{2}$

14 c The following pairings or parallel sides are possible:
1) AD&BC, 2)BD&AC, 3)CD&AB

$$1):\ \overrightarrow{OD}=\overrightarrow{OA}+\overrightarrow{BC}=\begin{pmatrix}9\\-6\\-6\end{pmatrix}$$

$\Rightarrow D(9,-6,-6)$

$$2):\ \overrightarrow{OD}=\overrightarrow{OB}+\overrightarrow{CA}=\begin{pmatrix}1\\4\\6\end{pmatrix}$$

$\Rightarrow D(1,4,6)$

$$3):\ \overrightarrow{OD}=\overrightarrow{OC}+\overrightarrow{AB}=\begin{pmatrix}3\\4\\6\end{pmatrix}$$

$\Rightarrow D(3,4,6)$

$\therefore (9,2,-6);(1,4,6);(3,4,6)$ all possible coordinates of D

d $3\sqrt{101}$

15 a $\overrightarrow{PQ}=\begin{pmatrix}-3\\6\\-2\end{pmatrix}$, $\overrightarrow{QR}=\begin{pmatrix}2\\-2\\-9\end{pmatrix}$

$$\therefore\ \overrightarrow{PQ}\cdot\overrightarrow{QR}=\begin{pmatrix}-3\\6\\-2\end{pmatrix}\cdot\begin{pmatrix}2\\-2\\-9\end{pmatrix}=-6-12+18=0$$

$\therefore PQ$ is perpendicular to QR

b Since PQ is perpendicular to QR, the three points lie on a semi-circle with diameter PR. Therefore the centre of the circle containing the three points is the midpoint of PR:

$$\overrightarrow{OP}+\frac{1}{2}\overrightarrow{PR}=\begin{pmatrix}1\\-1\\6\end{pmatrix}+\frac{1}{2}\begin{pmatrix}-1\\4\\-11\end{pmatrix}=\begin{pmatrix}\frac{1}{2}\\1\\\frac{1}{2}\end{pmatrix}$$

$\therefore$ Centre $\left(\frac{1}{2},1,\frac{1}{2}\right)$

$$\text{Radius}=\frac{1}{2}|\overrightarrow{PR}|\ =\frac{1}{2}\sqrt{(-1)^2+4^2+(-11)^2}$$

$$=\frac{\sqrt{138}}{2}$$

Challenge

1 $\mathbf{a}\cdot\mathbf{b}=|\mathbf{a}||\mathbf{b}|\cos\theta=|\mathbf{b}||\mathbf{a}|\cos\theta=\mathbf{b}\cdot\mathbf{a}$

2 a i **a**.(**b** + **c**) = |**a**||**b** + **c**| cosθ, but cosθ $=\dfrac{PQ}{|\mathbf{b}+\mathbf{c}|}$

so a.(b + c) = |a| × PQ

ii **a.b** = |**a**||**b**| cosα, but cosα = $\dfrac{PR}{|\mathbf{b}|}$

so **a.b** = |**a**| × PR

iii **a.c** = |**a**||**c**| cosβ, but cosβ = $\dfrac{MN}{|\mathbf{c}|}=\dfrac{RQ}{|\mathbf{c}|}$

so **a.c** = |**a**| × RQ

b **a.(b + c) = |a|** × PQ = |**a**| × (PR + RQ)
= (|**a**| × PR) + (|**a**| × RQ) = **a.b** + **a.c**;
so **a.(b + c)** = **a.b** + **a.c**