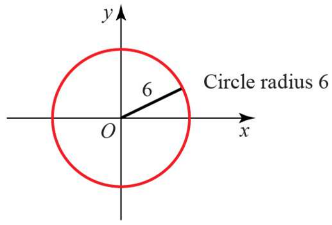
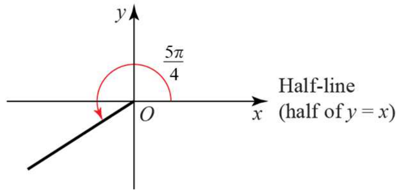
Polar coordinates 5B

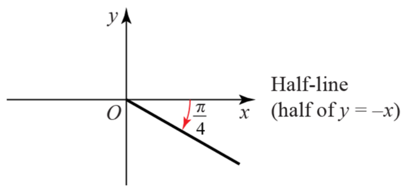
1 a $r = 6$



b $\theta = \dfrac{5\pi}{4}$



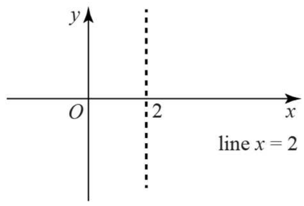
c $\theta = -\dfrac{\pi}{4}$



d $r = 2\sec\theta$

$\Rightarrow \quad r\cos\theta = 2$

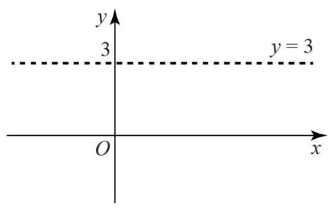
i.e. $\quad x = 2$



e $r = 3\operatorname{cosec}\theta$

$\Rightarrow \quad r\sin\theta = 3$

i.e. $\quad y = 3$



1 f

$$r = 2\sec\left(\theta - \frac{\pi}{3}\right)$$

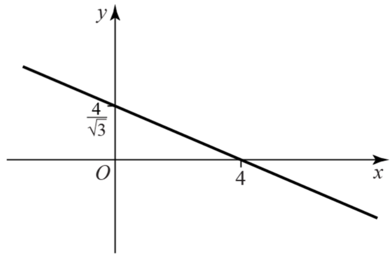
$$r\cos\left(\theta - \frac{\pi}{3}\right) = 2$$

$$\Rightarrow \quad r\cos\theta\cos\frac{\pi}{3} + r\sin\theta\sin\frac{\pi}{3} = 2$$

$$\Rightarrow \quad \frac{x}{2} + y\frac{\sqrt{3}}{2} = 2$$

$$x + y\sqrt{3} = 4$$

or $$y = \frac{4}{\sqrt{3}} - \frac{1}{\sqrt{3}}x$$



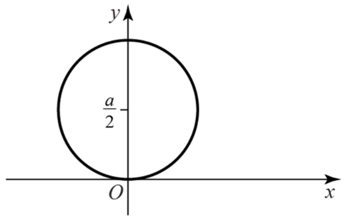
g

$$r = a\sin\theta$$

$$\Rightarrow \quad r^2 = ar\sin\theta$$

$$x^2 + y^2 = ay$$

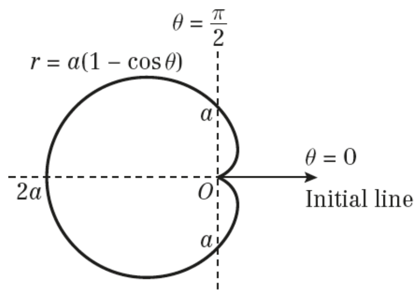
$$x^2 + \left(y - \frac{a}{2}\right)^2 = \frac{a^2}{4}$$



Circle centre $\left(0, \frac{a}{2}\right)$ radius $\frac{a}{2}$

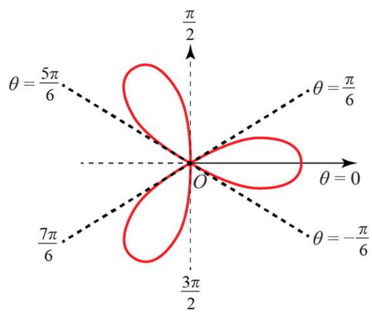
h $r = a(1 - \cos\theta)$

$\boldsymbol{\theta}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\boldsymbol{r}$	0	a	$2a$	a	0



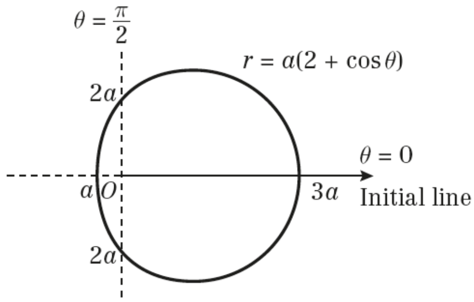
i $r = a\cos 3\theta$

$\boldsymbol{\theta}$	0	$\frac{\pi}{6}$	$-\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$
$\boldsymbol{r}$	a	0	0	0	a	0	0	a	0



1 j $r = a(2+\cos\theta)$

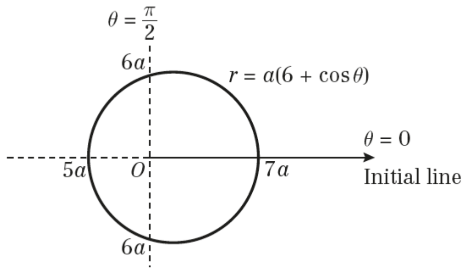
$\boldsymbol{\theta}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\boldsymbol{r}$	$3a$	$2a$	a	$2a$	$3a$



$2 = 2 \times 1 \quad \therefore \quad$ no dimple.

k $r = a(6+\cos\theta)$

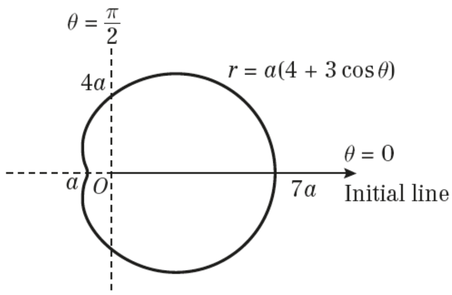
$\boldsymbol{\theta}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\boldsymbol{r}$	$7a$	$6a$	$5a$	$6a$	$7a$



$6 > 2 \times 1 \quad \therefore \quad$ no dimple.

l $r = a(4+3\cos\theta)$

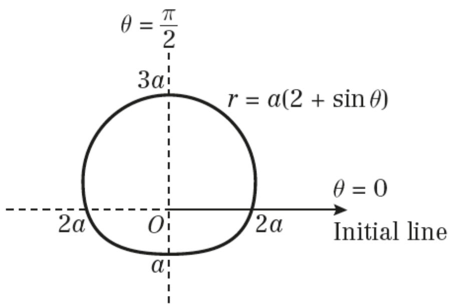
$\boldsymbol{\theta}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\boldsymbol{r}$	$7a$	$4a$	a	$4a$	$7a$



$4 < 2 \times 3 \quad \therefore \quad$ a dimple at $\theta = \pi$.

m $r = a(2+\sin\theta)$

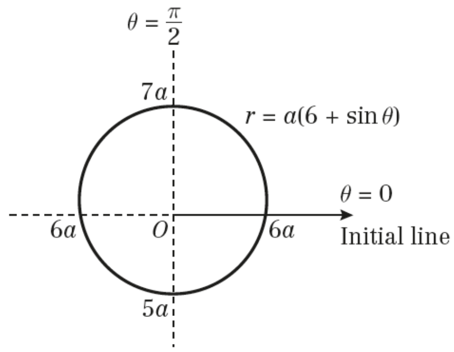
$\boldsymbol{\theta}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\boldsymbol{r}$	$2a$	$3a$	$2a$	a	$2a$



$2 = 2 \times 1$ so no dimple

1 n $r = a(6 + \sin\theta)$

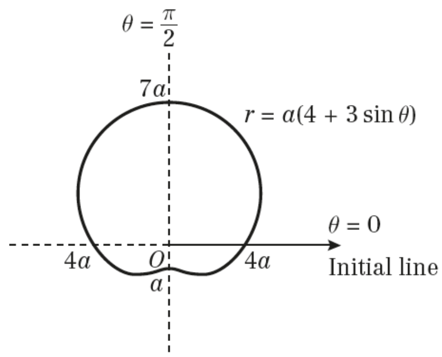
$\boldsymbol{\theta}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\boldsymbol{r}$	$6a$	$7a$	$6a$	$5a$	$6a$



$6 > 2 \times 1$ so no dimple

o $r = a(4 + 3\sin\theta)$

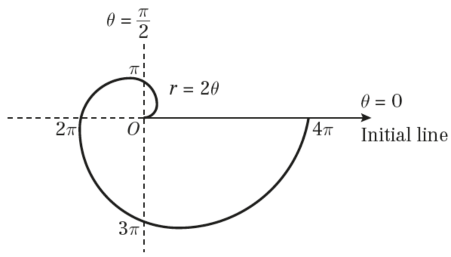
$\boldsymbol{\theta}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\boldsymbol{r}$	$4a$	$7a$	$4a$	a	$4a$



$4 < 2 \times 3 \therefore$ there is a dimple at $\theta = \frac{3\pi}{2}$

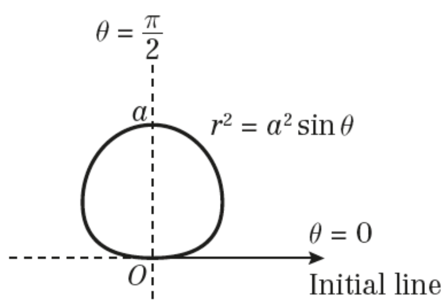
p $r = 2\theta$

$\boldsymbol{\theta}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\boldsymbol{r}$	0	π	2π	3π	4π



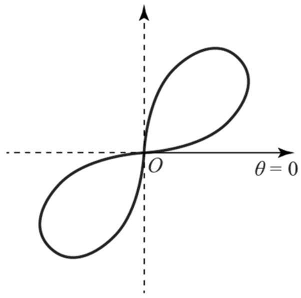
q $r^2 = a^2\sin\theta$

$\boldsymbol{\theta}$	0	$\frac{\pi}{2}$	π
$\boldsymbol{r}$	0	a	0



1 r $r^2 = a^2 \sin 2\theta$

$\boldsymbol{\theta}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$
$\boldsymbol{r}$	0	a	0	0	a	0

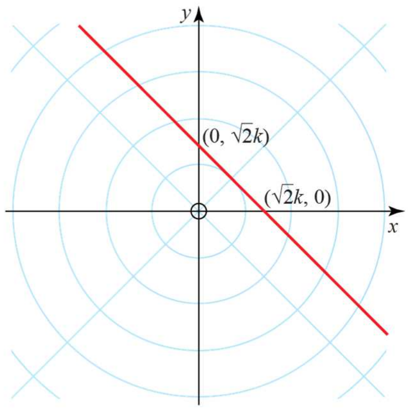


2 First we rearrange by multiplying both sides of $r = k\sec\left(\frac{\pi}{4} - \theta\right)$ by $\cos\left(\frac{\pi}{4} - \theta\right)$ to obtain $r\cos\left(\frac{\pi}{4} - \theta\right) = k$.

We then use the compound angle formula and obtain $r\cos\left(\frac{\pi}{4}\right)\cos\theta + r\sin\left(\frac{\pi}{4}\right)\sin\theta = k$,

which is equivalent to $\frac{r\cos\theta}{\sqrt{2}} + \frac{r\sin\theta}{\sqrt{2}} = k$.

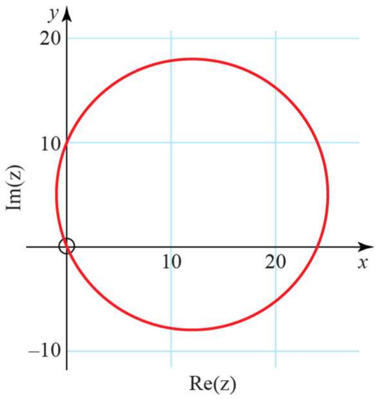
Setting $r\cos\theta = x$ and $r\sin\theta = y$, we obtain $x + y = \sqrt{2}k$,a Cartesian coordinate representation of the equation. Now we plot $y = \sqrt{2}k - x$.



3 a $|z-12-5i|=13$

So $|z-(12+5i)|=13$

This is a circle centred at $z=12+5i$ with radius 13.



b Letting $z=x+iy$ denote a complex number, the Cartesian equation for a circle centred at $(12,5)$ with radius 13 is $(x-12)^2+(y-5)^2=169$.

Converting this equation to polar coordinates we get

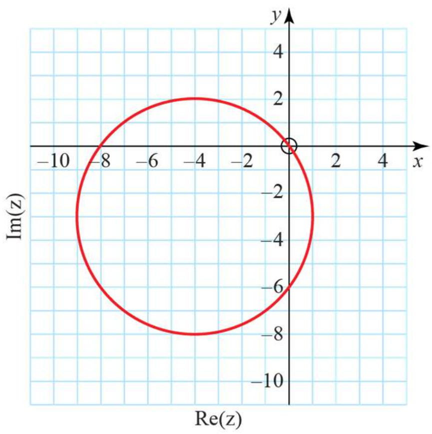
$(r\cos\theta-12)^2+(r\sin\theta-5)^2=169$

Then we expand and simplify to get $r^2-24r\cos\theta-10r\sin\theta=0$ which can be written as $r=24\cos\theta+10\sin\theta$ when $r\neq 0$.

4 a $|z+4+3i|=5$

So $|z-(-4-3i)|=5$

This is a circle centred at $z=-4-3i$ with radius 5.



b Letting $z=x+iy$ denote a complex number, the Cartesian equation for a circle centred at $(-4,-3)$ with radius 5 is $(x+4)^2+(y+3)^2=25$.

Converting this equation to polar coordinates we get

$(r\cos\theta+4)^2+(r\sin\theta+3)^2=25$.

Then we expand and simplify to get $r^2+8r\cos\theta+6r\sin\theta=0$ which can be written as $r=-8\cos\theta-6\sin\theta$ when $r\neq 0$.