Algorithms on graphs 3E

1 a

Initial tables

	P	*Q*	*R*	*S*
P	–	9	5	12
Q	9	–	3	7
R	5	3	–	∞
S	12	7	∞	–

	P	*Q*	*R*	*S*
P	*P*	*Q*	*R*	*S*
Q	*P*	*Q*	*R*	*S*
R	*P*	*Q*	*R*	*S*
S	*P*	*Q*	*R*	*S*

Initial distance table Initial route table

∞: no direct route

1st iteration

	P	*Q*	*R*	*S*
P	–	9	5	12
Q	9	–	3	7
R	5	3	–	17
S	12	7	17	–

	P	*Q*	*R*	*S*
P	*P*	*Q*	*R*	*S*
Q	*P*	*Q*	*R*	*S*
R	*P*	*Q*	*R*	*P*
S	*P*	*Q*	*P*	*S*

2nd iteration

	P	*Q*	*R*	*S*
P	–	9	5	12
Q	9	–	3	7
R	5	3	–	10
S	12	7	10	–

	P	*Q*	*R*	*S*
P	*P*	*Q*	*R*	*S*
Q	*P*	*Q*	*R*	*S*
R	*P*	*Q*	*R*	*Q*
S	*P*	*Q*	*Q*	*S*

3rd iteration

	P	*Q*	*R*	*S*
P	–	8	5	12
Q	8	–	3	7
R	5	3	–	10
S	12	7	10	–

	P	*Q*	*R*	*S*
P	*P*	*R*	*R*	*S*
Q	*R*	*Q*	*R*	*S*
R	*P*	*Q*	*R*	*Q*
S	*P*	*Q*	*Q*	*S*

4th iteration

	P	*Q*	*R*	*S*
P	–	8	5	12
Q	8	–	3	7
R	5	3	–	10
S	12	7	10	–

	P	*Q*	*R*	*S*
P	*P*	*R*	*R*	*S*
Q	*R*	*Q*	*R*	*S*
R	*P*	*Q*	*R*	*Q*
S	*P*	*Q*	*Q*	*S*

b The 4th iteration route table shows that the shortest route from *R* to *S* is via *Q*. It also shows that the shortest route from *R* to *Q* is direct and the shortest route from *Q* to *S* is direct. It follows that the shortest route from *R* to *S* is *RQS*.

2

	A	*B*	*C*	*D*
A	–	17	13	8
B	3	–	4	6
C	7	4	–	5
D	8	9	5	–

3

	J	*K*	*L*	*M*
J	–	6	11	4
K	6	–	9	10
L	11	9	–	7
M	4	8	7	–

4 a The distance table is not symmetrical about the leading diagonal.

b From the table we have BC = 9 is via A, and as AC = 3 this implies that BA = 9 − 3 = 6 = x.
Similar arguments give the entire solution $x = 6, y = 6, z = 23$

5 a The output of Floyd's algorithm gives the shortest distance between every pair of nodes.
The output of Dijkstra's algorithm gives the shortest distance from the start node to every other.

b i

	A	*B*	*C*	*D*	*E*	*F*	*G*
A	–	7	10	∞	∞	4	11
B	7	–	2	5	∞	∞	∞
C	10	2	–	4	3	8	∞
D	∞	5	4	–	6	∞	∞
E	∞	∞	3	6	–	7	9
F	4	∞	8	∞	7	–	6
G	11	∞	∞	∞	9	6	–

ii

	A	*B*	*C*	*D*	*E*	*F*	*G*
A	–	7	10	∞	∞	4	11
B	7	–	2	5	∞	11	18
C	10	2	–	4	3	8	21
D	∞	5	4	–	6	∞	∞
E	∞	∞	3	6	–	7	9
F	4	11	8	∞	7	–	6
G	11	18	21	∞	9	6	–

	A	*B*	*C*	*D*	*E*	*F*	*G*
A	*A*	*B*	*C*	*D*	*E*	*F*	*G*
B	*A*	*B*	*C*	*D*	*E*	*A*	*A*
C	*A*	*B*	*C*	*D*	*E*	*F*	*A*
D	*A*	*B*	*C*	*D*	*E*	*F*	*G*
E	*A*	*B*	*C*	*D*	*E*	*F*	*G*
F	*A*	*A*	*C*	*D*	*E*	*F*	*G*
G	*A*	*A*	*A*	*D*	*E*	*F*	*G*

6 a Jared: Dijkstra's Amy: Floyd's

b 1st iteration (no change)

	A	*B*	*C*	*D*	*E*	*F*
A	–	15	∞	∞	25	∞
B	15	–	22	∞	8	11
C	∞	22	–	14	∞	10
D	∞	∞	14	–	∞	6
E	25	8	∞	∞	–	21
F	∞	11	10	6	21	–

	A	*B*	*C*	*D*	*E*	*F*
A	*A*	*B*	*C*	*D*	*E*	*F*
B	*A*	*B*	*C*	*D*	*E*	*F*
C	*A*	*B*	*C*	*D*	*E*	*F*
D	*A*	*B*	*C*	*D*	*E*	*F*
E	*A*	*B*	*C*	*D*	*E*	*F*
F	*A*	*B*	*C*	*D*	*E*	*F*

2nd iteration

	A	*B*	*C*	*D*	*E*	*F*
A	–	15	37	∞	23	26
B	15	–	22	∞	8	11
C	37	22	–	14	30	10
D	∞	∞	14	–	∞	6
E	23	8	30	∞	–	19
F	26	11	10	6	19	–

	A	*B*	*C*	*D*	*E*	*F*
A	*A*	*B*	*B*	*D*	*B*	*B*
B	*A*	*B*	*C*	*D*	*E*	*F*
C	*B*	*B*	*C*	*D*	*B*	*F*
D	*A*	*B*	*C*	*D*	*E*	*F*
E	*B*	*B*	*B*	*D*	*E*	*B*
F	*B*	*B*	*C*	*D*	*B*	*F*

7 a $(n-1)(n-2) = n^2 - 3n + 2$

b Cubic

c $0.012 \times \left(\frac{100}{30}\right)^3 = 0.44$ seconds (2 d.p.)