The travelling salesman problem 5D

1 a $D_7B_{12}C_8E_{14}A_{19}D = 60$

b $E_8C_{11}A_{13}B_7D_{14}E = 53$
or
$E_8C_{11}D_7B_{13}A_{14}E = 53$

c The better upper bound is 53 since this is lower.

2 a $Z_{10}X_{15}Y_{40}V_{55}W_{30}S_{70}Z = 220$

b $X_{10}Z_{15}V_{40}Y_{45}W_{30}S_{55}X = 195$
$V_{15}Z_{10}X_{15}Y_{45}W_{30}S_{75}V = 190$

c The better upper bound is 190 because it is lower.

3 The application to a single vertex has order n^2, so application to entire network has order $n \times n^2 = n^3$
Hence the estimate for running time is $0.27 \times \left(\frac{20}{12}\right)^3 = 1.25\text{s}$

4 a $R_{150}S_{210}T_{120}U_{180}V_{300}W_{240}R = 1200$ minutes

b $U_{120}S_{150}R_{120}V_{150}T_{180}W_{270}U = 990$
and
$U_{120}T_{150}V_{120}R_{150}S_{240}W_{270}U = 1050$

c $V_{120}R_{150}S_{120}U_{120}T_{180}W_{300}V = 990$
and
$V_{120}R_{150}U_{120}S_{210}T_{180}W_{300}V = 1080$
and
$V_{120}R_{150}U_{120}T_{180}W_{240}S_{210}V = 1020$

d The better upper bound is 990 because it is lower.

5 a Since $x < 126$, x is the smallest entry in column A, and the nearest neighbour route from A is ADEGCBFA, of total weight $639 + x$.

Since $x < 126$, x is the smallest remaining entry in column G after rows B, E, G and D have been deleted, so the nearest neighbour route from B is BEGDAFCB, of total weight $648 + x$.

So $1419 = 639 + x + 648 + x \Rightarrow 2x = 132 \Rightarrow x = 66$

b 705 miles is an upper bound. In the previous part we have found upper bounds $648 + x = 714$ and $639 + x = 705$ The second one is better.