### Critical path analysis Mixed exercise

1 a Activity D depends on activities A and C, whereas activity E depends only on activity A. This shows that a dummy is required.

Activity J depends on activities G and I, whereas activity H depends only on activity G. This shows that a second dummy is required.



**b** Dummy 1 is needed to show *dependency*. *E* and *F* depend on *C* and *B*, but *D* depends on *B* only.

Dummy 2 is needed so that each activity can be *uniquely* represented in terms of its event.





**b** There are *two* critical paths: A - C - G - I - M and A - C - H - K

The critical activities are A, C, G, H, I, K

**c** Total float on *D* is 21-5-14=2Total float on *F* is 42-20-14=8

#### d



e Day 15: *C* Day 25: *G*, *H*, *E*, *F* 

## **Decision Mathematics 1**

- 4 a J depends on H alone, but L depends on H and I.
  - b



**c** Total float on D = 20 - 7 - 8 = 5Total float on E = 20 - 11 - 9 = 0Total float on F = 29 - 5 - 8 = 16



- e  $\frac{95}{38} = 2.5$  so 3 workers
- **f** For example

<u> </u>	2	4	0	0	10	12	14	10	10	2	υ.	22	24	20	20	50	32	. 54	30	3	04
		С				E					H	r			J			Λ	1		
		B				(	G							K							
	A	4			L				F			I		L							
																				Π	

2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40

## **Decision Mathematics 1**

5 a



- **b** A critical path is a continuous path from the source node to the sink node such that a delay in any activity results in a corresponding delay in the whole project.
- **c** The critical paths are: A E H K and A E L.
- **d**  $\frac{\text{Sum of all of the activity times}}{\text{critical time of the project}} = \frac{110}{30}$

Lower bound for number of workers is 4.

- **e** *D*, *H*, *I*, *J*, *L*
- **f** The answers to part **e** show that 5 workers are needed on day 20 in order to complete the project in the minimum time.

g	0	1	2	2 3	4	1 3	5 (	6	7	8	9	10	11	12	13	1	41	51	61	71	81	92	02	12	22	32	24.2	25 :	262	27.2	28.2	<u>93</u> 0
	Γ					A	4							Ē							Н								K			
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# **Decision Mathematics 1**

6 a





**b**  $\frac{83}{28} = 2.96$  so the lower bound is 3.





7 a Minimum time required to complete the project is 17 days.



**b** B-E-H-K and B-E-H-J-L

c

15																		
14																		
13	С	С	С															
12	С	С	С			G					Ι							
1	С	С	С	D	D	G					Ι							
10	С	С	С	D	D	G					Ι	Ι						
9	С	С	С	С	С	G	F	F	F	F	Ι	Ι						
8	Α	Α	A	С	С	G	F	F	F	F	Ι	Ι	Ι					
7	Α	Α	Α	С	С	G	D	D	D	D	D	Ι	Ι					
6	Α	Α	A	С	С	D	D	D	D	D	D	Ι	Ι			L	L	
5	Α	Α	A	С	С	D	Ε	Ε	Ε	Ε	E	E	Ι		J	L	L	
4	В	В	В	В	В	В	Ε	Ε	Ε	Ε	E	E	Ι		J	L	L	
3	В	В	В	В	В	В	Ε	Ε	Ε	E	E	E	H	H	J	L	L	
2	В	В	В	В	В	В	Ε	Ε	Ε	Ε	E	E	H	H	K	Κ	K	
1	В	В	В	В	В	В	Ε	Ε	Ε	Ε	Ε	E	Η	H	K	K	K	
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18

9	Γ			Γ									1		[	<u> </u>		[						<u> </u>	
8	A	A	A									F							Η	Н					,
7	A	A	A	Γ			D	D	D	D	D	F	F	F	F				Η	Η					
6	A	A	A	D	D	D	D	D	D	D	D	G	F	F	F				H	H			L	L	
5	A	A	A	D	D	D	С	C	C	C	C	G	E	E	E	E	E	E	I	I	I	J	L	L	
4	B	B	B	B	B	B	С	C	C	C	С	G	E	E	E	E	E	E	I	1	1	J	L	L	
3	B	B	B	B	B	B	С	С	С	С	С	G	E	E	E	E	E	E	I	I	I	J	L	L	
2	B	B	B	B	B	B	C	C	C	C	C	G	Ε	E	E	E	E	E	I	I	1	K	K	K	
1	B	B	B	B	B	B	C	C	C	С	С	G	E	E	E	E	E	E	1	I	I	K	K	K	
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	ļ

#### Challenge

- **a** A, D, F critical. Total floats are B: 2, C: 8, E: 8
- **b** i Reduce F by 3 days and D by 1 day, total cost £650.
  - ii Reduce F by 3 days, D by 4 days and B by two days. Total cost £1450.
- **c** 27 days. Activity *E* has already been reduced by 2 days, and critical path is now *ACE*. No further reduction possible on critical path so no further reduction in total project time possible.

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d P = 100y_{\rm B} + 200y_{\rm D} + 400y_{\rm E} + 150y_{\rm F}
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e x_1 \ge 12

x_2 + y_B \ge 25

x_3 - x_1 + y_D \ge 15

x_4 - x_1 \ge 8

x_5 - x_4 + y_E \ge 9

x_5 - x_3 + y_F \ge 10

x_3 - x_2 \ge 0

y_B \le 8

y_D \le 10

y_E \le 2

y_F \le 3

x_5 \le 28

x_1, x_2, x_3, x_4, x_5, y_B, y_D, y_E, y_F \ge 0
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