### **A Level Practice paper**

- 1 a Starting with AB means that H cannot be included without repeating either B or A.
  - **b** AHBCEFGDA
  - c Draw a polygon matching the Hamiltonian cycle and look at the inside edges one at a time.



*AB*(I), *AG*, *BF*, *BG*, *CD*, *CF AB*(I), *AG*(I), *BF*, *BG*, *CD*, *CF AB*(I), *AG*(I), *BF*(I), *BG*(I), *CD*(O), *CF AB*(I), *AG*(I), *BF*(I), *BG*(I), *CD*(O), *CF*(I) The graph is planar.



2	a	42	31	36	18	27	33	41	47	12	24	16
		42	36	41	47	33	31	18	27	12	24	16
		42	47	41	36	33	31	18	27	24	16	12
		47	42	41	36	33	31	27	18	24	16	12
		47	42	41	36	33	31	27	24	18	16	12
		47	42	41	36	33	31	27	24	18	16	12

All of the numbers have now been selected as pivots, so the list is in order.

- **b** Using the first fit decreasing algorithm with reels of size 80
  - $47 \rightarrow 1$
  - $42 \rightarrow 2$
  - $41 \rightarrow 3$
  - $36 \rightarrow 4$
  - $33 \rightarrow 1$  making it 80
  - $31 \rightarrow 2$  making it 73
  - $27 \rightarrow 3$  making it 68
  - $24 \rightarrow 4$  making it 60
  - $18 \rightarrow 4$  making it 78
  - $16 \rightarrow 5$
  - $12 \rightarrow 5$  making it 28

			18	
33	31	27	24	12
47	42	41	36	16
5 reels	s are req	uired.		

 $\mathbf{c} \quad 0.034 \times \frac{5000 \log 5000}{800 \log 800} = 0.27 \text{ seconds}$ 

This is only an estimate because the time taken is only approximately proportional to  $n \log n$ 

- **3** a A network is semi-Eulerian if it has exactly two odd nodes.
  - **b** The odd nodes are A, B, C, D, E, F but we can ignore E and B. Possible pairings: AC + DF = 10 + 10 = 20AD + CF = 11 + 9 = 20AF + CD = 2 + 10 = 12So we use AF and CD Minimum length = 72 + 12 = 84 miles
  - **c** *AF*, *CD*
  - **d** This time we can ignore *E* and *C* Possible pairings: AB + DF = 4 + 10 = 14AD + BF = 11 + 5 = 16AF + BD = 2 + 13 = 15So we use *AB* and *DF* The route is extended by 14 - 12 = 2 miles

### 4 a Dijkstra's or Floyd's Algorithm

- **b** Arc AC because of lack of symmetry in the table
- c 1st iteration (no change)

_	5	00	00	00	A	В	С	D
5	_	11	16	00	A	В	С	D
10	11	_	8	6	A	В	С	D
00	16	8	_	10	A	В	С	D
00	00	6	10	_	A	В	С	D
nd ite	eration	l						
_	5	16	21	00	A	В	В	В
5	_	11	16	~~~	A	В	С	D
10	11	_	8	6	A	В	С	D
21	16	8	_	10	В	В	С	D
00	00	6	10	-	A	В	С	D
rd ite	ration							
_	5	16	21	22	A	В	В	В
5	_	11	16	17	A	В	С	D
10	11	_	8	6	A	В	С	D
18	16	8	_	10	C	В	С	D
16	17	6	10	_	С	С	С	D
th ite	ration	(no ch	ange)					
_	5	16	21	22	A	В	В	В
5	_	11	16	17	A	В	С	D
10	11	_	8	6	A	В	С	D
18	16	8	-	10	С	В	С	D
	4 7	6	10	-	С	С	С	D
16	17							
16 oth ite	17 ration	(no ch	ange)					
16 5th ite _	17 ration 5	(no ch 16	ange) 21	22	A	В	В	В
16 5 th ite - 5	17 ration 5 -	(no ch 16 11	ange) 21 16	22 17	A A	B B	B C	B D
16 5 - 5 10	17 ration 5 - 11	(no ch 16 11 -	ange) 21 16 8	22 17 6	A A A	B B B	B C C	B D D
16 5 5 10 18	17 ration 5 - 11 16	(no ch 16 11 - 8	ange) 21 16 8 -	22 17 6 10	A A A C	B B B B	B C C C	B D D D

 d In final iteration look at row A column E – goes through C Now look at row A column C – goes through B So ABCE and length 22

## **Decision Mathematics 1**



So Angie should make 5 mini-packs and 6 mega-packs and the corresponding profit will be £166.

6 a



So 4 workers are required

c Delay the start of activity H by 2 days and the start of activity J by 2 days.

# **Decision Mathematics 1**

- 7 a The purpose of the first stage is to provide a basic feasible solution as a starting point for the second stage.
  - **b** Using slack, surplus and artificial variables we obtain: 2u + u + z + z = 50

 $2x + y + z + s_1 = 50$   $x + 3y + z + s_2 = 60$   $x - s_3 + a_1 = 10$ This gives the first 3 lines of the tableau P - x - 2y - z = 0 gives the 4<sup>th</sup> line We want to maximise  $I = -a_1 = x - s_3 - 10$ 

$$\Rightarrow$$
  $I - x + s_3 = -10$ 

which gives the 5<sup>th</sup> line

b.v.	x	у	z	<b>s</b> <sub>1</sub>	<b>S</b> <sub>2</sub>	<b>S</b> 3	<b>a</b> 1	Value
<b>S</b> 1	2	1	1	1	0	0	0	50
<b>\$</b> 2	1	3	1	0	1	0	0	60
<b>a</b> <sub>1</sub>	1	0	0	0	0	-1	1	10
Р	-1	-2	-1	0	0	0	0	0
Ι	-1	0	0	0	0	1	0	-10

- **c** So setting  $s_3, y, z = 0$  we obtain the first solution of I = 0 when  $a_1 = 0, s_1 = 30, s_2 = 50$
- **d** x now replaces  $a_1$  as a basic variable so we obtain the following table

b.v.	x	У	z	<b>S</b> 1	<b>S</b> <sub>2</sub>	<b>\$</b> 3	Value
<b>s</b> <sub>1</sub>	0	1	1	1	0	2	30
\$ <u>2</u>	0	3	1	0	1	1	50
x	1	0	0	0	0	-1	10
Р	0	-2	-1	0	0	-1	10

The smallest negative value in the bottom row is -2So the *y*-column is the pivot column The smallest value of  $\theta$  is then  $50 \div 3$ So the pivot is the 3 in the *y*-column

### e The iterations are as follows:

$\begin{array}{c c c c c c c c c c c c c c c c c c c $	bv	x		y l	z	<b>S</b> <sub>1</sub>	<b>S</b> <sub>2</sub>	<b>\$</b> 3	Value
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	<b>S</b> 1	0		1	1	1	0	2	30
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	\$2	0		3	1	0	1	1	50
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	x	1		0	0	0	0	-1	10
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	P	0	-	-2	-1	0	0	-1	10
bv         x         y         z         s1         s2         s3         Value operations         Row operations           y         0         1         1         1         0         2         30           y         0         1 $\frac{1}{3}$ 0 $\frac{1}{3}$ $\frac{1}{3}$ $\frac{50}{3}$ $R_2 \div 3$ x         1         0         0         0         -1         10         P           bv         x         y         z         s1         s2         s3         Value 0         Row operations           s1         0         0 $\frac{2}{3}$ 1 $-\frac{1}{3}$ $\frac{5}{3}$ $\frac{40}{3}$ $R_1 - R_2$ y         0         1 $\frac{1}{3}$ 0 $\frac{1}{3}$ $\frac{1}{3}$ $\frac{5}{3}$ $\frac{40}{3}$ $R_1 - R_2$ y         0         1 $\frac{3}{3}$ 0 $\frac{1}{3}$ $\frac{1}{3}$ $\frac{5}{3}$ $\frac{40}{3}$ $R_1 - R_2$ y         0         1 $\frac{3}{2}$ $-\frac{1}{2}$ $\frac{5}{2}$ $20$ $R_1 \times \frac{3}{2}$ y         0         1 <th></th> <th></th> <th></th> <th></th> <th></th> <th></th> <th></th> <th></th> <th></th>									
s1         0         1         1         1         0         2         30           y         0         1 $\frac{1}{3}$ 0 $\frac{1}{3}$ $\frac{1}{3}$ $\frac{50}{3}$ $R_2 \div 3$ x         1         0         0         0         0         -1         10           P         0         -2         -1         0         0         -1         10           bv         x         y         z         s1         s2         s3         Value         Row operations           s1         0         0 $\frac{2}{3}$ 1 $-\frac{1}{3}$ $\frac{5}{3}$ $\frac{40}{3}$ $R_1 - R_2$ y         0         1 $\frac{1}{3}$ 0 $\frac{1}{3}$ $\frac{1}{3}$ $\frac{1}{3}$ $\frac{5}{3}$ $\frac{40}{3}$ $R_1 - R_2$ y         0         1 $\frac{1}{3}$ 0 $\frac{2}{3}$ $-\frac{1}{3}$ $\frac{1}{3}$ $R_1 + 2R_2$ bv         x         y         z         s1         s2         s3         Value         Row operations           z         0         0 $\frac{1}{3}$ $\frac{1}{3}$	bv	x	у	z	<b>S</b> 1	<b>s</b> <sub>2</sub>	\$3	Value	Row operations
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	<b>s</b> <sub>1</sub>	0	1	1	1	0	2	30	
x         1         0         0         0         0         -1         10           P         0         -2         -1         0         0         -1         10           bv         x         y         z         s1         s2         s3         Value         Row operations           s1         0         0 $\frac{2}{3}$ 1 $-\frac{1}{3}$ $\frac{5}{3}$ $\frac{40}{3}$ $R_1 - R_2$ y         0         1 $\frac{1}{3}$ 0 $\frac{1}{3}$ $\frac{1}{3}$ $\frac{50}{3}$ x         1         0         0         0         -1         10 $R_1 - R_2$ y         0         1 $\frac{1}{3}$ 0 $\frac{3}{3}$ $\frac{1}{3}$ $\frac{50}{3}$ x         1         0         0         0 $\frac{2}{3}$ $-\frac{1}{3}$ $\frac{130}{3}$ $R_4 + 2R_2$ bv         x         y         z         s1         s2         s3         Value         Row operations           z         0         0 $-\frac{1}{3}$ $\frac{1}{3}$ $\frac{50}{3}$ $\frac{1}{3}$ $\frac{3}{2}$ $-$	у	0	1	$\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{50}{3}$	$R_2 \div 3$
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	r	1	0	0	0	0	-1	10	
Image: constraint of the second system is second system in the second system is second system in the second system is second system.       Value is second system is second system.       Value is second system is second system is second system is second system is second system.       Reverse is second system is second system is second system.         by       x       y       z       s_1       s_2       s_3       Value is second system is second system is second system.       Reverse is second system is second system is second system.         by       x       y       z       s_1       s_2       s_3       Value is second system.       Reverse is second system.         y       0       1 $\frac{3}{2}$ $-\frac{1}{2}$ $\frac{5}{2}$ 20       y         y		0	-2	-1	0	0	-1	10	
bv         x         y         z         s1         s2         s3         Value         Row operations           s1         0         0 $\frac{2}{3}$ 1 $-\frac{1}{3}$ $\frac{5}{3}$ $\frac{40}{3}$ $R_1 - R_2$ y         0         1 $\frac{1}{3}$ 0 $\frac{1}{3}$ $\frac{1}{3}$ $\frac{50}{3}$ x         1         0         0         0         -1         10 $\frac{7}{3}$ P         0         0 $-\frac{1}{3}$ 0 $\frac{2}{3}$ $-\frac{1}{3}$ $\frac{100}{3}$ $R_4 + 2R_2$ bv         x         y         z         s1         s2         s3         Value         Row operations           z         0         0         1 $\frac{3}{2}$ $-\frac{1}{2}$ $\frac{5}{2}$ 20 $R_1 \times \frac{3}{2}$ y         0         1 $\frac{3}{3}$ 0 $\frac{1}{3}$ $\frac{1}{3}$ $\frac{50}{3}$ x         1         0         0         0         -1         10           P         0         0 $\frac{3}{2}$ $-\frac{1}{2}$ $\frac{5}{2}$ 20 <th>1</th> <th>0</th> <th></th> <th>1</th> <th>U</th> <th>U</th> <th>1</th> <th>10</th> <th></th>	1	0		1	U	U	1	10	
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y       0       1 $\frac{1}{3}$ 0 $\frac{1}{3}$ $\frac{1}{3}$ $\frac{5}{3}$ $\frac{5}{3}$ x       1       0       0       0       0       -1       10         P       0       0 $-\frac{1}{3}$ 0 $\frac{2}{3}$ $-\frac{1}{3}$ $\frac{130}{3}$ $R_4 + 2R_2$ bv       x       y       z       s_1       s_2       s_3       Value       Row operations         z       0       0       1 $\frac{3}{2}$ $-\frac{1}{2}$ $\frac{5}{2}$ 20 $R_1 \times \frac{3}{2}$ y       0       1 $\frac{1}{3}$ 0 $\frac{1}{3}$ $\frac{1}{3}$ $\frac{50}{3}$ y       0       1 $\frac{3}{2}$ $-\frac{1}{2}$ $\frac{5}{2}$ 20 $R_1 \times \frac{3}{2}$ y       0       1 $\frac{1}{3}$ 0 $\frac{1}{3}$ $\frac{1}{3}$ $\frac{50}{3}$ $R_1 \times \frac{3}{2}$ y       0       1 $\frac{1}{3}$ 0 $\frac{2}{3}$ $-\frac{1}{3}$ $\frac{130}{3}$ $R_1 \times \frac{3}{3}$ z       0       0 $-\frac{1}{2}$ $\frac{1}{2}$ $-\frac{1}{2}$ $10$ $R_2 - \frac{1}{3}R_1$ $R_2$	<b>s</b> <sub>1</sub>	0	0	$\frac{2}{3}$	1	$-\frac{1}{3}$	$\frac{5}{3}$	$\frac{40}{3}$	$R_{1} - R_{2}$
x       1       0       0       0       0       -1       10         P       0       0 $-\frac{1}{3}$ 0 $\frac{2}{3}$ $-\frac{1}{3}$ $\frac{130}{3}$ $R_4 + 2R_2$ bv       x       y       z       s_1       s_2       s_3       Value       Row operations         z       0       0       1 $\frac{3}{2}$ $-\frac{1}{2}$ $\frac{5}{2}$ 20 $R_1 \times \frac{3}{2}$ y       0       1 $\frac{3}{2}$ $-\frac{1}{2}$ $\frac{5}{2}$ 20 $R_1 \times \frac{3}{2}$ y       0       1 $\frac{3}{2}$ $-\frac{1}{2}$ $\frac{5}{2}$ 20 $R_1 \times \frac{3}{2}$ y       0       1 $\frac{3}{3}$ 0 $\frac{1}{3}$ $\frac{1}{3}$ $\frac{50}{3}$ $\frac{7}{3}$ x       1       0       0 $\frac{2}{3}$ $-\frac{1}{3}$ $\frac{130}{3}$ $\frac{1300}{3}$ $\frac{130}{3}$ $\frac{130}{3}$ bv       x       y       z       s_1       s_2       s_3       Value       Row operations         z       0       0 $-\frac{1}{2}$ $\frac{1}{2}$ $-\frac{1}{2}$ $10$ $R_2 - \frac{1}{3}R_1$	v	0	1	1	0	1	$\frac{1}{1}$	50	
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bv       x       y       z       s1       s2       s3       Value       Row operations         z       0       0       1 $\frac{3}{2}$ $-\frac{1}{2}$ $\frac{5}{2}$ 20 $R_1 \times \frac{3}{2}$ y       0       1 $\frac{1}{3}$ 0 $\frac{1}{3}$ $\frac{1}{3}$ $\frac{50}{3}$ x       1       0       0       0       -1       10       0         P       0       0 $-\frac{1}{3}$ 0 $\frac{2}{3}$ $-\frac{1}{3}$ $\frac{130}{3}$ bv       x       y       z       s1       s2       s3       Value       Row operations         z       0       0 $-\frac{1}{3}$ 0 $\frac{2}{3}$ $-\frac{1}{3}$ $\frac{130}{3}$ 3         bv       x       y       z       s1       s2       s3       Value       Row operations         z       0       0 $1$ $\frac{3}{2}$ $-\frac{1}{2}$ $\frac{5}{2}$ $20$ $20$ y       0       1       0 $-\frac{1}{2}$ $\frac{1}{2}$ $-\frac{1}{2}$ $10$ $R_2 - \frac{1}{3}R_1$ y       0       1       0 <td>Р</td> <td>0</td> <td>0</td> <td><math>-\frac{1}{3}</math></td> <td>0</td> <td><math>\frac{2}{3}</math></td> <td><math>-\frac{1}{3}</math></td> <td><math>\frac{130}{3}</math></td> <td><math>R_4 + 2R_2</math></td>	Р	0	0	$-\frac{1}{3}$	0	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{130}{3}$	$R_4 + 2R_2$
bv         x         y         z         s1         s2         s3         Value         Row operations           z         0         0         1 $\frac{3}{2}$ $-\frac{1}{2}$ $\frac{5}{2}$ 20 $R_1 \times \frac{3}{2}$ y         0         1 $\frac{1}{3}$ 0 $\frac{1}{3}$ $\frac{1}{3}$ $\frac{50}{3}$ x         1         0         0         0         -1         10 $R_1 \times \frac{3}{2}$ y         0         1 $\frac{1}{3}$ 0 $\frac{1}{3}$ $\frac{1}{3}$ $\frac{50}{3}$ x         1         0         0         0         -1         10 $R_1 \times \frac{3}{2}$ y         0         0 $-\frac{1}{3}$ 0 $\frac{2}{3}$ $-\frac{1}{3}$ $\frac{130}{3}$ $R_1 \times \frac{3}{3}$ bv         x         y         z         s1         s2         s3         Value         Row operations           z         0         0         1 $\frac{3}{2}$ $-\frac{1}{2}$ $\frac{5}{2}$ 20           y         0         1         0 $-\frac{1}{2}$ $\frac{1}{2}$ $-\frac{1}{2}$				I	1		1		L
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y       0       1 $\frac{1}{3}$ 0 $\frac{1}{3}$ $\frac{1}{3}$ $\frac{50}{3}$ x       1       0       0       0       0       -1       10         P       0       0 $-\frac{1}{3}$ 0 $\frac{2}{3}$ $-\frac{1}{3}$ $\frac{130}{3}$ bv       x       y       z       s_1       s_2       s_3       Value       Row operations         z       0       0       1 $\frac{3}{2}$ $-\frac{1}{2}$ $\frac{5}{2}$ 20       Row operations         y       0       1       0 $-\frac{1}{2}$ $\frac{1}{2}$ $-\frac{1}{2}$ $10$ $R_2 - \frac{1}{3}R_1$ y       0       1       0 $-\frac{1}{2}$ $\frac{1}{2}$ $-\frac{1}{2}$ $10$ $R_2 - \frac{1}{3}R_1$ p       0       0       0 $-\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $50$ $R_4 + \frac{1}{3}R_1$	z	0	0		3	1	5	• •	<b>D</b> 3
x       1       0       0       0       0       -1       10         P       0       0 $-\frac{1}{3}$ 0 $\frac{2}{3}$ $-\frac{1}{3}$ $\frac{130}{3}$ bv       x       y       z       s <sub>1</sub> s <sub>2</sub> s <sub>3</sub> Value       Row operations         z       0       0       1 $\frac{3}{2}$ $-\frac{1}{2}$ $\frac{5}{2}$ 20         y       0       1       0 $-\frac{1}{2}$ $\frac{1}{2}$ $-\frac{1}{2}$ 10 $R_2 - \frac{1}{3}R_1$ x       1       0       0 $-\frac{1}{2}$ $\frac{1}{2}$ $-\frac{1}{2}$ 10 $R_2 - \frac{1}{3}R_1$ P       0       0 $0$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $50$ $R_4 + \frac{1}{3}R_1$		Ŭ	0	1	$\frac{1}{2}$	$-\frac{1}{2}$	$\overline{2}$	20	$R_1 \times \frac{-}{2}$
x       1       0       0       0       0       1       10         P       0       0 $-\frac{1}{3}$ 0 $\frac{2}{3}$ $-\frac{1}{3}$ $\frac{130}{3}$ bv       x       y       z       s_1       s_2       s_3       Value       Row operations         z       0       0       1 $\frac{3}{2}$ $-\frac{1}{2}$ $\frac{5}{2}$ 20       Row operations         y       0       1       0 $-\frac{1}{2}$ $\frac{1}{2}$ $-\frac{1}{2}$ $\frac{10}{2}$ $\frac{10}{2}$ y       0       1       0 $-\frac{1}{2}$ $\frac{1}{2}$ $-\frac{1}{2}$ $\frac{10}{2}$ $R_2 - \frac{1}{3}R_1$ x       1       0       0       0       0 $-1$ $12$ $\frac{1}{2}$ $\frac{1}{2}$ $50$ $R_4 + \frac{1}{3}R_1$ P       0       0 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $50$ $R_4 + \frac{1}{3}R_1$	у	0	1	$\frac{1}{\frac{1}{3}}$	$\frac{1}{2}$	$\begin{array}{c} -\frac{1}{2} \\ \frac{1}{3} \end{array}$	$\frac{\overline{2}}{\frac{1}{3}}$	$\frac{20}{\frac{50}{3}}$	$\frac{R_1 \times -2}{2}$
P       0       0 $-\frac{1}{3}$ 0 $\frac{2}{3}$ $-\frac{1}{3}$ $\frac{150}{3}$ bv       x       y       z       s_1       s_2       s_3       Value       Row operations         z       0       0       1 $\frac{3}{2}$ $-\frac{1}{2}$ $\frac{5}{2}$ 20         y       0       1       0 $-\frac{1}{2}$ $\frac{1}{2}$ $-\frac{1}{2}$ $\frac{10}{2}$ $R_2 - \frac{1}{3}R_1$ x       1       0       0       0       0 $-1$ $1$ $2$ $-\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $10$ $R_2 - \frac{1}{3}R_1$ P       0       0       0 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $50$ $R_4 + \frac{1}{3}R_1$	y x	0	0 1 0	$\frac{1}{\frac{1}{3}}$	$\frac{1}{2}$ 0	$\begin{array}{c c} -\underline{-2} \\ \underline{1} \\ \underline{3} \\ 0 \end{array}$	$\frac{\overline{2}}{1}$ $\frac{1}{3}$ $-1$	$\frac{20}{\frac{50}{3}}$	$R_1 \times \frac{1}{2}$
bv         x         y         z         s1         s2         s3         Value         Row operations           z         0         0         1 $\frac{3}{2}$ $-\frac{1}{2}$ $\frac{5}{2}$ 20            y         0         1         0 $-\frac{1}{2}$ $\frac{1}{2}$ $-\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{$	y x	0	0 1 0	$ \begin{array}{c} 1\\ \hline 1\\ \hline 3\\ \hline 0\\ \hline 1 \end{array} $	$\begin{array}{c} \frac{1}{2} \\ 0 \\ 0 \\ \end{array}$	$\begin{array}{c c} -\frac{-}{2} \\ \frac{1}{3} \\ 0 \\ 2 \end{array}$	$\begin{array}{r} \overline{2} \\ \overline{1} \\ \overline{3} \\ \overline{-1} \\ 1 \end{array}$	$   \begin{array}{r}     20 \\     \underline{50} \\     \underline{3} \\     10 \\     130   \end{array} $	$R_1 \times \frac{-}{2}$
bv         x         y         z         s1         s2         s3         Value         Row operations           z         0         0         1 $\frac{3}{2}$ $-\frac{1}{2}$ $\frac{5}{2}$ 20         20           y         0         1         0 $-\frac{1}{2}$ $\frac{1}{2}$ $-\frac{1}{2}$ 20           x         1         0 $-\frac{1}{2}$ $\frac{1}{2}$ $-\frac{1}{2}$ 10 $R_2 - \frac{1}{3}R_1$ x         1         0         0         0 $-1$ 10 $R_4 + \frac{1}{3}R_1$ P         0         0 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $50$ $R_4 + \frac{1}{3}R_1$	у х Р	0 1 0	1 0 0	$ \begin{array}{r} 1\\ \hline 1\\ \hline 3\\ \hline 0\\ \hline -\frac{1}{3}\\ \end{array} $	0 0 0	$\begin{array}{c c} -\frac{-}{2} \\ \frac{1}{3} \\ 0 \\ \frac{2}{3} \end{array}$	$\begin{array}{r} -\frac{1}{2} \\ \frac{1}{3} \\ -1 \\ -\frac{1}{3} \end{array}$	$     \frac{20}{\frac{50}{3}}     \frac{10}{\frac{130}{3}} $	$R_1 \times \frac{1}{2}$
z     0     0     1 $\frac{3}{2}$ $-\frac{1}{2}$ $\frac{5}{2}$ 20       y     0     1     0 $-\frac{1}{2}$ $\frac{1}{2}$ $-\frac{1}{2}$ 10 $R_2 - \frac{1}{3}R_1$ x     1     0     0     0     0     -1     10       P     0     0     0 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $50$ $R_4 + \frac{1}{3}R_1$	у х Р	0	0 1 0 0	$ \begin{array}{c} 1\\ \hline 1\\ \hline 3\\ \hline 0\\ \hline -\frac{1}{3}\\ \end{array} $	$\begin{array}{c} - \\ - \\ 2 \\ 0 \\ 0 \\ 0 \\ \end{array}$	$\begin{array}{c c} -\frac{-}{2} \\ \frac{1}{3} \\ 0 \\ \frac{2}{3} \end{array}$	$\frac{\overline{2}}{\overline{3}}$ $\frac{1}{\overline{3}}$ $-\frac{1}{\overline{3}}$	$   \begin{array}{r}     20 \\     \overline{50} \\     \overline{3} \\     \overline{10} \\     \overline{130} \\     \overline{3} \\   \end{array} $	$R_1 \times \frac{1}{2}$
y     0     1     0 $-\frac{1}{2}$ $\frac{1}{2}$ $-\frac{1}{2}$ 10 $R_2 - \frac{1}{3}R_1$ x     1     0     0     0     0     -1     10       P     0     0     0 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{50}{2}$	y x P bv	0 1 0	0 1 0 0 y	$ \begin{array}{c} 1\\ \frac{1}{3}\\ 0\\ -\frac{1}{3}\\ \mathbf{z} \end{array} $	0 0 0 81	$ \begin{array}{c c} -\frac{-}{2} \\ \frac{1}{3} \\ 0 \\ \frac{2}{3} \\ \hline \mathbf{s}_2 \end{array} $	$ \frac{\overline{2}}{\overline{3}} $ $ -1 $ $ -\frac{1}{\overline{3}} $ $ \mathbf{s}_{3} $	$     \frac{50}{3}     10     \frac{130}{3}     Value $	$\frac{R_1 \times \frac{1}{2}}{Row}$
x         1         0         0         0         0         -1         10           P         0         0         0 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $50$ $R_4 + \frac{1}{3}R_1$	y x P bv z	0 1 0 <b>x</b> 0	0 1 0 0 <b>y</b> 0	$ \begin{array}{c} 1\\ \frac{1}{3}\\ 0\\ -\frac{1}{3}\\ \hline \mathbf{z}\\ 1 \end{array} $	$ \begin{array}{c} \frac{1}{2} \\ 0 \\ 0 \\ 0 \\ \hline \mathbf{s_1} \\ \frac{3}{2} \end{array} $	$ \begin{array}{c c} -\frac{-}{2} \\ \frac{1}{3} \\ 0 \\ \frac{2}{3} \\ \hline \mathbf{s_2} \\ -\frac{1}{2} \end{array} $	$ \frac{\overline{2}}{1} $ $ \frac{1}{3} $ $ -1 $ $ -\frac{1}{3} $ $ \frac{s_3}{5} $ $ \frac{5}{2} $	$20$ $\frac{50}{3}$ $10$ $\frac{130}{3}$ Value $20$	$\frac{R_1 \times \frac{1}{2}}{Row}$
<b>P</b> 0 0 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $50$ $R_4 + \frac{1}{3}R_1$	y x P bv z y	0 1 0 <b>x</b> 0	0 1 0 0 <b>y</b> 0 1	$ \begin{array}{c} 1\\ \frac{1}{3}\\ 0\\ -\frac{1}{3}\\ \hline 1\\ 0\\ \end{array} $	$ \begin{array}{c} \overline{2} \\ 0 \\ 0 \\ 0 \\ \hline \mathbf{s_1} \\ \overline{3} \\ 2 \\ -\frac{1}{2} \end{array} $	$\begin{array}{c c} -\frac{-1}{2} \\ \frac{1}{3} \\ 0 \\ \frac{2}{3} \\ \hline \\ \frac{1}{2} \\ \frac{1}{2} \\ \end{array}$	$ \frac{\overline{2}}{1} $ $ \frac{1}{3} $ $ -1 $ $ -\frac{1}{3} $ $ \frac{s_3}{5} $ $ \frac{5}{2} $ $ -\frac{1}{2} $	$     \frac{20}{\frac{50}{3}}     \frac{50}{3}     \frac{10}{3}     \frac{130}{3}     \overline{3}     $ Value 20 10	$\frac{R_1 \times \frac{1}{2}}{R_2 - \frac{1}{3}R_1}$
	y x P bv z y x	0 1 0 <b>x</b> 0 1	0 1 0 0 <b>y</b> 0 1 0	$ \begin{array}{c} 1\\ \frac{1}{3}\\ 0\\ -\frac{1}{3}\\ \hline \\ \mathbf{z}\\ 1\\ 0\\ 0\\ \hline \\ 0\\ \end{array} $	$ \begin{array}{c} \overline{2} \\ 0 \\ 0 \\ 0 \\ \hline 1 \\ \hline 2 \\ \hline 0 \\ \hline 1 \\ \hline 2 \\ \hline 0 \\ \hline \end{array} $	$ \begin{array}{c c} -\frac{-}{2} \\ \frac{1}{3} \\ 0 \\ \frac{2}{3} \\ \hline  \\  \\  \\  \\  \\  \\  \\  \\  \\  \\  \\  \\  \\ $	$\frac{\overline{2}}{1}$ $\frac{1}{3}$ $-1$ $-\frac{1}{3}$ $\mathbf{s_3}$ $\frac{5}{2}$ $-\frac{1}{2}$ $-1$	$     \begin{array}{r}       20 \\       \frac{50}{3} \\       10 \\       \frac{130}{3} \\       \hline       \overline{3} \\       \hline       Value \\       20 \\       10 \\       10 \\       10   \end{array} $	$\frac{R_{1} \times \frac{1}{2}}{R_{2} - \frac{1}{3}R_{1}}$

So solution is P = 50 when x = 10, y = 10, z = 20