Transportation problems 1B

1 a The total supply is 200, but the total demand is 180. A dummy is needed to absorb this excess, so that total supply equals total demand.

b Adding a dummy column E, where the demand is 20 (the amount by which the supply exceeds the demand) gives:

	A	*B*	*C*	*D*	Dummy	Supply
X	27	33	34	41	0	60
Y	31	29	37	30	0	60
Z	40	32	28	35	0	80
Demand	40	70	50	20	20	200

Finding a solution using the north-west corner method gives:

	A	*B*	*C*	*D*	Dummy	Supply
X	40	20				60
Y		50	10			60
Z			40	20	20	80
Demand	40	70	50	20	20	200

$\text{Cost} = 40\times 27 + 20\times 33 + 50\times 29 + 10\times 37 + 40\times 28 + 20\times 35 + 20\times 0 = 5380\text{ p} = £53.80$

2 a A degenerate solution occurs when the number of cells used in a solution is fewer than the number of rows + number of columns – 1. It will happen when an entry, other than the last, completes both the supply requirement of the row and the demand requirement of the column.

b Units transported are:

	K	*L*	*M*	*N*	Supply
A	20				20
B	5	10			15
C			18	2	20
D				20	20
Demand	25	10	18	22	75

There are 4 rows and 4 columns, so a non-degenerate solution would use 4 + 4 – 1 = 7 cells. This solution only uses 6 cells but fulfils all the supply and demand needs, so it is a degenerate solution.

2 c The table in part **b** has a diagonal 'move' from cell *BL* to cell *CM*. To make the solution in part **b** non-degenerate, a zero must be placed in either cell *CL* or cell *BM* so that 7 cells are filled. These are therefore both non-degenerate initial solutions:

	K	***L***	***M***	***N***	**Supply**
A	20				20
B	5	10			15
C		0	18	2	20
D				20	20
Demand	25	10	18	22	75

or:

	K	***L***	***M***	***N***	**Supply**
A	20				20
B	5	10	0		15
C			18	2	20
D				20	20
Demand	25	10	18	22	75

3 a There are four rows and three columns, so a non-degenerate solution will use $4 + 3 - 1 = 6$ cells. Since the initial north-west corner solution is degenerate, the solution must use 5 or less cells.

Start applying the north-west corner method. After two entries the table is:

	L	***M***	***N***	**Supply**
P	15	7		22
Q				a
R				11
S				b
Demand	15	17	20	

To fill in cell QM, suppose $a < 10$. Then:

	L	***M***	***N***	**Supply**
P	15	7		22
Q		a		a
R		$10 - a$	$a + 1$	11
S			$19 - a$	b
Demand	15	17	20	

As the solution is balanced $19 - a = b$, but this solution is non-degenerate as 6 cells are used. Hence $a \geqslant 10$.

So fill in cell QM, assuming $a \geqslant 10$. Then:

	L	***M***	***N***	**Supply**
P	15	7		22
Q		10	$a - 10$	a
R				11
S				b
Demand	15	17	20	

As the problem is balanced, the supply from R and S must be used to satisfy in part the demand from N, which means cells RN and SN must be non-zero. But if $a > 10$, this would give a solution that uses 6 cells, which would be non-degenerate.

A degenerate solution is therefore only obtained if $a = 10$. As the problem is balanced, total supply must equal total demand. So:

$15 + 17 + 20 = 22 + a + 11 + b$

$\Rightarrow a + b = 19$

As $a = 10$, this gives $b = 9$

3 b Using the preliminary work towards a solution using the north-west corner method in part **a**, the solution is:

	L	***M***	***N***	**Supply**
P	15	7		22
Q		10		10
R			11	11
S			9	9
Demand	15	17	20	52

c The solution is degenerate because the number of non-empty cells, $5 \neq 4 + 3 - 1$. Entering a zero into cell *QN* or cell *RM* will make the solution non-degenerate.

4 a Total demand = 24 + 30 + 45 = 99; Total supply = 28 + 26 + 31 = 85
As the total supply ≠ total demand, the problem is unbalanced.

To balance the problem, the total supply must be increased and this is achieved by adding an extra dummy supply point, *D*, with a supply of 99 – 85 =14.
The total demand and the total supply now balance.

b Units transported:

	P	***Q***	***R***	**Supply**
A	24	4		28
B		26	0	26
C			31	31
D			14	14
Demand	24	30	45	

As there are 4 rows and 3 columns, a non-degenerate solution has 4 + 3 – 1 = 6 filled cells. To avoid a degenerate solution, a zero has been entered in cell *BR*.

A solution with a zero in cell *CQ* and cell *BR* blank is also an acceptable non-degenerate solution.

c As row *D* is the dummy supply to balance the problem, this means that garage *R* will receive 14 fewer tyres than requested. It will receive 31 tyres rather than the 45 tyres demanded.