Transportation problems 1E

1 Let x_{ij} be the number of units transported from i to j where

$i \in \{A, B, C\}$

$j \in \{P, Q, R\}$

$x_{ij} \geqslant 0$

Minimise:

$$C = 150x_{AP} + 213x_{AQ} + 222x_{AR} + 175x_{BP} + 204x_{BQ} + 218x_{BR} + 188x_{CP} + 198x_{CQ} + 246x_{CR}$$

Subject to:

$$x_{AP} + x_{AQ} + x_{AR} \leqslant 32$$
$$x_{BP} + x_{BQ} + x_{BR} \leqslant 44$$
$$x_{CP} + x_{CQ} + x_{CR} \leqslant 34$$
$$x_{AP} + x_{BP} + x_{CP} \leqslant 28$$
$$x_{AQ} + x_{BQ} + x_{CQ} \geqslant 45$$
$$x_{AR} + x_{BR} + x_{CR} \geqslant 37$$

2 Let x_{ij} be the number of units transported from i to j where

$i \in \{A, B, C\}$

$j \in \{P, Q, R, S\}$

$x_{ij} \geqslant 0$

Minimise:

$$C = 27x_{AP} + 33x_{AQ} + 34x_{AR} + 41x_{AS} + 31x_{BP} + 29x_{BQ} + 37x_{BR} + 30x_{BS} + 40x_{CP} + 32x_{CQ} + 28x_{CR} + 35x_{CS}$$

Subject to:

$$x_{AP} + x_{AQ} + x_{AR} + x_{AS} \leqslant 54$$
$$x_{BP} + x_{BQ} + x_{BR} + x_{BS} \leqslant 67$$
$$x_{CP} + x_{CQ} + x_{CR} + x_{CS} \leqslant 29$$
$$x_{AP} + x_{BP} + x_{CP} \geqslant 21$$
$$x_{AQ} + x_{BQ} + x_{CQ} \geqslant 32$$
$$x_{AR} + x_{BR} + x_{CR} \geqslant 51$$
$$x_{AS} + x_{BS} + x_{CS} \geqslant 46$$

3 Let x_{ij} be the number of units transported from i to j where

$i \in \{A, B, C, D\}$

$j \in \{P, Q, R\}$

$x_{ij} \geqslant 0$

Minimise:

$$C = 17x_{AP} + 24x_{AQ} + 19x_{AR} + 15x_{BP} + 21x_{BQ} + 25x_{BR} + 19x_{CP} + 22x_{CQ} + 18x_{CR} + 20x_{DP} + 27x_{DQ} + 16x_{DR}$$

Subject to:

$$x_{AP} + x_{AQ} + x_{AR} \leqslant 123$$
$$x_{BP} + x_{BQ} + x_{BR} \leqslant 143$$
$$x_{CP} + x_{CQ} + x_{CR} \leqslant 84$$
$$x_{DP} + x_{DQ} + x_{DR} \leqslant 150$$
$$x_{AP} + x_{BP} + x_{CP} + x_{DP} \geqslant 200$$
$$x_{AQ} + x_{BQ} + x_{CQ} + x_{DQ} \geqslant 100$$
$$x_{AR} + x_{BR} + x_{CR} + x_{DR} \geqslant 200$$

4 Let x_{ij} be the number of units transported from i to j where

$i \in \{A, B, C, D\}$

$j \in \{P, Q, R, S\}$

$x_{ij} \geqslant 0$

Minimise:

$$C = 56x_{AP} + 86x_{AQ} + 80x_{AR} + 61x_{AS} + 59x_{BP} + 76x_{BQ} + 78x_{BR} + 65x_{BS} + 62x_{CP} + 70x_{CQ} + 57x_{CR} + 67x_{CS} + 60x_{DP} + 68x_{DQ} + 75x_{DR} + 71x_{DS}$$

Subject to:

$$x_{AP} + x_{AQ} + x_{AR} + x_{AS} \leqslant 134$$
$$x_{BP} + x_{BQ} + x_{BR} + x_{BS} \leqslant 203$$
$$x_{CP} + x_{CQ} + x_{CR} + x_{CS} \leqslant 176$$
$$x_{DP} + x_{DQ} + x_{DR} + x_{DS} \leqslant 187$$
$$x_{AP} + x_{BP} + x_{CP} + x_{DP} \geqslant 175$$
$$x_{AQ} + x_{BQ} + x_{CQ} + x_{DQ} \geqslant 175$$
$$x_{AR} + x_{BR} + x_{CR} + x_{DR} \geqslant 175$$
$$x_{AS} + x_{BS} + x_{CS} + x_{DS} \geqslant 175$$

5 a Total supply = 25 + 28 + 21 = 74
Total demand = 20 + 15 + 12 + 16 = 63
Total supply ≠ Total demand, so the problem is unbalanced.

b As supply is greater than demand, create a dummy demand point, *D*. This point has demand of 11 televisions making the problem balanced. So the problem becomes:

	W	***X***	***Y***	***Z***	***D***	**Supply**
A	8	11	7	9	0	25
B	12	10	8	7	0	28
C	10	12	9	8	0	21
Demand	20	15	12	16	11	74

Let x_{ij} be the number of units transported from *i* to *j* where

$i \in \{A, B, C\}$

$j \in \{W, X, Y, Z, D\}$

$x_{ij} \geqslant 0$

Minimise:

$$C = 8x_{AW} + 11x_{AX} + 7x_{AY} + 9x_{AZ} + 12x_{BW} + 10x_{BX} + 8x_{BY} + 7x_{BZ} + 10x_{CW} + 12x_{CX} + 9x_{CY} + 8x_{CZ}$$

This objective function is not affected by including a dummy demand point as the associated costs of the dummy location are 0.

Subject to:

$$\sum x_{Aj} \leqslant 25$$
$$\sum x_{Bj} \leqslant 28$$
$$\sum x_{Cj} \leqslant 21$$
$$\sum x_{iW} \geqslant 20$$
$$\sum x_{iX} \geqslant 15$$
$$\sum x_{iY} \geqslant 12$$
$$\sum x_{iZ} \geqslant 16$$
$$\sum x_{iD} \geqslant 11$$

Note that the dummy demand point must be included in the constraints.
These constraints have been shown using sigma notation. They could also be given in full.
For example:

$\sum x_{Aj} \leqslant 25$ can be written as $x_{AW} + x_{AX} + x_{AY} + x_{AZ} + x_{AD} \leqslant 25$

6 a The decision variables have not been defined.
The formulation should state that the objective function is to be minimised.
The third constraint should be $\leqslant 10$ not $\leqslant 20$.
The last three constraints should all be $\geqslant$ not $\leqslant$.

b Let x_{ij} represent the number of cars transported from i to j where
$i \in \{A, B, C\}$
$j \in \{X, Y, Z\}$
$x_{ij} \geqslant 0$

Minimise:
$$C = 70x_{AX} + 50x_{AY} + 60x_{AZ} + 85x_{BX} + 60x_{BY} + 74x_{BZ} + 68x_{CX} + 73x_{CY} + 80x_{CZ}$$

Subject to:
$$x_{AX} + x_{AY} + x_{AZ} \leqslant 12$$
$$x_{BX} + x_{BY} + x_{BZ} \leqslant 8$$
$$x_{CX} + x_{CY} + x_{CZ} \leqslant 10$$
$$x_{AX} + x_{BX} + x_{CX} \geqslant 11$$
$$x_{AY} + x_{BY} + x_{CY} \geqslant 9$$
$$x_{AZ} + x_{BZ} + x_{CZ} \geqslant 10$$