Flow in networks 3B

1 Cut $C_1 = 19 + 8 + 10 + 4 = 41$
Cut $C_2 = 20 + 7 + 20 = 47$

2 Cut $C_1 = 7 + 9 + 4 + 19 = 39$
Cut $C_2 = 15 + 3 + 16 = 34$

3 Cut $C_1 = 15 + 45 + 18 + 15 + 10 = 103$
Cut $C_2 = 15 + 10 + 20 + 10 + 15 + 8 = 78$
Cut $C_3 = 20 + 45 + 18 + 15 + 8 = 106$

4 Cut $C_1 = 16 + 16 + 4 + 25 = 61$
Cut $C_2 = 30 + 6 + 23 + 10 = 69$

5 Cut $C_1 = 30 + 32 + 18 + 30 = 110$
Cut $C_2 = 20 + 50 + 18 + 35 = 123$
Cut $C_3 = 20 + 15 + 8 + 15 + 10 + 18 + 14 = 100$

6 a A cut, in a network with source S and sink T, is a set of arcs whose removal separates the network into two parts X and Y, where X contains at least S and Y contains at least T.

b i To find the value of the flow consider either the flow out of S or the flow into T:
Out of $S = 26 + 16 = 42$
Into $T = 24 + 18 = 42$
So the value of the flow is 42

ii To find the capacity of each cut, we need to sum up the capacities flowing **out** of each cut. Thus:
$C_1 = SW + VW + VY + VX$
$= 18 + 4 + 14 + 14 = 50$
So capacity of cut C_1 is 50
Similarly: $C_2 = SW + VW + VY + XT$
Note that YX flows **into** the cut, so it does not contribute to the capacity.
Hence: $C_2 = 18 + 4 + 14 + 24 = 60$
So the capacity of C_2 is 60

7 a The capacity of a cut is the sum of capacities flowing out of the cut.
For cut C we have
$C = SK + JM + LT$
Note that KJ and ML flow into the cut and so they don't contribute to the capacity.
Hence: $C = 12 + x + x = 12 + 2x$
We know that the capacity of C is 28, thus
$12 + 2x = 28$
$x = 8$

b Since KJ flows out of the cut, we have no information about its value. Additionally, the network is only described with capacities and not flows, so we cannot use the fact that flow into a node = flow out either.