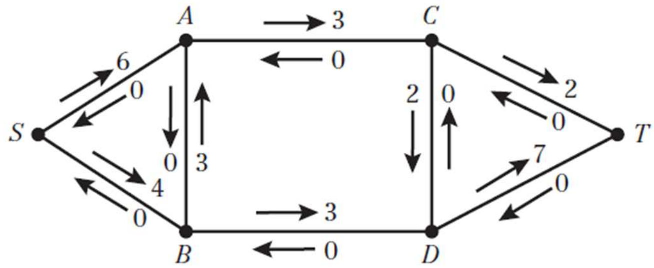
Flow in networks Mixed exercise

1 a



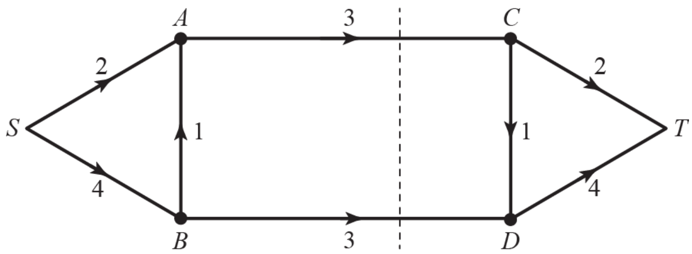
For example (many different flow combinations are possible)

$S\,B\,D\,T - 3$

$S\,A\,C\,T - 2$

$S\,B\,A\,C\,D\,T - 1$

b

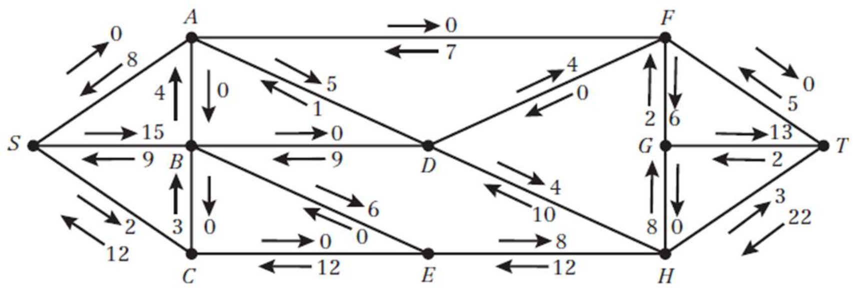


Minimum cut = 6 so by maximum flow = minimum cut theorem, flow is maximum.

2 a <u>Applied.</u> Idea of flow through a system, idea of directed flow.

e.g. traffic moving through a one-way system of roads

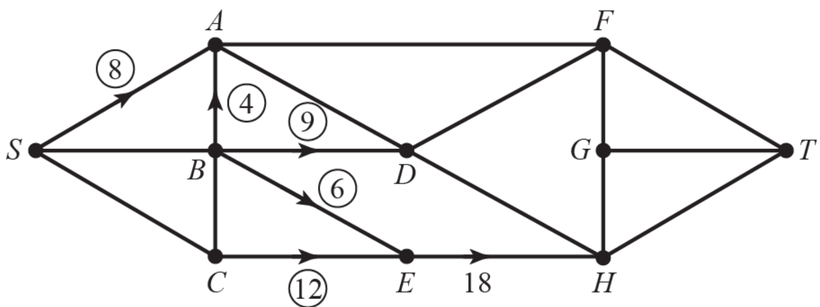
b



e.g. $S\,B\,E\,H\,G\,T - 6$ and $S\,B\,A\,D\,F\,G\,T - 4$

or $S\,B\,A\,D\,H\,G\,T - 4$ and $S\,C\,B\,E\,H\,T - 2$ and $S\,B\,E\,H\,G\,T - 4$ etc.

c Many solution are possible, but the following values must be given. Flows must be consistent.



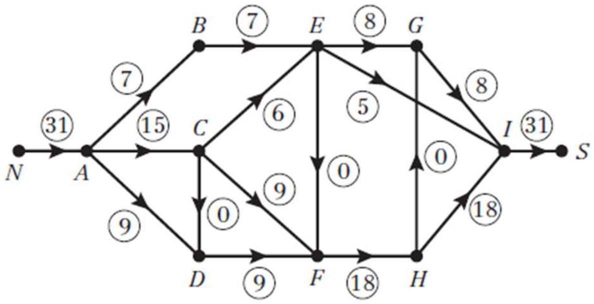
Final flow is 39

2 d Cut through SA, BA, BD, BE and CE

e The arcs are saturated.

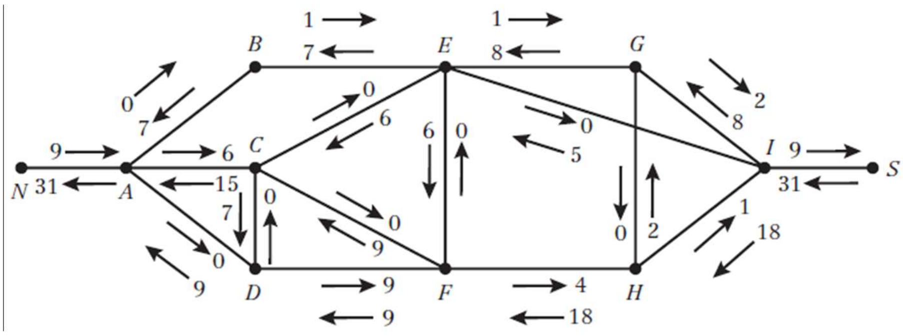
3 a A F G and H, possible flow in > possible flow out.

b e.g.



c Using labelling procedure

e.g.

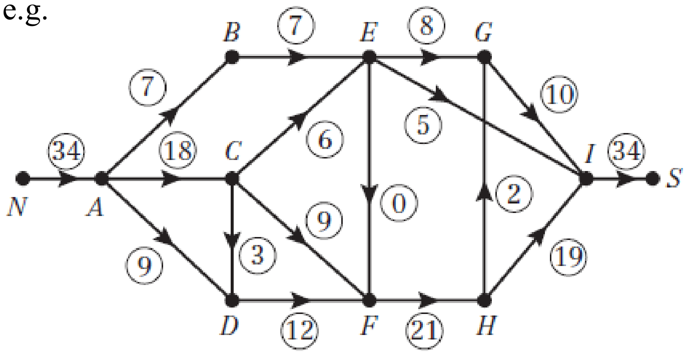


e.g.

If $HI = 16$	If $HI = 17$	If $HI = 18$	If $HI = 19$
$NACDFHIS - 3$	$NACDFHGIS - 1$	$NACDFHGIS - 2$	$NACDFHGIS - 1$
	$NACDFHIS - 2$	$NACDFHIS - 1$	$NACDFEGIS - 2$

Final flow 34

d e.g.



e and f GI EI and HI

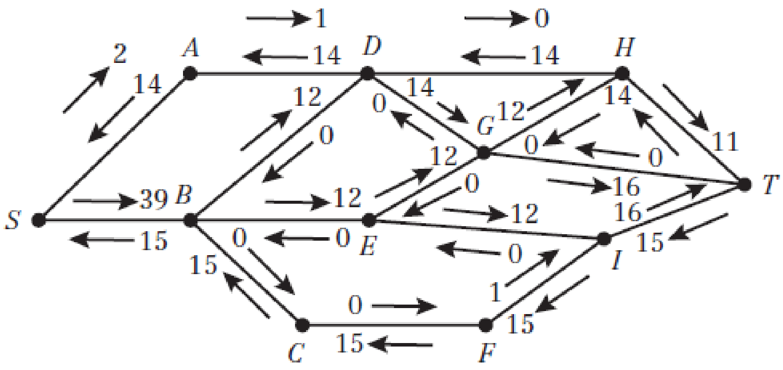
AB CE (EF) HG and HI

4 a i Flow along $S\,B\,C\,F\,I\,T = 15$

ii Flow along $S\,A\,D\,H\,T = 14$

b A diagram showing the 2 flows correctly

c



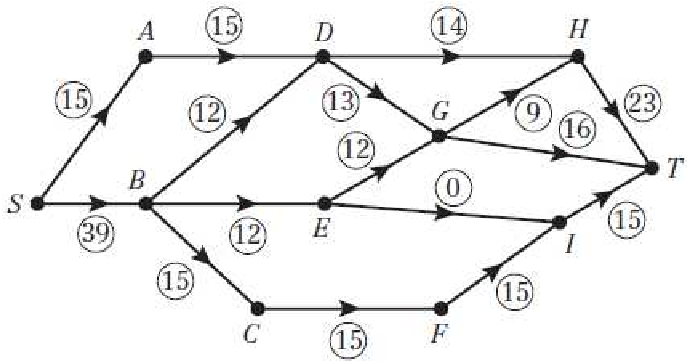
e.g. $S\,A\,D\,G\,T - 1$

$S\,B\,D\,G\,T - 12$

with $S\,B\,E\,I\,T - 12$ or $S\,B\,E\,G\,H\,T - 9$ and $S\,B\,E\,G\,T - 3$

giving a total flow of 54

d e.g.

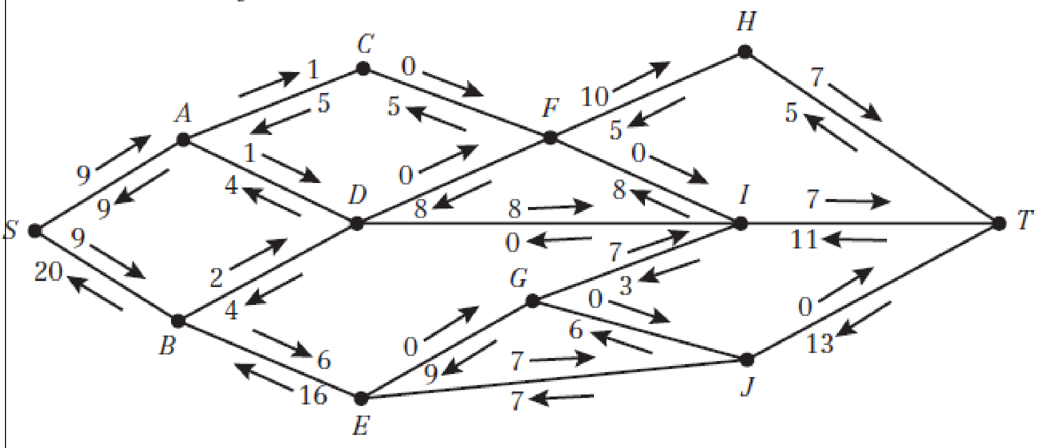


e Max flow – min cut theorem, cut through AD, BD, BE and BC or CF

f The flow into D and into C could not increase,
so increase the flow along BE

5 a $v = 7,\ w = 6,\ x = 8,\ y = 3,\ z = 11$ (conservation of flow)

b



Increasing flow by an additional 3

e.g. $S\,B\,D\,I\,T$ (2)

$S\,A\,D\,I\,T$ (1)

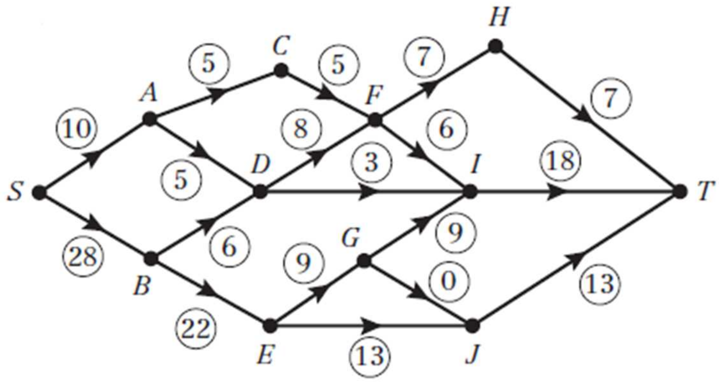
Additional flow increase (reversing initial flow)

e.g. $S\,B\,E\,J\,G\,I\,T$ (4)

$S\,B\,E\,J\,G\,I\,F\,H\,T$ (2)

Flow up to maximum (38)

c e.g.



Complete, consistent flow ringed numbers (flow of 38)

5 d To find the capacity of a cut we need to sum up the capacities (**not the flows!**) of all the arcs flowing **into** of the cut. The arcs which flow **out of** the cut contribute **zero** to the capacity of that cut.

5 e i $12+18+13=43$

ii Cut through CF, AD, BD, BE has capacity $5+5+6+22=38$

iii Max flow – min cut theorem – e.g.

The minimum cut separates the source from the sink. Any additional flow must cross this cut at some point. Since all arcs in the minimum cut are saturated no additional flow can be transported along these arcs. Hence no additional flow is possible.

6 a A cut, in a capacitated network with source S and sink T, is a set of arcs whose removal separates the network into two parts X and Y, where X contains at least S and Y contains at least T.

b $C_1=40 \quad C_2=56$

c max flow = min cut = 40

d e.g. Flow into F is 16 $\therefore$ flow into G is 24. The flow along DG is 8 $\therefore$ Flow along GT is 24

e e.g. Flow into A = flow out of A $\therefore$ flow along $AD \leqslant 12$

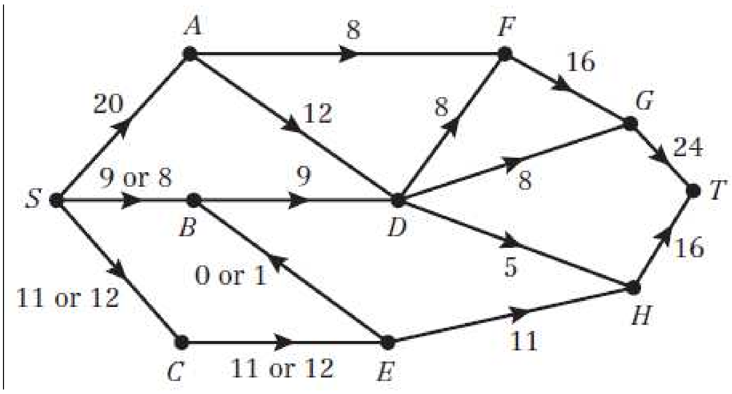
Flow into D = flow out of D = 21

So flow along AD + flow along BD = 21

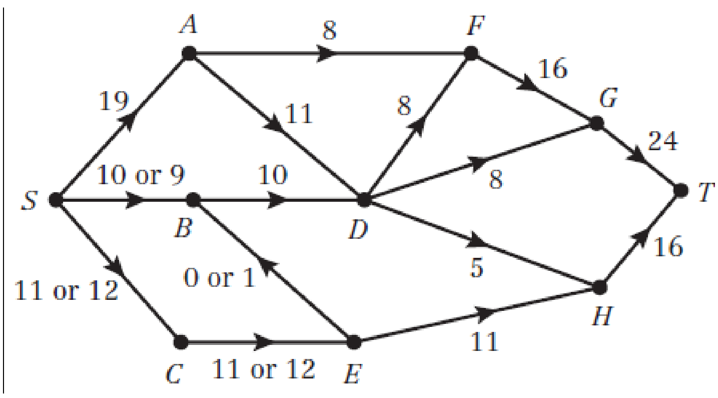
$\therefore$ flow along AD and BD could be 12 + 9 or 11 + 10

$\therefore$ possible flows are 20 and 19

f $SA=20$



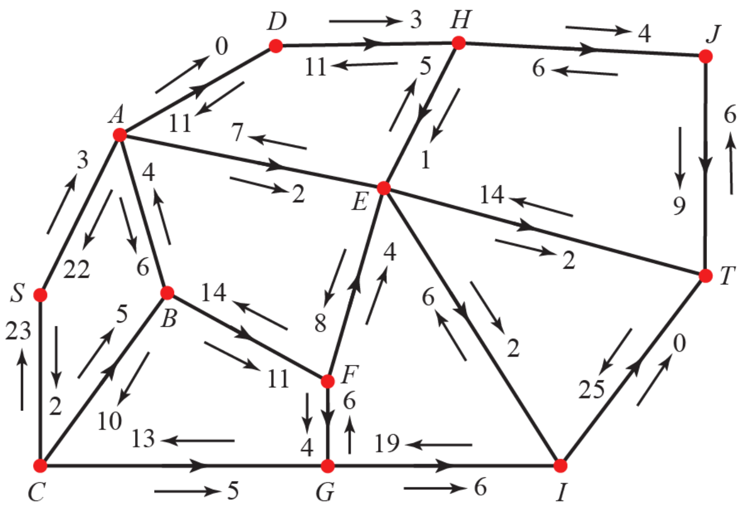
$SA=19$



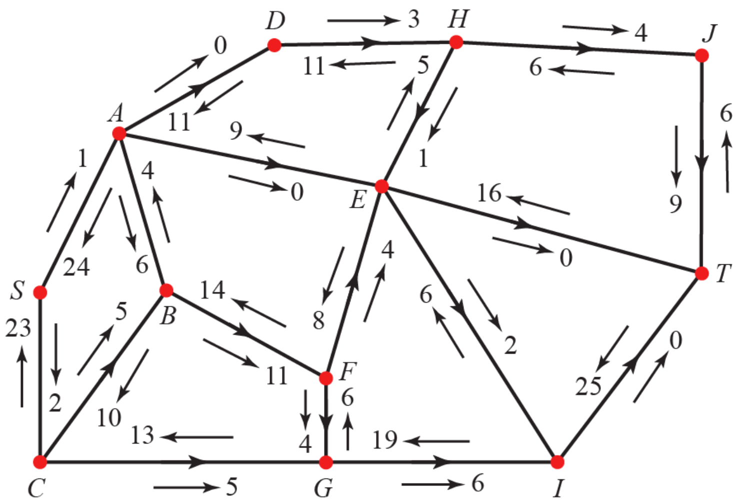
g There are 2 more – CE could be 11 or 12 in each case, for example.

Challenge

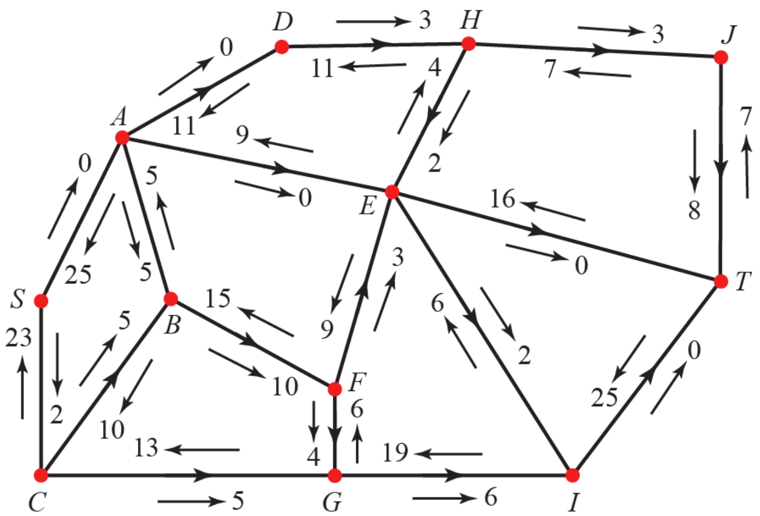
a Begin by identifying spare flows. Remember that in this particular network, the maximum capacity at each node is 25!



We see that there is a spare capacity of 2 along $SAET$. Updated diagram:



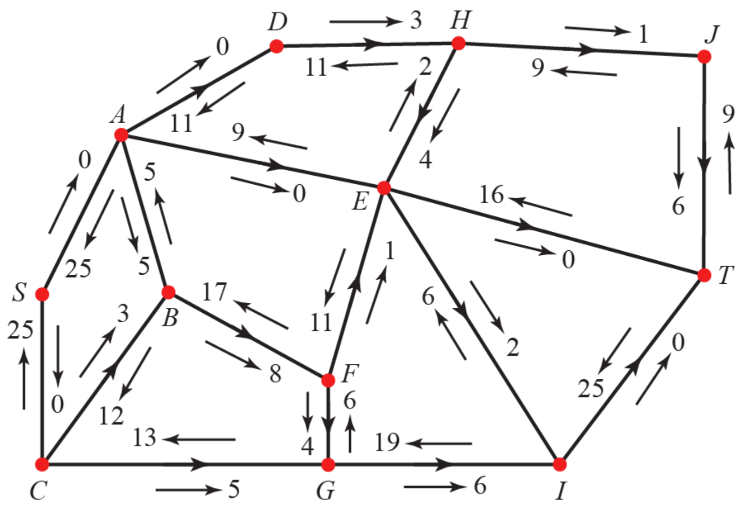
From node A we can no longer go along D or E, so the only option is to follow B. Send 1 amp along $SABFEHJT$. Updated diagram:



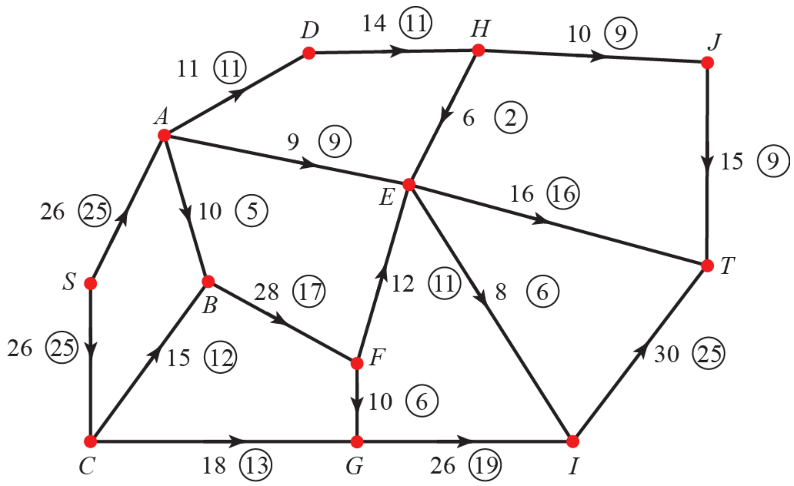
Challenge

a (continued)

There is no spare capacity left in the direction of diode A, but we can send extra 2 amps along $SCBFEHJT$. Updated diagram:



There is no more capacity along C nor A, so we cannot add more amps to this network. Hence the final flow looks as follows:



And we see that the maximum flow is 50.

b By inspecting the diagram above, we find that we cannot draw a cut through saturated or empty arcs. This is because of the additional restriction of a maximum current flow through any component being 25 amps, which is less than the original maximum capacity for some diodes. Thus we cannot apply the maximum flow-minimum cut theorem here.