Dynamic programming 5A

1 a Shortest

Stage	State	Action	Destination	Value
1	F	FT	T	23*
	G	GT	T	24*
	H	HT	T	21*
2	D	DF	F	$24+23=47$
		DG	G	$22+24=46$
		DH	H	$17+21=38$*
	E	EF	F	$20+23=43$
		EG	G	$25+24=49$
		EH	H	$20+21=41$*
3	A	AD	D	$34+38=72$*
		AE	E	$33+41=74$
	B	BD	D	$32+38=70$*
		BE	E	$31+41=72$
	C	CD	D	$30+38=68$*
		CE	E	$30+41=71$
4	S	SA	A	$18+72=90$
		SB	B	$17+70=87$*
		SC	C	$20+68=88$

Shortest route SBDHT length 87

1 b Longest

Stage	State	Action	Destination	Value
1	F	FT	T	23*
	G	GT	T	24*
	H	HT	T	21*
2	D	DF	F	$24+23=47$*
		DG	G	$22+24=46$
		DH	H	$17+21=38$
	E	EF	F	$20+23=43$
		EG	G	$25+24=49$*
		EH	H	$20+21=41$
3	A	AD	D	$34+47=81$
		AE	E	$33+49=82$*
	B	BD	D	$32+47=79$
		BE	E	$31+49=80$*
	C	CD	D	$30+47=77$
		CE	E	$30+49=79$*
4	S	SA	A	$18+82=100$*
		SB	B	$17+80=97$
		SC	C	$20+79=99$

Longest route SAEGT length 100

2 a Shortest

Stage	State	Action	Destination	Value
1	G	GT	T	21*
	H	HT	T	23*
	I	IT	T	24*
2	C	CG	G	$19+21=40$*
		CH	H	$21+23=44$
	D	DG	G	$20+21=41$*
		DH	H	$21+23=44$
	E	EH	H	$22+23=45$*
		EI	I	$23+24=47$
	F	FH	H	$25+23=48$
		FI	I	$20+24=44$*
3	A	AC	C	$34+40=74$*
		AD	D	$37+41=78$
		AE	E	$35+45=80$
	B	BD	D	$38+41=79$
		BE	E	$33+45=78$*
		BF	F	$36+44=80$
4	S	SA	A	$14+74=88$*
		SB	B	$13+78=91$

Shortest route length is 88 with route SACGT

2 b Longest

Stage	State	Action	Destination	Value
1	G	GT	T	21*
	H	HT	T	23*
	I	IT	T	24*
2	C	CG	G	$19+21=40$
		CH	H	$21+23=44$*
	D	DG	G	$20+21=41$
		DH	H	$21+23=44$*
	E	EH	H	$22+23=45$
		EI	I	$23+24=47$*
	F	FH	H	$25+23=48$*
		FI	I	$20+24=44$
3	A	AC	C	$34+44=78$
		AD	D	$37+44=81$
		AE	E	$35+47=82$*
	B	BD	D	$38+44=82$
		BE	E	$33+47=80$
		BF	F	$36+48=84$*
4	S	SA	A	$14+82=96$
		SB	B	$13+84=97$*

Longest route length is 97 with route SBFHT

3 a Shortest

Stage	State	Action	Destination	Value
1	H	HT	T	16*
	I	IT	T	18*
	J	JT	T	17*
2	D	DH	H	$30+16=46$*
	E	EH	H	$27+16=43$*
		EI	I	$26+18=44$
	F	FI	I	$26+18=44$*
		FJ	J	$28+17=45$
	G	GJ	J	$25+17=42$*
3	A	AD	D	$28+46=74$
		AE	E	$29+43=72$*
	B	BD	D	$26+46=72$
		BE	E	$23+43=66$*
		BF	F	$25+44=69$
	C	CE	E	$22+43=65$*
		CF	F	$24+44=68$
		CG	G	$24+42=66$
4	S	SA	A	$26+72=98$
		SB	B	$27+66=93$*
		SC	C	$32+65=97$

Shortest route length is 93 with route SBEHT

3 b Longest

Stage	State	Action	Destination	Value
1	H	HT	T	16*
	I	IT	T	18*
	J	JT	T	17*
2	D	DH	H	$30+16=46$*
	E	EH	H	$27+16=43$
		EI	I	$26+18=44$*
	F	FI	I	$26+18=44$
		FJ	J	$28+17=45$*
	G	GJ	J	$25+17=42$*
3	A	AD	D	$28+46=74$*
		AE	E	$29+44=73$
	B	BD	D	$26+46=72$*
		BE	E	$23+44=67$
		BF	F	$25+45=70$
	C	CE	E	$22+44=66$
		CF	F	$24+45=69$*
		CG	G	$24+42=66$
4	S	SA	A	$26+74=100$
		SB	B	$27+72=99$
		SC	C	$32+69=101$*

Longest route length is 101 with route SCFJT

4 a

Stage	State	Action	Value
1	G	GT	20*
	H	HT	24*
2	D	DG	18 + 20 = 38*
		DH	17 + 24 = 41
	E	EG	15 + 20 = 35*
		EH	14 + 24 = 38
	F	FG	16 + 20 = 36*
		FH	16 + 24 = 40
3	A	AD	22 + 38 = 60*
		AE	25 + 35 = 60*
	B	BD	27 + 38 = 65
		BE	28 + 35 = 63
		BF	21 + 36 = 57*
	C	CE	23 + 35 = 58*
		CF	24 + 36 = 60
4	S	SA	28 + 60 = 88
		SB	27 + 57 = 84*
		SC	30 + 58 = 88

Decisions are SBFGT

b Total cost is £84 000, average cost (per year) is £21 000

5 Maximise

Stage	State	Action	Destination	Value
1 Assets	E	ET	T	5*
	F	FT	T	30*
	G	GT	T	20*
	H	HT	T	55*
2 Year two	A	AE (No)	E	$5+5=10$
		AF (TV)	F	$-13+30=17$
		AG (Radio)	G	$-3+20=17$
		AH (Both)	H	$-16+55=39$*
	B	BE (No)	E	$25+5=30$
		BF (TV)	F	$3+30=33$
		BG (Radio)	G	$8+20=28$
		BH (Both)	H	$-2+55=53$*
	C	CE (No)	E	$8+5=13$
		CF (TV)	F	$-7+30=23$
		CG (Radio)	G	$5+20=25$
		CH (Both)	H	$-4+55=51$*
	D	DE (No)	E	$55+5=60$
		DF (TV)	F	$40+30=70$*
		DG (Radio)	G	$20+20=40$
		DH (Both)	H	$6+55=61$
3 Year one	S	SA (No)	A	$5+39=44$
		SB (TV)	B	$30+53=83$
		SC (Radio)	C	$20+51=71$
		SD (Both)	D	$75+70=145$*

The maximum profit is £145 000

The maximum route is SDFT

In practical terms the company's strategy is:

Year 1 – advertise in both TV and Radio

Year 2 – advertise on TV only