Dynamic programming 5B

1 a Minimax

Stage	State	Action	Destination	Value
1	F	FT	T	23*
	G	GT	T	24*
	H	HT	T	21*
2	D	DF	F	$\text{Max}(24, 23) = 24$
		DG	G	$\text{Max}(22, 24) = 24$
		DH	H	$\text{Max}(17, 21) = 21^*$
	E	EF	F	$\text{Max}(20, 23) = 23$
		EG	G	$\text{Max}(25, 24) = 25$
		EH	H	$\text{Max}(20, 21) = 21^*$
3	A	AD	D	$\text{Max}(34, 21) = 34$
		AE	E	$\text{Max}(33, 21) = 33^*$
	B	BD	D	$\text{Max}(32, 21) = 32$
		BE	E	$\text{Max}(31, 21) = 31^*$
	C	CD	D	$\text{Max}(30, 21) = 30^*$
		CE	E	$\text{Max}(30, 21) = 30^*$
4	S	SA	A	$\text{Max}(18, 33) = 33$
		SB	B	$\text{Max}(17, 31) = 31$
		SC	C	$\text{Max}(20, 30) = 30^*$

Minimax route SCDHT or SCEHT – both of value 30

1 b Maximin

Stage	State	Action	Destination	Value
1	F	FT	T	23*
	G	GT	T	24*
	H	HT	T	21*
2	D	DF	F	$\text{Min}(24,23)=23$*
		DG	G	$\text{Min}(22,24)=22$
		DH	H	$\text{Min}(17,21)=17$
	E	EF	F	$\text{Min}(20,23)=20$
		EG	G	$\text{Min}(25,24)=24$*
		EH	H	$\text{Min}(20,21)=20$
3	A	AD	D	$\text{Min}(34,23)=23$
		AE	E	$\text{Min}(33,24)=24$*
	B	BD	D	$\text{Min}(32,23)=23$
		BE	E	$\text{Min}(31,24)=24$*
	C	CD	D	$\text{Min}(30,23)=23$
		CE	E	$\text{Min}(30,24)=24$*
4	S	SA	A	$\text{Min}(18,24)=18$
		SB	B	$\text{Min}(17,24)=17$
		SC	C	$\text{Min}(20,24)=20$*

Maximin route SCEGT of value 20

2 a Minimax

Stage	State	Action	Destination	Value
1	G	GT	T	21*
	H	HT	T	23*
	I	IT	T	24*
2	C	CG	G	$\text{Max}(19,21)=21$*
		CH	H	$\text{Max}(21,23)=23$
	D	DG	G	$\text{Max}(20,21)=21$*
		DH	H	$\text{Max}(21,23)=23$
	E	EH	H	$\text{Max}(22,23)=23$*
		EI	I	$\text{Max}(23,24)=24$
	F	FH	H	$\text{Max}(25,23)=25$
		FI	I	$\text{Max}(20,24)=24$*
3	A	AC	C	$\text{Max}(34,21)=34$*
		AD	D	$\text{Max}(37,21)=37$
		AE	E	$\text{Max}(35,23)=35$
	B	BD	D	$\text{Max}(38,21)=38$
		BE	E	$\text{Max}(33,23)=33$*
		BF	F	$\text{Max}(36,24)=36$
4	S	SA	A	$\text{Max}(14,34)=34$
		SB	B	$\text{Max}(13,33)=33$*

Minimax route SBEHT of value 33

2 b Maximin

Stage	State	Action	Destination	Value
1	G	GT	T	21*
	H	HT	T	23*
	I	IT	T	24*
2	C	CG	G	$\text{Min}(19,21)=19$
		CH	H	$\text{Min}(21,23)=21^*$
	D	DG	G	$\text{Min}(20,21)=20$
		DH	H	$\text{Min}(21,23)=21^*$
	E	EH	H	$\text{Min}(22,23)=22$
		EI	I	$\text{Min}(23,24)=23^*$
	F	FH	H	$\text{Min}(25,23)=23^*$
		FI	I	$\text{Min}(20,24)=20$
3	A	AC	C	$\text{Min}(34,21)=21$
		AD	D	$\text{Min}(37,21)=21$
		AE	E	$\text{Min}(35,23)=23^*$
	B	BD	D	$\text{Min}(38,21)=21$
		BE	E	$\text{Min}(33,23)=23^*$
		BF	F	$\text{Min}(36,23)=23^*$
4	S	SA	A	$\text{Min}(14,23)=14^*$
		SB	B	$\text{Max}(13,23)=13$

Maximin route SAEIT of value 14

3 a Minimax

Stage	State	Action	Destination	Value
1	H	HT	T	16*
	I	IT	T	18*
	J	JT	T	17*
2	D	DH	H	$\text{Max}(30,16) = 30$*
	E	EH	H	$\text{Max}(27,16) = 27$
		EI	I	$\text{Max}(26,18) = 26$*
	F	FI	I	$\text{Max}(26,18) = 26$*
		FJ	J	$\text{Max}(28,17) = 28$
	G	GJ	J	$\text{Max}(25,17) = 25$*
3	A	AD	D	$\text{Max}(28,30) = 30$
		AE	E	$\text{Max}(29,26) = 29$*
	B	BD	D	$\text{Max}(26,30) = 30$
		BE	E	$\text{Max}(23,26) = 26$*
		BF	F	$\text{Max}(25,26) = 26$*
	C	CE	E	$\text{Max}(22,26) = 26$
		CF	F	$\text{Max}(24,26) = 26$
		CG	G	$\text{Max}(24,25) = 25$*
4	S	SA	A	$\text{Max}(26,29) = 29$
		SB	B	$\text{Max}(27,26) = 27$*
		SC	C	$\text{Max}(32,35) = 32$

Minimax route SBEIT or SBFIT – both of value 27

3 b Maximin

Stage	State	Action	Destination	Value
1	H	HT	T	16*
	I	IT	T	18*
	J	JT	T	17*
2	D	DH	H	Min(30,16) = 16*
	E	EH	H	Min(27,16) = 16
		EI	I	Min(26,18) = 18*
	F	FI	I	Min(26,18) = 18*
		FJ	J	Min(28,17) = 17
	G	GJ	J	Min(25,17) = 17*
3	A	AD	D	Min(28,16) = 16
		AE	E	Min(29,18) = 18*
	B	BD	D	Min(26,16) = 16
		BE	E	Min(23,18) = 18*
		BF	F	Min(25,18) = 18*
	C	CE	E	Min(22,18) = 18*
		CF	F	Min(24,18) = 18*
		CG	G	Min(24,17) = 17
4	S	SA	A	Min(26,18) = 18*
		SB	B	Min(27,18) = 18*
		SC	C	Min(32,18) = 18*

Maximin route SAEIT, SBEIT, SBFIT, SCEIT, SCFIT all of value 18

4 a Maximin problem

4 b

Stage	State	Action	Value
1	G	GT	10*
	H	HT	14*
2	D	DG	Min(8,10) = 8*
		DH	Min(7,14) = 7
	E	EG	Min(5,10) = 5*
		EH	Min(4,14) = 4
	F	FG	Min(6,10) = 6*
		FH	Min(6,10) = 6*

All action values in stage 3 and 4 are greater than the current value so the minimum value attained by any route through D is 8, E is 5 and F is 6 – the optimal route will go through D and any route including DGT will be optimal (SADGT or SBDGT).

c Minimum clearing is 8 feet

5 a Minimax problem

5 b

Stage	State	Action	Value
1	G	GT	12*
	H	HT	14*
	I	IT	11*
2	D	DG	Max(10,12) = 12*
		DH	Max(9,14) = 14
		DI	Max(12,11) = 12*
	E	EG	Max(11,12) = 12*
		EH	Max(11,14) = 14
		EI	Max(13,11) = 13
	F	FG	Max(14,14) = 14*
		FH	Max(14,11) = 14*
3	A	AD	Max(14,14) = 14*
	B	BD	Max(12,12) = 12*
		BE	Max(16,12) = 16
		BF	Max(14,14) = 14
	C	CE	Max(15,12) = 15*
		CF	Max(17,14) = 17
4	S	SA	Max(11,14) = 14
		SB	Max(10,12) = 12*
		SC	Max(12,15) = 15

Two possible routes are SBDFT (length 44 miles) and SBDIT (length 45 miles)

Challenge

SACT is an optimal path (as is SADT) but ACT is not optimal (while ADT is)