Dynamic Programming 5C

1 a Minimax problem

b Stage – time, in years, remaining

State – resorts already created

Action – resort to be opened and added to brochure

c

Stage	State	Action	Value
1	BC	A	60*
	AC	B	85*
	AB	C	75*
2	A	AB	max(65,75) = 75*
		AC	max(75,85) = 85
	B	BA	max(70,75) = 75
		BC	max(65,60) = 65*
	C	CA	max(75,85) = 85
		CB	max(80,60) = 80*
3	N	A	max(55,75) = 75
		B	max(70,65) = 70*
		C	max(60,80) = 80

Costs are in £100s

d The resorts should be added in the order BCA with maximum annual cost £7000

2 a Stage – phase being considered

State – number of days remaining

Action – number of days allocated

Destination – number of days remaining

Value – total costs

2 b

Stage	State	Action	Destination	Value
Decorating	5	5	0	14*
	10	10	0	12*
	15	15	0	9*
Modernisation	10	5	5	$22+14=36$*
	15	10	5	$19+14=33$*
		5	10	$22+12=34$
	20	15	5	$15+14=29$*
		10	10	$19+12=31$
		5	15	$22+9=31$
Repairing	15	5	10	$24+36=60$*
	20	10	10	$20+36=56$*
		5	15	$24+33=57$
	25	5	20	$24+29=53$
		10	15	$20+33=53$
		15	10	$15+36=51$*
Clearance	30	5	25	$15+51=66$*
		10	20	$13+56=69$
		15	15	$8+60=68$

The minimum cost £66 000. The time should be allocated as follows:

Activity	Clearance	Repairing	Modernisation	Decorating
Number of days	5	15	5	5

3 a Stage – Month

State – number in storage

Action – number to be made

Stage	State	Action	Destination	Value (in £10 000)
June (2)	2	0	0	$2=2^*$
	1	1	0	$5+1=6^*$
	0	2	0	$5=5^*$
May (3)	2	1	0	$5+2+5=12$
		2	1	$5+2+6=13$
		3	2	$2+5+2+2=11^*$
	1	2	0	$5+1+5=11^*$
		3	1	$2+5+1+6=14$
	0	3	0	$2+5+3+2=12^*$
April (2)	2	0	0	$2+12=14^*$
		1	1	$5+2+11=18$
		2	2	$5+2+11=18$
	1	1	0	$5+1+12=18^*$
		2	1	$5+1+11=17$
		3	2	$2+5+1+11=19$
	0	2	0	$5+12=17^*$
		3	1	$2+5+11=18$
March (1)	0	1	0	$5+17=22$
		2	1	$5+16=21$
		3	2	$2+5+14=21^*$

The minimum cost is £210 000. The aircraft should be built as follows:

Month	March	April	May	June
Number of aircraft built in each month	3	0	3	2

b Bellman's principle of optimality is that any part of an optimal path is optimal.

c If a maximum of 1 aircraft can be made in March, then 1,2,3,2 is the optimal schedule with cost £220 000

4 Stage – day

State – shop being visited

Action – next journey to be undertaken

Stage	State	Action	Destination	Value, in £100
Thursday	H	H – home	home	$14-6=8^*$
	I	I – home	home	$13-4=9^*$
	J	J – home	home	$11-3=8^*$
Wednesday	F	FH	H	$10-5+8=13$
		FI	I	$10-4+9=15^*$
		FJ	J	$10-4+8=14$
	G	GH	H	$11-5+8=14$
		GI	I	$11-5+9=15^*$
		GJ	J	$11-4+8=15^*$
Tuesday	D	DF	F	$12-5+15=22^*$
		DG	G	$12-5+15=22^*$
	E	EF	F	$14-4+15=25^*$
		EG	G	$14-7+15=22$
Monday	A	AD	D	$8-3+22=27$
		AE	E	$8-4+25=29^*$
	B	BD	D	$9-4+22=27$
		BE	E	$9-6+25=28^*$
	C	CD	D	$8-4+22=26$
		CE	E	$8-4+25=29^*$
Sunday	Home	Home – A		$-2+29=27^*$
		Home – B		$-2+28=26$
		Home – C		$-3+29=26$

The minimum route is:

Home – A – E – F – I – Home

With a value of £2700