Momentum and impulse 1B
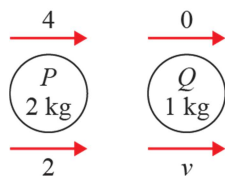
1 Conservation of momentum ($\rightarrow$)
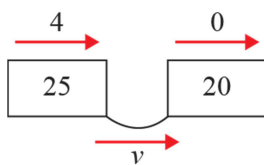
$(2 \times 4) + (1 \times 0) = (2 \times 2) + (1 \times v)$

$\qquad 8 = 4 + v$

$\qquad 4 = v$

The speed of Q after the collision is 4 m s^{-1}

2 Conservation of momentum ($\rightarrow$)

$(4 \times 25) + (20 \times 0) = 45v$

$\qquad 100 = 45v$

$\qquad \dfrac{20}{9} = v$

The common speed of the trucks is $\dfrac{20}{9}$ m s^{-1}

3 Conservation of momentum ($\rightarrow$)

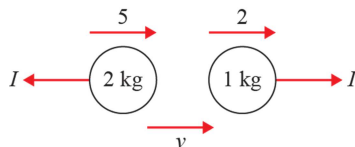$(0.5 \times 5) + (0.2 \times 2) = (0.5 \times 4) + (0.2 \times v)$

$\qquad 2.5 + 0.4 = 2.0 + 0.2v$

$\qquad 0.9 = 0.2v$

$\qquad 4.5 = v$
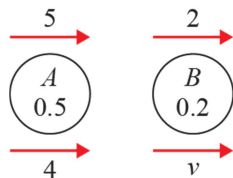
The speed of B after the collision is 4.5 m s^{-1}

4 a Conservation of momentum ($\rightarrow$)

$(2 \times 4) + (1 \times 0) = 3v$

$\qquad 8 = 3v$

$\qquad \dfrac{8}{3} = v$

The common speed of the particles after the collision is $\dfrac{8}{3}$ m s^{-1}

4 b For the 1 kg particle:
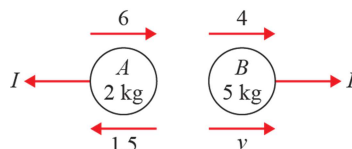
$$(\rightarrow) I = 1 \times v = \frac{8}{3}$$

The magnitude of the impulse in the collision is $\frac{8}{3}$ N s

[Alternatively, for the 2 kg particle:

$$(\leftarrow) I = 2(-v-(-4))$$
$$= 2(-\frac{8}{3}+4)$$
$$= 2 \times \frac{4}{3} = \frac{8}{3} \,]$$

5 a Conservation of momentum $(\rightarrow)$

$$(2 \times 6) + (5 \times (-4)) = (2 \times (-1.5)) + 5v$$
$$12 - 20 = -3 + 5v$$
$$-5 = 5v$$
$$v = -1$$

The speed of B is 1 m s^{-1} and its direction of motion is unchanged by the collision.
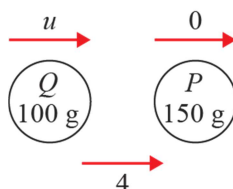
b For $A : (\leftarrow) I = 2(1.5-(-6))$
$$= 2 \times 7.5$$
$$= 15$$

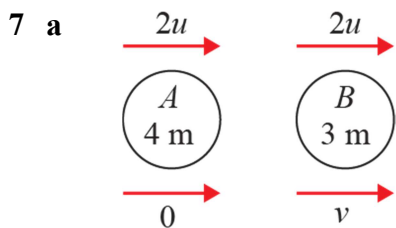[or for $B : (\rightarrow) I = 5(v-(-4))$
$$= 5(-1+4)$$
$$= 15]$$

The magnitude of the impulse given to B is 15 N s

6 Conservation of momentum $(\rightarrow)$

$$100u + (150 \times 0) = 250 \times 4$$
$$100u = 1000$$
$$u = 10$$

The value of u is 10.

7 a

Conservation of momentum $(\rightarrow)$

$$\left(4m \times 2u\right) + \left(3m \times (-2u)\right) = \left(4m \times 0\right) + \left(3m \times v\right)$$
$$8mu - 6mu = 3mv$$
$$2mu = 3mv$$
$$v = \frac{2u}{3}$$

The velocity of B after the collision is $\dfrac{2u}{3}$ m s^{-1} in the opposite direction.

b For $A : (\leftarrow)\ I = 4m(0 - (-2u))$
$$= 8mu$$

The magnitude of the impulse given by A to B is $8mu$

[or for B: $(\rightarrow)\ I = 3m(v - (-2u))$
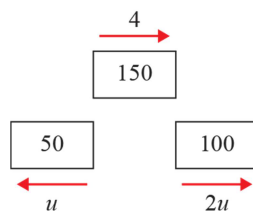$$= 3m\left(\frac{2u}{3} + 2u\right)$$
$$= 2mu + 6mu = 8mu]$$

8 Conservation of momentum $(\rightarrow)$

$$(150 \times 4) = (100 \times 2u) + (50 \times (-u))$$
$$600 = 200u - 50u$$
$$600 = 150u$$
$$4 = u$$

The larger has speed 8 m s^{-1} and the smaller part has speed 4 m s^{-1}

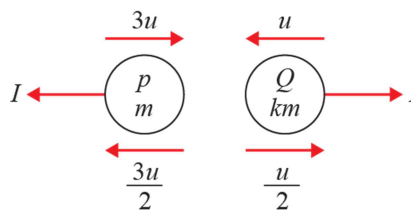**9 a** Conservation of momentum $(\rightarrow)$

$$(m \times 3u) + (km \times (-u)) = \left(m \times \frac{-3u}{2}\right) + \left(km \times \frac{u}{2}\right)$$

$$3mu - kmu = -\frac{3mu}{2} + \frac{kmu}{2}$$

$$6 - 2k = -3 + k$$

$$3 = k$$

The value of k is 3.

b For P: $(\leftarrow)$ $I = m\left(\frac{3u}{2} - (-3u)\right)$ [or for Q: $(\rightarrow)$ $I = km\left(\frac{u}{2} - (-u)\right)$

$$= \frac{9mu}{2}$$

$$= 3m \times \frac{3u}{2}$$

$$= \frac{9mu}{2}]$$

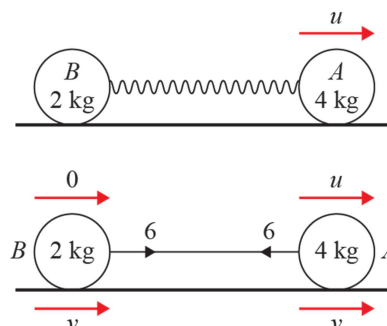The magnitude of the impulse is $\dfrac{9mu}{2}$

10 a For $B : (\rightarrow)$

impulse–momentum principle

$$6 = 2(v - 0)$$
$$3 = v$$

The common speed is 3 m s^{-1}

b Conservation of momentum $(\rightarrow)$

$$4u = 2v + 4v = -6 \times 3 = 18$$
$$u = 4.5$$

[or impulse–momentum principle for A: $(\rightarrow) -6 = 4(3 - u)$
$$-1.5 = 3 - u$$
$$u = 4.5]$$

The value of u is 4.5.
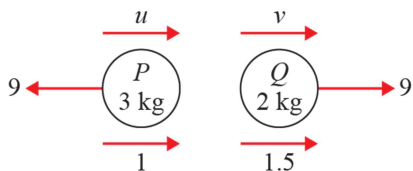
11 a For $P:(\leftarrow)$

impulse–momentum principle

$9 = 3(-1-(-u))$

$9 = 3(-1+u)$

$3 = -1+u$

$4 = u$

The speed of P before the collision is 4 m s^{-1} and it was moving in the same direction as it was after the collision.

b For $Q:(\rightarrow)$

impulse–momentum principle

$9 = 2(1.5-v)$

$9 = 3-2v$

$2v = -6$

$v = -3$

[or conservation of momentum $(\rightarrow)$

$3u+2v = (3\times1)+(2\times1.5)$

$12+2v = 3+3 = 6$

$\quad\quad 2v = -6$

$\quad\quad v = -3]$

The speed of Q before the collision was 3 m s^{-1} and it was moving in the opposite direction to its direction after the collision.
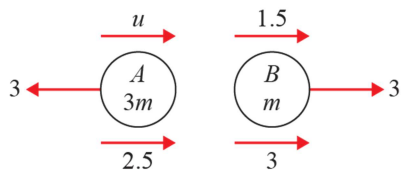
12 a Conservation of momentum $(\rightarrow)$

$3mu+1.5m = (3m\times2.5)+(m\times3)$

$3mu+1.5m = 7.5m+3m$

$\quad\quad 3u = 9$

$\quad\quad u = 3$

The speed of A before the collision is 3 m s^{-1}

12 b Using the impulse–momentum principle:

for $B : (\rightarrow)$

$3 = m(3 - 1.5)$
$2 = m$

[or for $A : (\leftarrow)$

$3 = 3m(-2.5 - (-u))$
$3 = 3m(-2.5 + 3)$
$1 = 0.5m$
$2 = m$]

The mass of A is 6 kg.

Challenge

$P : I = \dfrac{9mu_1}{4} \qquad Q : I = \dfrac{3mu_2}{2}$

$\dfrac{9mu_1}{4} = \dfrac{3mu_2}{2}$ gives $u_1 = \dfrac{2u_2}{3}$