Work, energy and power 2C

1 a P.E. lost $= mgh = 0.4 \times 9.8 \times 7$

$= 27.44$

The P.E. lost is 27.4 J (3 s.f.)

b K.E. gained $= \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$= \frac{1}{2} \times 0.4 \times v^2 - 0$

P.E. lost = K.E. gained

$27.44 = \frac{1}{2} \times 0.4 \times v^2$

$v^2 = \frac{27.44}{0.2}$

$v = 11.71\ldots$

The final speed of the particle is 11.7 m s^{-1} (3 s.f.)

2 a K.E. gained $= \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$= \frac{1}{2} \times 0.5 \times 12^2 - 0$

$= 36$

The K.E. gained by the stone is 36 J

b P.E. lost = K.E. gained

= 36 J

The P.E. lost by the stone is 36 J

c P.E. lost $= mgh$

$36 = 0.5 \times 9.8 \times h$

$h = \frac{36}{0.5 \times 9.8}$

$h = 7.346\ldots$

The height of the tower is 7.35 m (3 s.f.)

3

a Increase in K.E. $= \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$= \frac{1}{2} \times 6 \times 5^2 - \frac{1}{2} \times 6 \times 2.5^2$

$= 56.25$

The increase in K.E. of the box is 56.3 J (3 s.f.)

b The work done by the force is 56.3 J

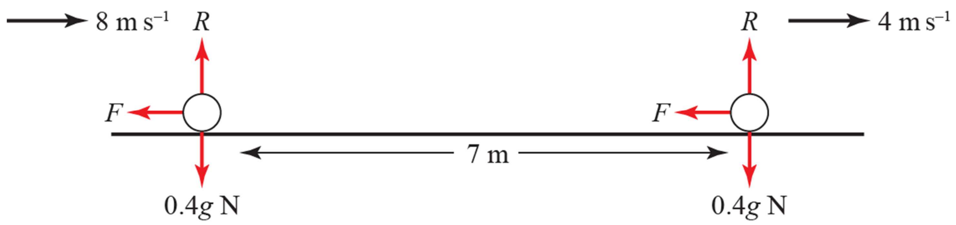
3 c Work done $= Fs$ ← Work done = change in energy

$56.25 = 10 \times s$

$s = \dfrac{56.25}{10} = 5.625$

The distance PQ is 5.63 m (3 s.f.)

4



a K.E. lost $= \frac{1}{2}mu^2 - \frac{1}{2}mv^2$

$= \frac{1}{2} \times 0.4 \times 8^2 - \frac{1}{2} \times 0.4 \times 4^2$

$= 9.6$

The K.E. lost by the particle is 9.6 J

b The work done against friction is 9.6 J ← Work done = change in energy

c Resolving perpendicular to the surface: $R = 0.4g$

Friction is limiting: $F = \mu R$

$F = 0.4g \times \mu$

Work done $= Fs$

$9.6 = 0.4g \times \mu \times 7$

$\mu = \dfrac{9.6}{0.4 \times 9.8 \times 7} = 0.3498\ldots$

The coefficient of friction is 0.350 (3 s.f.)

5

$\mu = 0.4$

a K.E. lost $= \frac{1}{2}mu^2 - \frac{1}{2}mv^2$

$= \frac{1}{2} \times 3 \times 6^2 - 0$

$= 54$

The kinetic energy lost by the box is 54 J

b The work done against friction is 54 J

5 c Resolving perpendicular to the floor: $R = 3g$

Friction is limiting: $F = \mu R$

$$F = 0.4 \times 3g$$

Work done $= Fs$

$$54 = 0.4 \times 3g \times s$$

$$s = \frac{54}{0.4 \times 3g} = 4.591\ldots$$

The distance AB is 4.59 m (3 s.f.)

6 P.E. lost $= mgh$

$$= 0.8 \times 9.8 \times 5$$

$$= 39.2$$

K.E. gained $=$ P.E. lost

$$= 39.2$$

K.E. gained $= \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$$39.2 = \frac{1}{2} \times 0.8v^2 - 0$$

$$v^2 = \frac{39.2 \times 2}{0.8}$$

$$v = 9.899\ldots$$

The particle hits the ground at a speed of 9.90 m s^{-1} (3 s.f.)

7 K.E. gained $= \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$$= \frac{1}{2} \times 0.3 \times 20^2 - 0$$

$$= 60$$

P.E. lost $=$ K.E. gained

$$= 60$$

P.E. lost $= mgh$

$$60 = 0.3 \times 9.8 \times h$$

$$h = \frac{60}{0.3 \times 9.8}$$

$$h = 20.40\ldots$$

The cliff is 20.4 m high (3 s.f.)

8 P.E. gained $= mgh$

$= 0.3 \times 9.8 \times 5$

K.E. lost = initial K.E. − final K.E.

$= \frac{1}{2} \times mu^2 - 2.1$

$= \frac{1}{2} \times 0.3u^2 - 2.1$

K.E. lost = P.E. gained

$\frac{1}{2} \times 0.3u^2 - 2.1 = 0.3 \times 9.8 \times 5$

$u^2 = \frac{0.3 \times 9.8 \times 5 + 2.1}{\frac{1}{2} \times 0.3}$

$u = 10.58\ldots$

The value of u is 10.6 (3 s.f.)

9 Loss of K.E. $= \frac{1}{2}mu^2 - \frac{1}{2}mv^2$

$= \frac{1}{2} \times 0.1 \times 500^2 - 0$

Work done by resistance $= Fs$

$= F \times 0.05$ ← 1 mm = 0.001 m

Work done by resistance = loss of K.E.

$F \times 0.05 = \frac{1}{2} \times 0.1 \times 500^2$

$F = \frac{\frac{1}{2} \times 0.1 \times 500^2}{0.05}$

$= 250\ 000$

The magnitude of the resistive force is 250 000 N (or 250 kN)

10 a Loss of K.E $= \frac{1}{2}mu^2 - \frac{1}{2}mv^2$

$= \frac{1}{2} \times 0.15 \times 500^2 - 0$ ← 150 g = 0.15 kg

Work done by resistance $= Fs$

$= 250\,000s$

Work done by resistance = loss of K.E.

$250\,000s = \frac{1}{2} \times 0.15 \times 500^2$

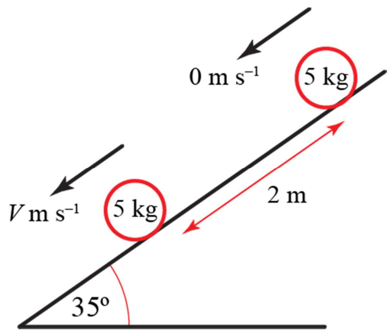
$s = \frac{\frac{1}{2} \times 0.15 \times 500^2}{250\ 000}$

$= 0.075$

The distance the bullet penetrates the wall is 0.075 m (or 75 mm)

b The resistive force could depend on the speed of the bullet.

11



a P.E. lost = mgh

$$= 5 \times 9.8 \times (2 \sin 35°)$$
$$= 56.21\ldots$$

The P.E. lost is 56.2 J (3 s.f.)

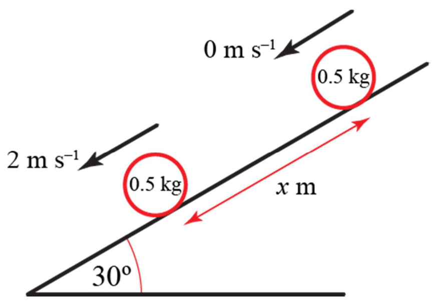
b The K.E. gained is 56.2 J

c K.E. gained $= \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$$56.21 = \frac{1}{2} \times 5 \times v^2 - 0$$
$$v^2 = \frac{56.21 \times 2}{5}$$
$$v = 4.741\ldots$$

The final speed of the package is 4.74 m s^{-1} (3 s.f.)

12



K.E. gained $= \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$$= \frac{1}{2} \times 0.5 \times 2^2 - 0$$
$$= 1$$

P.E. lost $= mgh = 0.5 \times 9.8 \times (x \sin 30°)$

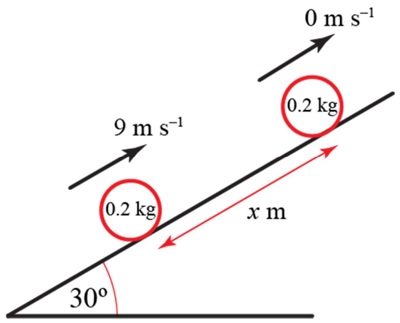
P.E. lost = K.E. gained

$0.5 \times 9.8 \times (x \sin 30°) = 1$

$$x = \frac{1}{0.5 \times 9.8 \times \sin 30°}$$
$$= 0.4081\ldots$$

The value of x is 0.408 (3 s.f.)

13



K.E. lost $= \frac{1}{2}mu^2 - \frac{1}{2}mv^2$

$= \frac{1}{2}\times 0.2\times 9^2 - 0$

P.E. gained $= mgh$

$= 0.2\times 9.8\times (x\sin 30°)$

P.E. gained = K.E. lost

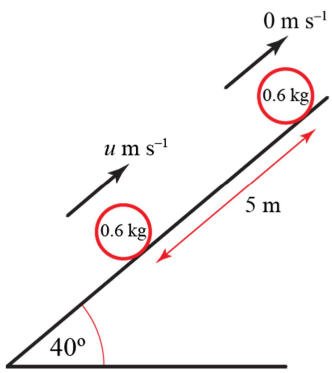
$0.2\times 9.8\times (x\sin 30°) = \frac{1}{2}\times 0.2\times 9^2$

$$x = \frac{\frac{1}{2}\times 0.2\times 9^2}{0.2\times 9.8\sin 30°}$$

$= 8.265\ldots$

The value of x is 8.27 (3 s.f.)

14



K.E. lost $= \frac{1}{2}mu^2 - \frac{1}{2}mv^2$

$= \frac{1}{2}\times 0.6u^2 - 0$

P.E. gained $= mgh$

$= 0.6\times 9.8\times (5\sin 40°)$

K.E. lost = P.E. gained

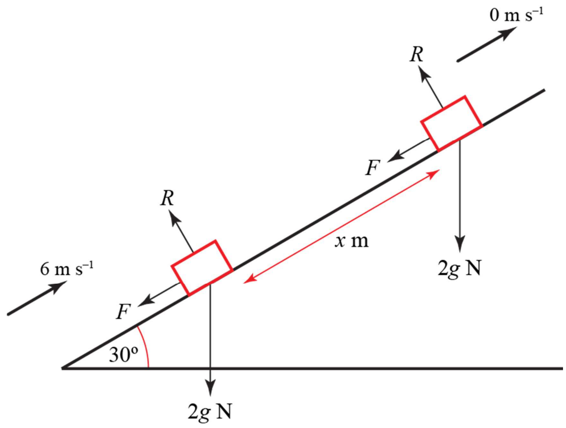
$\frac{1}{2}\times 0.6u^2 = 0.6\times 9.8\times 5\sin 40°$

$$u^2 = \frac{0.6\times 9.8\times 5\sin 40°}{\frac{1}{2}\times 0.6}$$

$u = 7.936\ldots$

The speed of projection is 7.94 m s^{-1} (3 s.f.)

15



$$\text{K.E. lost } = \frac{1}{2}mu^2 - \frac{1}{2}mv^2$$
$$= \frac{1}{2} \times 2 \times 6^2 - 0$$
$$= 36$$

$$\text{P.E. gained} = mgh$$
$$= 2 \times 9.8 \times (x \sin 30°)$$
$$= 9.8x$$

Resolving perpendicular to the plane: $R = 2g \cos 30°$

Friction is limiting: $F = \mu R$

$$F = \frac{1}{3} \times 2g \cos 30° = \frac{2}{3} g \cos 30°$$

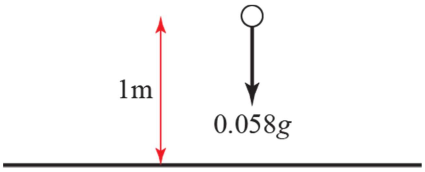
Work done against friction $= Fx = \frac{2}{3} gx \cos 30°$

K.E. lost = P.E. gained + work done against friction

$$\Rightarrow 36 = 9.8x + \frac{2}{3} gx \cos 30°$$
$$36 = 9.8x\left(1 + \frac{2}{3}\cos 30°\right)$$
$$x = \frac{36}{9.8\left(1 + \frac{2}{3}\cos 30°\right)} = 2.328\ldots$$

The particle moves 2.33 m up the plane (3 s.f.)

16



a Resolving vertically:

$I = mv - mu$

$1.7 = 0.058v - 0.058 \times 0$

$v = \frac{1.7}{0.058}$

$v = 29.31\ldots$

The initial speed of the ball is 29.3 m s^{-1} (3 s.f.)

b Energy lost = initial kinetic energy – final potential energy

$= \frac{1}{2}mv^2 - mgh$

$= \frac{1}{2} \times 0.058 \times 29.31^2 - 0.058 \times 9.8 \times 27$

$= 9.566\ldots$

The energy lost due to air resistance is 9.57 J (3 s.f.)

The ball started 1 m above the ground, so the increase in height is 27 m.

c Work done against air resistance = energy lost

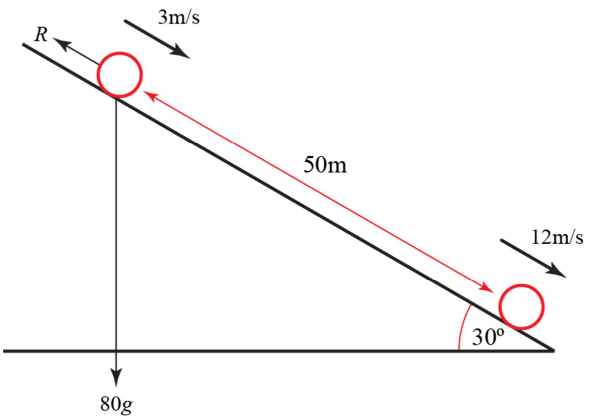
$R \times 27 = 9.566\ldots$

$R = \frac{9.566\ldots}{27}$

$R = 0.3543\ldots$

The value of R is 0.354 (3 s.f.)

17



a Work done by resistive forces on the skier = change in total energy of the skier

Loss in P.E. = mgh

Increase in K.E. = $\frac{1}{2}mv^2 - \frac{1}{2}mu^2$

Total loss of energy = P.E. lost – K.E. gained

$= mgh + \frac{1}{2}mu^2 - \frac{1}{2}mv^2$

Force × distance = $mgh + \frac{1}{2}mu^2 - \frac{1}{2}mv^2$

$50R = (80 \times 9.8 \times 50 \sin 30°) + \left(\frac{1}{2} \times 80 \times 3^2\right) - \left(\frac{1}{2} \times 80 \times 12^2\right)$

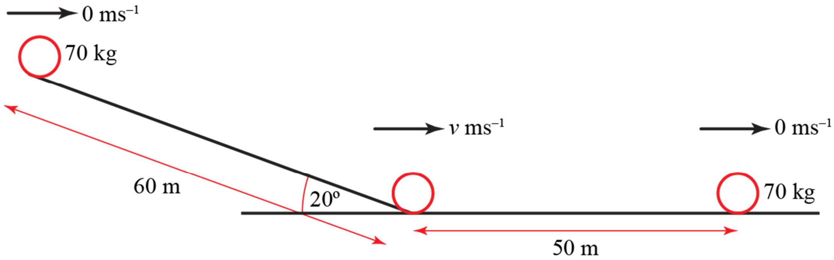
$50R = 14\,200$

$R = 284$

The value of R is 284.

17 b The resistive force may not be constant, and could depend on speed, for example.

18



Consider energy changes from start to end – do not divide the motion into two parts.

Change in K.E. $= \frac{1}{2}mv^2 - \frac{1}{2}mu^2$
$= 0 - 0$

Loss of P.E. $= mgh$
$= 70 \times 9.8 \times (60 \sin 20°)$

Work done against resistance $= Fs$
$= R \times (60 + 50)$
$= 110R$

Work done against resistance = loss of P.E.

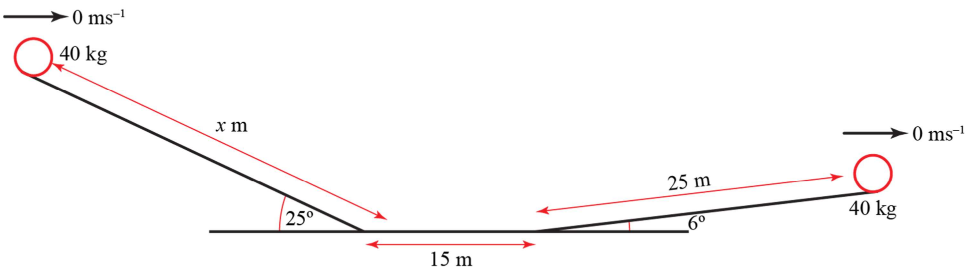
$$110R = 70 \times 9.8 \times (60 \sin 20°)$$

$$R = \frac{70 \times 9.8 \times 60 \sin 20°}{110}$$

$$R = 127.9\ldots$$

The value of R is 128 (3 s.f.)

19



Consider energy changes from start to end – do not divide the motion into three parts.

Loss of P.E. $= mgh$
$= 40 \times 9.8 \times (x \sin 25° - 25 \sin 6°)$

Change in K.E. $= \frac{1}{2}mv^2 - \frac{1}{2}mu^2$
$= 0 - 0$

Work done against resistance $= Fs$
$= 18 \times (x + 15 + 25)$
$= 18 \times (x + 40)$

Work done against resistance = loss of P.E.

$$18x + 18 \times 40 = 40 \times 9.8 \times x \sin 25° - 40 \times 9.8 \times 25 \sin 6°$$

$$(40 \times 9.8 \sin 25° - 18)x = 18 \times 40 + 40 \times 9.8 \times 25 \sin 6°$$

$$x = \frac{18 \times 40 + 40 \times 9.8 \times 25 \sin 6°}{40 \times 9.8 \sin 25° - 18}$$

$$x = 11.81\ldots$$

The girl travels 11.8 m down the slope.

Challenge
Let the mass of a hydrogen molecule $= m$
So the mass of an oxygen molecule $= 8m$

Consider the average kinetic energy of the oxygen molecules:
$\frac{1}{2}mv^2 = \frac{1}{2} \times 8m \times 400^2 = \frac{3}{2}kT$

Consider the average kinetic energy of the hydrogen molecules:
Average K.E. $= \frac{3}{2}kT = \frac{1}{2} \times 8m \times 400^2 = \frac{1}{2}mv^2$
So $\frac{1}{2} \times 8m \times 400^2 = \frac{1}{2}mv^2$
$8 \times 400^2 = v^2$
$v = \sqrt{1\,280\,000}$
$= 1131.3\ldots$
The average speed of the hydrogen molecules is 1130 m s^{-1}