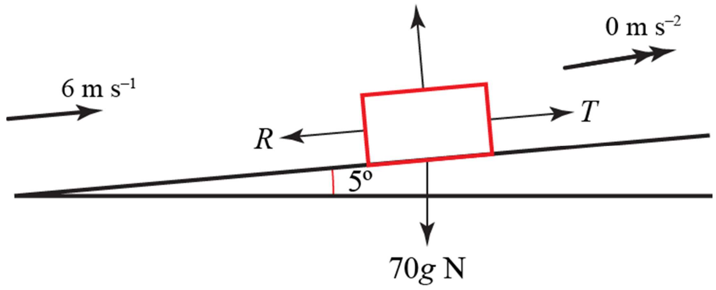
Work, energy and power Mixed exercise 2

1



Power $= Fv$

$480 = T \times 6$

$T = \frac{480}{6} = 80$

Resolving parallel to the slope:

$T = R + 70g\sin 5°$

$80 = R + 70 \times 9.8\sin 5°$

$R = 80 - 70 \times 9.8\sin 5°$

$R = 20.21\ldots$

The magnitude of the resistance is 20.2 N (3 s.f.)

2 **a** P.E. gained by water and bucket $= mgh$

$= 12 \times 9.8 \times 25$

$= 2940$

Initial K.E. = final K.E. = 0

Work done by the boy = P.E. gained by bucket

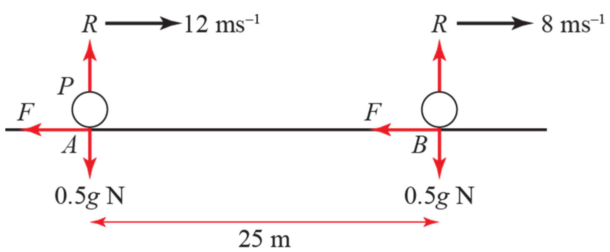
= 2940 J

b Average rate of working $= \frac{\text{work done}}{\text{time taken}} = \frac{2940}{30}$

$= 98$

The average rate of working of the boy is 98 J s^{-1} (or 98 W)

3



a K.E. lost by particle $= \frac{1}{2} \times 0.5 \times 12^2 - \frac{1}{2} \times 0.5 \times 8^2$

$= 20$

Work done by friction = K.E. lost by particle

$\therefore$ Work done by friction = 20 J

3 b Resolving vertically: $R = 0.5g$

Friction is limiting:

$F = \mu R = \mu \times 0.5g$

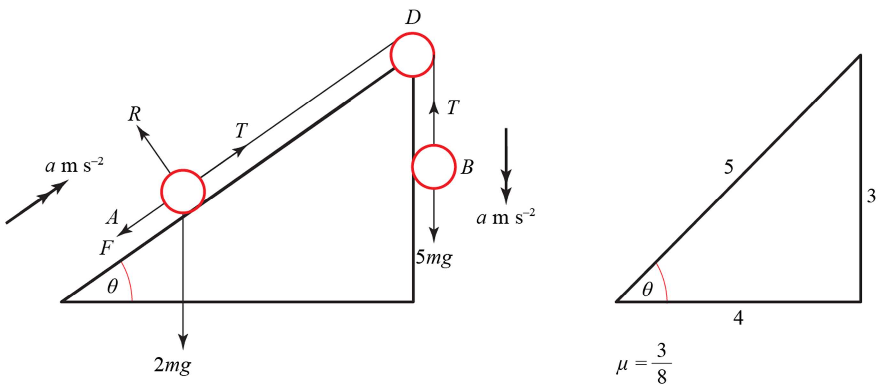
Work done by friction $= F \times s$

$$20 = \mu \times 0.5g \times 25$$

$$\mu = \frac{20}{0.5g \times 25} = 0.1632\ldots$$

The coefficient of friction is 0.163 (3 s.f.)

4



a Resolving perpendicular to the plane for A:

$R = 2mg\cos\theta$

Friction is limiting:

$F = \mu R$

$$F = \frac{3}{8} \times 2mg\cos\theta$$

$$= \frac{3}{8} \times 2mg \times \frac{4}{5}$$

$$= \frac{3}{5}mg$$

$F = ma$ for A: $\quad T - (F + 2mg\sin\theta) = 2ma$

$$T - \left(\frac{3}{5}mg + 2mg \times \frac{3}{5}\right) = 2ma$$

$$T - \frac{9mg}{5} = 2ma \quad (1)$$

$F = ma$ for B: $\quad 5mg - T = 5ma \quad (2)$

$(1) + (2)$: $\quad 5mg - \frac{9mg}{5} = 7ma$

$$\frac{16mg}{5} = 7ma$$

$$a = \frac{16g}{35} = \frac{16 \times 9.8}{35}$$

$$a = 4.48$$

The initial acceleration of A is 4.48 m s^{-2}

4 b For the first 1 m A travels

> The motion must be considered in two parts, before and after the string breaks. The friction force acting on A is the same throughout the motion.

$u = 0$
$a = 4.48 \text{ m s}^{-2}$
$s = 1 \text{ m}$
$v = ?$

$v^2 = u^2 + 2as$
$v^2 = 2 \times 4.48 \times 1$
$v^2 = 8.96$

After string breaks:

$$\text{Loss of K.E. (of A)} = \frac{1}{2}mu^2 - \frac{1}{2}mv^2$$
$$= \frac{1}{2} \times 2m \times 8.96 - 0$$
$$= 8.96m$$

$$\text{Gain of P.E. (of A)} = mgh$$
$$= 2mg \times (x \sin\theta)$$
$$= 2mg \times x \times \frac{3}{5}$$
$$= \frac{6mgx}{5}$$

where x is the distance moved up the plane.

$$\text{Work done by friction} = \frac{3mg}{5} \times x$$

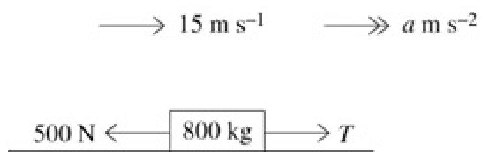
Work–energy principle:

$$\frac{3mgx}{5} + \frac{6mgx}{5} = 8.96m$$
$$\frac{9gx}{5} = 8.96$$
$$x = \frac{8.96 \times 5}{9 \times 9.8}$$
$$x = 0.5079\ldots$$

$$\text{Total distance moved} = 1 + 0.5079\ldots$$
$$= 1.51$$

The total distance moved by A before it first comes to rest is 1.51 m (3 s.f.)

5 a



Ensure units are consistent.

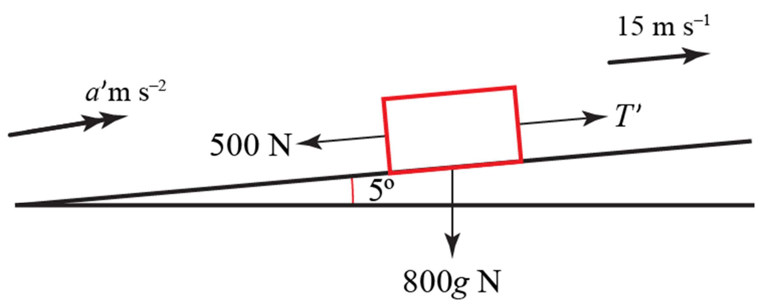
$$\text{Power} = Fv$$
$$16000 = T \times 15$$
$$T = \frac{16000}{15}$$

Using $F = ma$:

$$T - 500 = 800a$$
$$\frac{16000}{15} - 500 = 800a$$
$$a = \frac{\frac{16000}{15} - 500}{800}$$
$$a = 0.7083\ldots$$

The acceleration is 0.708 m s^{-2}

b



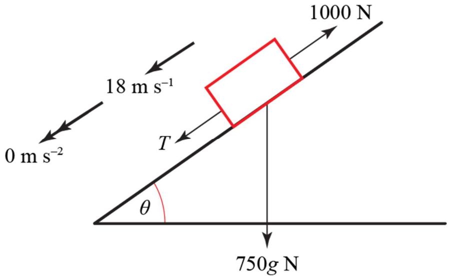
$$\text{Power} = Fv$$
$$24000 = T' \times 15$$
$$T' = \frac{24000}{15}$$

Resolving parallel to the slope and using $F = ma$:

$$T' - 500 - 800g\sin 5° = 800a'$$
$$\frac{24000}{15} - 500 - 800 \times 9.8\sin 5° = 800a'$$
$$800a' = 416.698\ldots$$
$$a' = 0.5208\ldots$$

The new acceleration is 0.521 m s^{-2} (3 s.f.)

6 a



$\tan\theta = \frac{1}{20}$ so $\theta = 2.8624°$

Resolving parallel to the slope:

$T + 750g\sin\theta = 1000$

$T = 1000 - 750 \times 9.8\sin 2.8624°$

$T = 632.95$

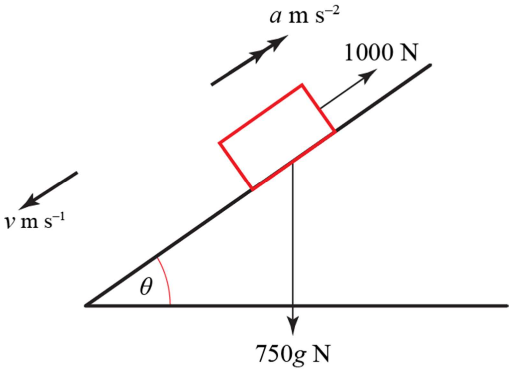
Power $= Fv$

$= 632.95 \times 18$

$= 11393.2\ldots$

The rate of working of the car's engine is 11.4 kW (3 s.f.)

b



The tractive force is zero.

Resolving parallel to the slope and using $F = ma$:

$1000 - 750 \times 9.8 \times \sin\theta = 750a$

$a = \frac{1000 - 750 \times 9.8\sin 2.8624°}{750}$

$a = 0.8439$

Consider motion down the slope:

$a = -0.8439$ m s^{-2}, $u = 18$ m s^{-1}, $v = 0$ m s^{-1}, $t = ?$

$v = u + at$

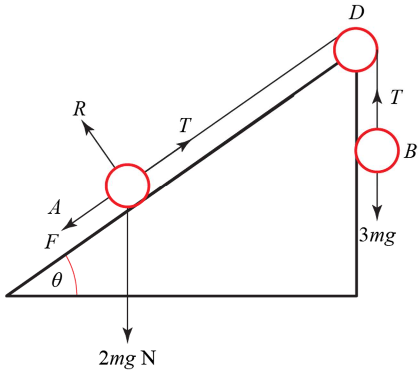
$0 = 18 - 0.8439 \times t$

$t = \frac{18}{0.8439}$

$t = 21.32\ldots$

The value of t is 21.3 (3 s.f.)

7



a P.E. gained by $A = mgh$

$$= 2mg \times (s \times \sin\theta)$$

$$= 2mg \times s \times \frac{3}{5}$$

$$= \frac{6mgs}{5}$$

P.E. lost by $B = mgh$

$$= 3mgs$$

$\therefore$ P.E. lost by system $= 3mgs - \frac{6mgs}{5} = \frac{9mgs}{5}$

7 b Consider A:

Find the frictional force and use the work–energy principle.

Resolving perpendicular to the slope:

$$R = 2mg\cos\theta$$
$$= 2mg \times \frac{4}{5}$$
$$= \frac{8mg}{5}$$

Friction is limiting:

$$F = \mu R$$
$$= \frac{1}{4} \times \frac{8mg}{5}$$
$$= \frac{2mg}{5}$$

Work done against friction $= Fs$

$$= \frac{2mgs}{5}$$

K.E. gained by A and B $= \frac{1}{2}(2m)v^2 + \frac{1}{2}(3m)v^2$

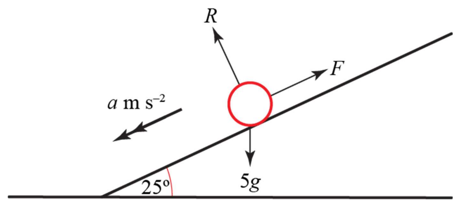
$$= \frac{5mv^2}{2}$$

Work–energy principle:

K.E. gained + work done against friction = P.E. lost

$$\frac{5mv^2}{2} + \frac{2mgs}{5} = \frac{9mgs}{5}$$
$$\frac{5mv^2}{2} = \frac{7mgs}{5}$$
$$v^2 = \frac{2 \times 7mgs}{5 \times 5m}$$
$$v^2 = \frac{14gs}{25}$$

8 a



Resolving parallel to the slope and using $F = ma$:

$5g\sin 25° - F = 5a$

Friction is limiting:

$F = \mu R$

$F = 0.3 \times 5g\cos 25°$

So $5g\sin 25° - 5\times 0.3\times g\cos 25° = 5a$

$$a = g(\sin 25° - 0.3\cos 25°)$$

Consider the motion down the slope.

$u = 0$ and $t = 2$

$v = u + at$

$= 0 + 2g(\sin 25° - 0.3\cos 25°)$

$= 2g(\sin 25° - 0.3\cos 25°)$

$= 2.9542\ldots$

After it has been moving for 2 s the parcel has speed 2.95 m s^{-1} (3 s.f.)

b In 2 s the parcel slides a distance s m down the sloping platform.

Loss of P.E. $= mgh$

$= mg \times s\sin 25°$

$= 5g \times s\sin 25°$

$u = 0$, $v = 2.954$ m s^{-1}, $t = 2$ s

Using $s = \dfrac{u+v}{2}\times t$

$s = \dfrac{0 + 2.954}{2}\times 2 = 2.954$

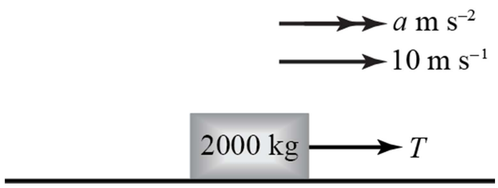
So, loss of P.E. $= 5g \times 2.954 \times \sin 25°$

$= 5\times 9.8\times 2.954\times \sin 25°$

$= 61.17\ldots$

During the 2 s, the parcel loses 61.2 J of kinetic energy (3 s.f.)

9



Power $= 4000$ W
Power $= Tv = 10T$

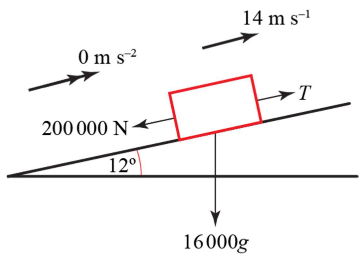
So $T = \dfrac{4000}{10} = 400$ N

Using $F = ma$:

$$T = 2000 \times a$$
$$400 = 2000a$$

So $a = \dfrac{400}{2000} = 0.2 \text{ m s}^{-2}$

10



Resolving parallel to the slope:

$T = 200000 + 16000g \sin 12°$

$T = 232600.5\cdots$

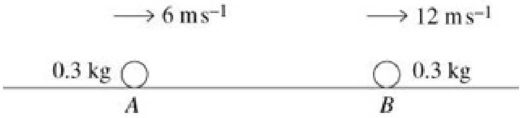
Work done in 10 s = force $\times$ distance moved

$= 232\ 600\ldots \times (14 \times 10)$

$= 32\ 564\ 000$ (3 s.f.)

The work done in 10s is 32 600 000 J (or 32 600 kJ) (3 s.f.)

11



a K.E. gained $= \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$= \frac{1}{2}\times 0.3\times 12^2 - \frac{1}{2}\times 0.3\times 6^2$

$= 16.2$

The K.E. gained is 16.2 J

b The work done by the force is 16.2 J

c Work done $= Fs$

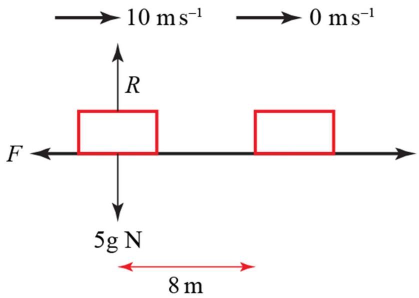
$16.2 = F\times 4$

$F = \frac{16.2}{4}$

$F = 4.05$

The force has magnitude 4.05 N

12



a K.E. lost $= \frac{1}{2}mu^2 - \frac{1}{2}mv^2$

$= \frac{1}{2}\times 5\times 10^2 - 0$

$= 250$

The K.E. lost is 250 J

b Work done against friction = 250 J

Work done $= Fs$

$250 = F\times 8$

$F = \frac{250}{8}$

Resolving perpendicular to the slope: $R = 5g$

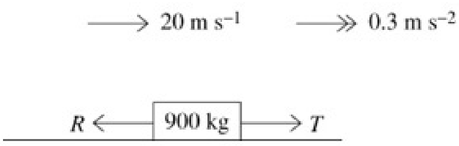
Friction is limiting: $F = \mu R$

$\frac{250}{8} = \mu\times 5g$

$\mu = \frac{250}{8\times 5g}$

The coefficient of friction is 0.638 (3 s.f.)

13



a Power $= Fv$

$15000 = T \times 20$

$$T = \frac{15000}{20} = 750$$

Using $F = ma$:

$T - R = 900 \times 0.3$

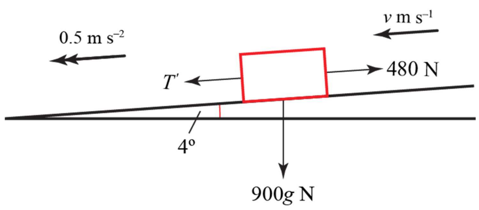
$750 - R = 270$

$R = 750 - 270$

$R = 480$

The magnitude of the resistance is 480 N

b



Resolving along the slope and using $F = ma$:

$T' + 900g\sin 4° - 480 = 900 \times 0.5$

$T' = 450 + 480 - 900g\sin 4°$

Power $= Fv$

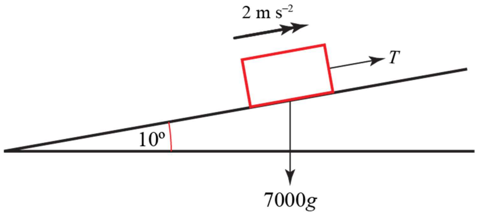
$8000 = (450 + 480 - 900g\sin 4°)v$

$$v = \frac{8000}{(450 + 480 - 900g\sin 4°)}$$

$v = 25.41\ldots$

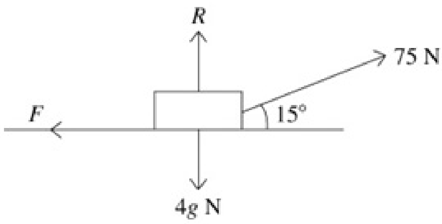
The speed of the car is 25.4 m s^{-1} (3 s.f.)

14



Power $= Fv$
Power $= 4000$ W
$T = \frac{4000}{v}$
Resolving along the slope and using $F = ma$:
$\frac{4000}{v} - 7000g\sin 10° = 7000 \times 2$
$\frac{4000}{v} = 25912$
So $v = \frac{4000}{25912} = 0.154\ldots$
The speed of the bus is 0.15 m s^{-1} (2 s.f.)

15



$\mu = \frac{3}{8}$

a Resolving perpendicular to the floor:
$R + 75\sin 15° = 4g$
$R = 4g - 75\sin 15°$
Friction is limiting: $F = \mu R$
$F = \frac{3}{8} \times (4 \times 9.8 - 75\sin 15°)$
$F = 7.420\ldots$
The magnitude of the frictional force is 7.42 N (3 s.f.)

b Work done $= Fs$
$= 75\cos 15° \times 6$
$= 434.66\ldots$
The work done is 435 J (3 s.f.)

15 c Using the work–energy principle:

K.E. gained = work done by tension – work done against friction

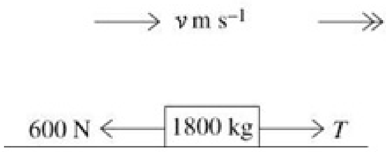
$$\frac{1}{2}\times 4v^2 = 434.66 - 7.420\times 6$$

$$v^2 = \frac{1}{2}(434.66 - 7.420\times 6)$$

$$v = 13.96\ldots$$

The block is moving at 14.0 m s^{-1} (3 s.f.)

16 a



At maximum speed, $a = 0$

Resolving along the road and using $F = ma$:

$$T - 600 = 0$$

$$T = 600$$

Power $= Fv$

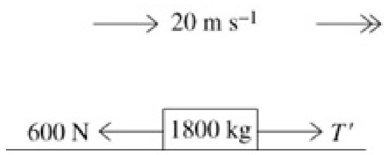
$$20\,000 = 600v$$

$$v = \frac{20\,000}{600}$$

$$v = 33.33$$

The lorry's maximum speed is 33.3 m s^{-1} (3 s.f.)

b



Power $= Fv$

$$20\,000 = T'\times 20$$

$$T' = 1000$$

Using $F = ma$:

$$T' - 600 = 1800a$$

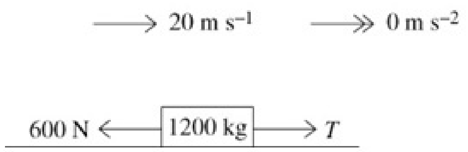
$$1000 - 600 = 1800a$$

$$a = \frac{400}{1800}$$

$$a = 0.2222\ldots$$

The acceleration of the lorry is 0.222 m s^{-2} (3 s.f.)

17

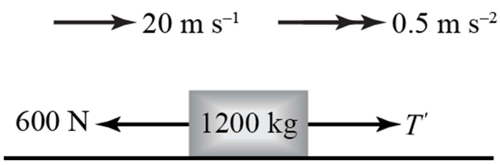


a Resolving along the road: $T = 600$

$$\begin{aligned}\text{Power } &= Fv\\ &= 600 \times 20\\ &= 12\ 000\text{ W}\\ &= 12\text{ kW}\end{aligned}$$

The power is 12 kW

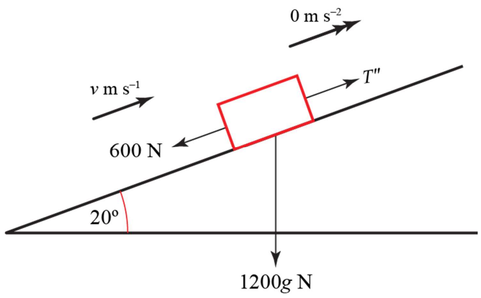
b



$$\begin{aligned}F &= ma\\ T' - 600 &= 1200 \times 0.5\\ T' &= 600 + 600\\ T' &= 1200\\ \text{Power} &= F \times v\\ &= 1200 \times 20\\ &= 24\,000\end{aligned}$$

The new rate of working is 24 kW

c



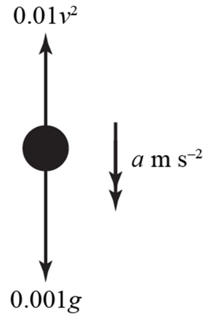
Resolving along the slope:

$T'' = 600 + 1200g\sin 20°$

$$\begin{aligned}\text{Power} &= Fv\\ 50\,000 &= (600 + 1200g\sin 20°)v\\ v &= \frac{50\,000}{(600 + 1200g\sin 20°)}\\ v &= 10.82\ldots\end{aligned}$$

The value of v is 10.8 (3.s.f.)

18

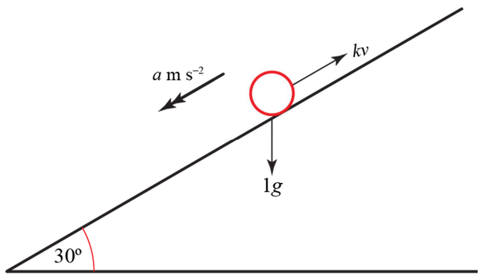


a Applying $F = ma$ vertically downwards:
$0.001g - 0.01v^2 = 0.001a$

When $v = 0.5$:
$0.001 \times 9.8 - 0.01 \times 0.5^2 = 0.001a$
$0.0073 = 0.001a$
So $a = \dfrac{0.0073}{0.001} = 7.3 \text{ m s}^{-2}$

b At the maximum velocity, the resultant force is zero.
So $0.001g = 0.01v^2$
$\dfrac{0.001 \times 9.8}{0.01} = v^2$
So $v^2 = 0.98$
So $v = \sqrt{0.98} = 0.99 \text{ m s}^{-1}$ (2 s.f.)

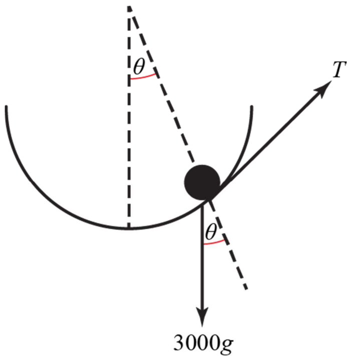
19



a Applying $F = ma$ down the plane:
$1g\sin 30° - kv = 1a$
When $v = 1$:
$9.8\sin 30° - k = a$
So $a = (4.9 - k) \text{ m s}^{-2}$

b At the maximum velocity, forces parallel to the plane are in equilibrium.
So $1g\sin 30° = kv$
$1g\sin 30° = 5k$
So $k = \dfrac{g\sin 30°}{5} = 0.98$

Challenge



a Car is moving with constant speed in a direction along the tangent to the cylinder.
Resolving along the path of the car:

$T = 3000g\sin\theta$

Power $= Tv$
Power $= 3000g\sin\theta \times 20 = 60\,000g\sin\theta = 588\,000\sin\theta$ W

b When $\theta = 0°$, there is no force to act against, so no power is required.
When $\theta = 90°$, maximum power is needed.