Elastic strings and springs 3A

1

$(\uparrow)\, T = 4$ Hooke's law

$$T = \frac{\lambda x}{3}$$

So, $\dfrac{\lambda x}{3} = 4$

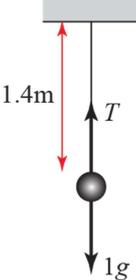$$\Rightarrow x = \frac{12}{\lambda}$$

$\therefore$ Total length of string, $L = 3 + \dfrac{12}{\lambda}$

a $\lambda = 30:\ L = 3 + \dfrac{12}{30}$
$$= 3.4 \text{ m}$$

b $\lambda = 12:\ L = 3 + \dfrac{12}{12}$
$$= 4$$

c $\lambda = 16:\ L = 3 + \dfrac{12}{16}$
$$= 3.75 \text{ m}$$

2 By Hooke's law,
$$20 = \frac{25(l - 0.8)}{l}$$
$$4l = 5l - 4$$
$$4 = l$$
Natural length is 4 m

3

Let natural length be l

$(\uparrow)\, T = g = 9.8$

$$T = \frac{20(1.4 - l)}{l}$$

$$9.8 = 20\frac{(1.4 - l)}{l}$$

$$9.8l = 28 - 20l$$

$$29.8l = 28 \Rightarrow l = \frac{28}{29.8} = \frac{140}{149}$$

Let the new extension be x

$$0.8g = \frac{20x}{\left(\dfrac{140}{149}\right)}$$

$$0.8g = \frac{\cancel{20}\,x \times 149}{\cancel{140}^{\,7}}$$

$$\frac{5.6g}{149} = x$$

$$x \approx 0.3683\ldots$$

Total length of string is $0.3683 + \dfrac{140}{149}$

$$= 1.31 \text{ m} \,(3 \text{ s.f.})$$

4 Let the initial extension be x_1

$(\uparrow)\ T = Mg$

$$Mg = \frac{\lambda x_1}{a} \Rightarrow x_1 = \frac{Mga}{\lambda}$$

When the mass m is added to the scale pan, extension is x_2

$(\uparrow)\ T = (M + m)g$

$$(M + m)g = \frac{\lambda x_2}{a} \Rightarrow x_2 = \frac{(M + m)ga}{\lambda}$$

$$\therefore\ x_2 - x_1 = \frac{ga}{\lambda}(M + m - M) = \frac{mga}{\lambda}$$

New equilibrium is $\dfrac{mga}{\lambda}$ below the old one.

5 $m_1 g = \dfrac{\lambda(a_1 - l)}{l}$ **(1)**

$m_2 g = \dfrac{\lambda(a_2 - l)}{l}$ **(2)**

Dividing **(1)** by **(2)**:

$$\frac{m_1}{m_2} = \frac{a_1 - l}{a_2 - l}$$

$$m_1(a_2 - l) = m_2(a_1 - l)$$

$$m_1 a_2 - m_1 l = m_2 a_1 - m_2 l$$

$$m_1 a_2 - m_2 a_1 = l(m_1 - m_2)$$

$$l = \frac{m_1 a_2 - m_2 a_1}{m_1 - m_2}$$

The natural length $l = \dfrac{m_1 a_2 - m_2 a_1}{m_1 - m_2}$

Subtracting **(2)** from **(1)**:

$$m_1 g - m_2 g = \frac{\lambda a_1}{l} - \lambda - \left(\frac{\lambda a_2}{l} - \lambda \right)$$
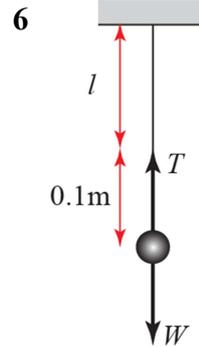
$$lg(m_1 - m_2) = \lambda(a_1 - a_2)$$

$$\lambda = gl \frac{(m_1 - m_2)}{(a_1 - a_2)}$$

Substituting for l:

$$\lambda = g \frac{(m_1 - m_2)}{(a_1 - a_2)} \frac{(m_1 a_2 - m_2 a_1)}{(m_1 - m_2)}$$

$$= g \frac{(m_1 a_2 - m_2 a_1)}{(a_1 - a_2)}$$

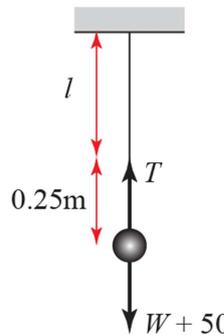The modulus of elasticity is $g \dfrac{(m_1 a_2 - m_2 a_1)}{(a_1 - a_2)}$

6

$\left(\uparrow\right)\ T = W$

$$T = \frac{\lambda \times 0.1}{l}$$

$$W = \frac{\lambda \times 0.1}{l}$$

So $\lambda = 10Wl$

$\left(\uparrow\right)\ T = W + 50$

$$T = \frac{\lambda \times 0.25}{l}$$

$$W + 50 = \frac{\lambda \times 0.25}{l}$$

$$W + 50 = \frac{10Wl \times 0.25}{l}$$

$$W + 50 = \frac{10W}{4}$$

$$50 = \frac{3W}{2}$$

So $W = \dfrac{100}{3}$ N

7 a

$a + x + y + a = 5a$

$y = 3a - x$

$\lambda = 2mg$

$(\uparrow)\ T_1 = T_2 + mg$

$\dfrac{2mgx}{a} = \dfrac{2mg(3a - x)}{a} + mg$

$2x = 2(3a - x) + a$

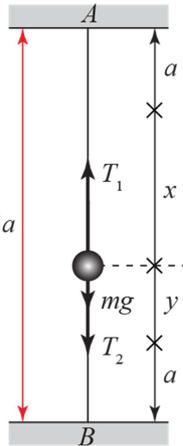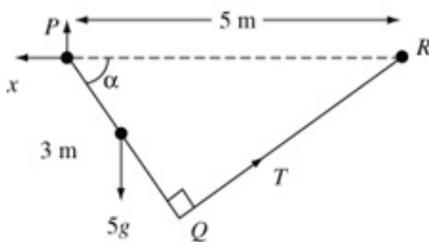$2x = 6a - 2x + a$

$4x = 7a$

$x = \dfrac{7a}{4}$

Distance of particle from ceiling is

$a + x = a + \dfrac{7a}{4} = \dfrac{11a}{4}$

b If the spring is not light, then in effect the mass would increase, the extension would increase and hence the distance of the particle below the ceiling would increase.

8

$PQR = 90° \Rightarrow QR = 4\,\text{m}$

$\cos\alpha = \dfrac{3}{5};\ \sin\alpha = \dfrac{4}{5}$

a Taking moments about P:

$5g \times \dfrac{3}{2}\cos\alpha = 3T$

$5g \times \dfrac{3}{2} \times \dfrac{3}{5} = 3T$

$T = \dfrac{3g}{2} = 14.7$

Tension is 14.7 N

8 b Using Hooke's law:

$14.7 = \dfrac{30(4 - l)}{l}$

$14.7l = 120 - 30l$

$44.7l = 120$

$l = 2.68\ldots$
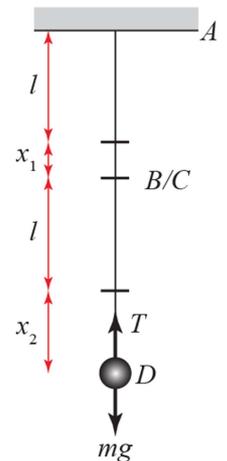
Natural length is 2.7 m (2 s.f.)

9 $(\uparrow)\ T = mg$ throughout the length.

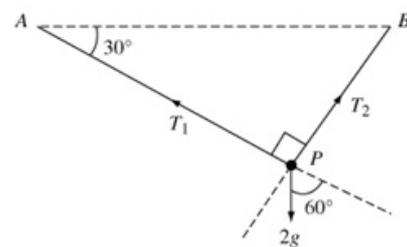So, $mg = \dfrac{2mgx_1}{l} \Rightarrow x_1 = \dfrac{1}{2}l$

and $mg = \dfrac{4mgx_2}{l} \Rightarrow x_2 = \dfrac{1}{4}l$

$\therefore\quad AD = 2l + x_1 + x_2$

$\qquad = \dfrac{11l}{4}$

The length AD is $\dfrac{11l}{4}$

10

a $(\nwarrow)$ (along PA)

$T_1 = 2g\cos 60° = g = 9.8\text{N}$

so $\dfrac{9.8x_1}{0.5} = 9.8$

$x_1 = 0.5$

$\therefore\quad AP = 0.5 + 0.5$

$\qquad = 1\ \text{m}$

b $\dfrac{PB}{AP} = \tan 30°$

$PB = AP\tan 30° = \tan 30° = \dfrac{1}{\sqrt{3}}$

$PB \approx 0.577\ \text{m}$

$\qquad = 0.58\ \text{m} \ (2\ \text{s.f.})$

10 c $\left(\nearrow\right)$ (along PB)

$T_2 = 2g\cos 30°$

$\quad = 2g\dfrac{\sqrt{3}}{2}$

$\quad = g\sqrt{3}$ N

$\quad \approx 17$ N (2 s.f.)

The tension in PB is 17 N (2 s.f.)

11 a
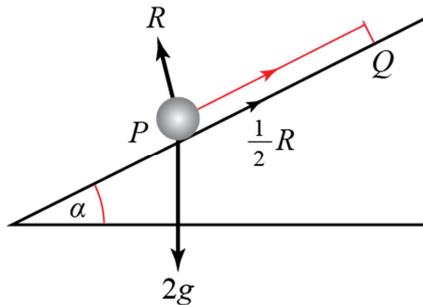
$\tan\alpha = \dfrac{3}{4}$ so $\cos\alpha = \dfrac{4}{5}$ and $\sin\alpha = \dfrac{3}{5}$

$(\nwarrow)\ R = 2g\cos\alpha = \dfrac{8g}{5}$

$\therefore\quad F = \mu R = \dfrac{1}{2} \times \dfrac{8g}{5} = \dfrac{4g}{5}$

$(\nearrow)\ T + F = 2g\sin\alpha$

$\qquad T = 2g\sin\alpha - F$

$\qquad\quad = \left(2g \times \dfrac{3}{5}\right) - \dfrac{4g}{5} = \dfrac{2g}{5}$

$\qquad\quad = 3.92$

The tension in the string is 3.9 N (2 s.f.)

b $\quad T = \dfrac{\lambda x}{l}$

$3.92 = \dfrac{20x}{0.8}$

$\quad x = 0.1568 = 0.16$ (2 s.f.)

Length of the string $= 0.16 + 0.8$

$\qquad\qquad\qquad\quad = 0.96$ m (2 s.f.)