Circular motion 1A

1 a $5\,\text{rev min}^{-1} = 5 \times 2\pi\,\text{rad min}^{-1} = \dfrac{5 \times 2\pi}{60}\,\text{rad s}^{-1} \approx 0.524\ \text{rad s}^{-1}$

b $120\,\text{rev min}^{-1} = 120 \times 2\pi\,\text{rad min}^{-1} = \dfrac{120 \times 2\pi}{60}\,\text{rad s}^{-1} \approx 12.6\,\text{rad s}^{-1}$

c $4\ \text{rad s}^{-1} = 4 \times 60\,\text{rad min}^{-1} = \dfrac{4 \times 60}{2\pi}\,\text{rev min}^{-1} \approx 38.2\,\text{rev min}^{-1}$

d $3\ \text{rad s}^{-1} = 3 \times 60 \times 60\,\text{rad h}^{-1} = \dfrac{3 \times 60 \times 60}{2\pi}\,\text{rev h}^{-1} \approx 1720\ \text{rev h}^{-1}$

2 a $v = r\omega : v = 20 \times 4 = 80\,\text{m s}^{-1}$

b Distance per minute $= 40 \times 2\pi \times 20 = 1600\pi\,\text{m}$

Distance per second $= \dfrac{1600\pi}{60} \approx 83.8\text{m}$, $v = 83.8\,\text{m s}^{-1}$

3 a $\omega = \dfrac{v}{r} = \dfrac{2}{0.25} = 8\,\text{rad s}^{-1}$ ← Need to convert cm to m.

b $8\,\text{rad s}^{-1} = 8 \times 60\,\text{rad min}^{-1} = \dfrac{8 \times 60}{2\pi} \approx 76.4\,\text{rev min}^{-1}$

4 a $v = r\omega : v = 0.8 \times 2.5 = 2\,\text{m s}^{-1}$

b $25\,\text{rev min}^{-1} = 25 \times 2\pi\,\text{rad min}^{-1} = \dfrac{25 \times 2\pi}{60}\,\text{rad s}^{-1} = 2.617\ldots\ \text{rad s}^{-1}$

$v = r\omega : v = 0.8 \times 2.617\ldots \simeq 2.09\,\text{m s}^{-1}$

5 a $\text{time} = \dfrac{\text{distance}}{\text{speed}} = \dfrac{2\pi \times 50}{7} \approx 44.9\,\text{s}$

b $\omega = \dfrac{v}{r} = \dfrac{7}{50} = 0.14\ \text{rad s}^{-1}$

6 a $1\,\text{rev in}\,10\,\text{s} = 0.1\,\text{rev s}^{-1} = 0.1 \times 2\pi\,\text{rad s}^{-1} \approx 0.628\ \text{rad s}^{-1}$

b $v = r\omega : v = 0.12 \times 0.628\ldots \approx 0.0754\,\text{m s}^{-1}$

c $v = r\omega : v = 0.08 \times 0.628\ldots \approx 0.0503\,\text{m s}^{-1}$

7 a $2\,\text{circuits} = 2 \times 2\pi\,\text{radians in}\,45\,\text{seconds} = \dfrac{4\pi}{45}\,\text{rad s}^{-1} \approx 0.279\,\text{rad s}^{-1}$

b $40\,\text{km h}^{-1} = \dfrac{40 \times 1000}{3600} = 11.1\ldots\,\text{m s}^{-1}$

$r = \dfrac{v}{\omega} = \dfrac{11.111}{0.279} \approx 39.8\,\text{m}$

8 $10\,\text{rev min}^{-1} = 10 \times 2\pi\,\text{rad min}^{-1} = \dfrac{10 \times 2\pi}{60}\,\text{rad s}^{-1} = 1.047...\,\text{rad s}^{-1}$

$v = r\omega$: Anish's speed $= 3 \times 1.047\ldots \approx 3.14\,\text{m s}^{-1}$,

Bethany's speed $= 5 \times 1.047\ldots \approx 5.24\,\text{m s}^{-1}$

9 a 1 circuit in 26 seconds $= \dfrac{2\pi}{26}\,\text{rad s}^{-1} = 0.2416...\,\text{rad s}^{-1}$

b $v = r\omega$: $v = 1.5 \times 0.2416\ldots = 0.362\,\text{m s}^{-1}$

10 $150\,\text{km h}^{-1} = \dfrac{150 \times 1000}{3600}\,\text{m s}^{-1} \approx 41.666...\,\text{m s}^{-1}$

$$\omega = \frac{v}{r} = \frac{41.666...}{750} \approx 0.056\,\text{rad s}^{-1}$$

11 a Hour hand : 2π radians in 12 hours $= \dfrac{2\pi}{12 \times 3600}\,\text{rad s}^{-1} \approx 0.000145\,\text{rad s}^{-1}$

Minute hand : 2π radians in 1 hour $= \dfrac{2\pi}{3600}\,\text{rad s}^{-1} \approx 0.00175\,\text{rad s}^{-1}$

b $v = r\omega$: End of hour hand moves at $0.1 \times 0.00145 = 1.45 \times 10^{-5}\,\text{m s}^{-1}$

End of minute hand moves at $0.15 \times 0.00175 = 2.62 \times 10^{-4}\,\text{m s}^{-1}$

12 $1200\,\text{rev min}^{-1} = 1200 \times 2\pi\,\text{rad min}^{-1} = \dfrac{1200 \times 2\pi}{60}\,\text{rad s}^{-1} = 125.66...\,\text{rad s}^{-1}$

$v = r\omega$: $v = 125.66.. \times 0.5 = 62.8\,\text{m s}^{-1}$

13 a $45\,\text{rev min}^{-1} = 45 \times 2\pi\,\text{rad min}^{-1} = \dfrac{45 \times 2\pi}{60}\,\text{rad s}^{-1} = 4.712...\,\text{rad s}^{-1} = 4.71\,\text{rad s}^{-1}$

b $r = \dfrac{v}{\omega} = \dfrac{12}{4.712} \approx 2.55\,\text{cm}$ ← Working is in cm, not m here.

14 Distance travelled in one year $= 2\pi \times 1.5 \times 10^{11}\,\text{m}$,

So speed $= \dfrac{2\pi \times 1.5 \times 10^{11}}{365 \times 24 \times 3600} \approx 29900\,\text{m s}^{-1}$ ← We cannot achieve more than 2 s.f. accuracy in the answer because one of the original values only contained 2 s.f.

15 $v > 5\ \text{ms}^{-1}$

$r = \dfrac{v}{\omega} > \dfrac{5}{\omega}$

$r > 5\ \text{m}$

Challenge

The dots are diametrically opposite each other when they are at their maximum distance.

$$\text{Angular speed} = \frac{2\pi}{190}$$

$$\omega = \left(\frac{2\pi}{190}\right) \div 0.2$$

$$= \frac{\pi}{19} \text{ rads}^{-1}$$