Centres of mass of plane figures 2A

1 $(1\times6)+(4\times3)+(3\times2)+(2\times4)=\bar{x}(1+4+3+2)$ — Use $\sum m_1x_1=\bar{x}\sum m_1$

$6+12+6+8=10\bar{x}$ — Simplify.

$32=10\bar{x}$

$3.2=\bar{x}$ — Solve for $\bar{x}$.

Centre of mass is at (3.2, 0). — Give both coordinates.

2 $(1\times2)+(2\times5)+(3\times1)=\bar{y}(1+2+3)$ — Use $\sum m_iy_i=\bar{y}\sum m_i$

$2+10+3=6\bar{y}$ — Simplify.

$15=6\bar{y}$

$2.5=\bar{y}$ — Solve for $\bar{y}$.

Centre of mass is at (0, 2.5). — Give both coordinates.

3 $(2\times-1)+(3\times-4)+(5\times5)=\bar{x}(2+3+5)$ — Use $\sum m_ix_i=\bar{x}\sum m_i$

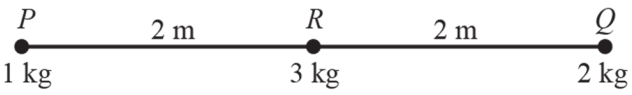
$-2+-12+25=10\bar{x}$ — Simplify.

$11=10\bar{x}$

$1.1=\bar{x}$ — Solve for $\bar{x}$.

Centre of mass is at (1.1, 0). — Give both coordinates.

4



Draw a diagram. The rod has no mass.

$(1\times0)+(3\times2)+(2\times4)=\bar{x}(1+3+2)$ — Take P as the origin and use $\sum m_ix_i=\bar{x}\sum m_i$

$0+6+8=6\bar{x}$ — Simplify.

$\frac{7}{3}=\bar{x}$ — Solve for

$PG=2\frac{1}{3}\text{m}$

5 $(5\times4)+(3\times2)+(m\times5)=4(5+3+m)$ — Use $\sum m_iy_i=\bar{y}\sum m_i$

$20+6+5m=32+4m$ — Simplify.

$m=6$ — Solve for m.

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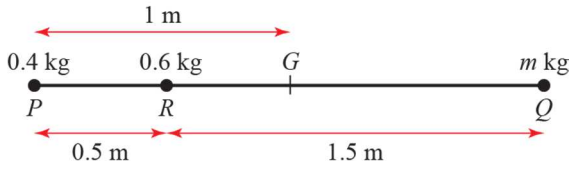
6



The rod, being light, has no mass.

Draw a diagram showing all the information.
G is the centre of mass. Assume the mass of the particle required is m kg.

Take P as the origin.

$(0.4\times 0)+(0.6\times 0.5)+(m\times 2)=1\times(0.4+0.6+m)$ ← Use $\sum m_i x_i = \bar{x}\sum m_i$

$0.3+2m=1.0+m$ ← Simplify.

$m=0.7$ ← Solve for m.

The mass of the particle is 0.7 kg.

7 $(2m\times a)+(3m\times 2)+(7m\times -1)+(8m\times 1)=1\times(2m+3m+7m+8m)$ ← Use $\sum m_i y_i = \bar{y}\sum m_i$

$2ma+6m-7m+8m=20m$ ← Divide by m.

$2a+7=20$

$a=6\frac{1}{2}$ ← Solve for a.

8 Suppose the particle is placed at $(0, y)$.

$(3\times -2)+(2\times 7)+(1\times 4)+(6\times y)=0\times(3+2+1+6)$ ← Use $\sum m_i y_i = \bar{y}\sum m_i$

$-6+14+4+6y=0$ ← Simplify.

$6y=-12$

$y=-2$ ← Solve for y.

The particle must be placed at $(0, -2)$. ← Give both coordinates.

9 $5+m_1+m_2=10$ ← Use the total mass.

$m_1+m_2=5$ **(1)**

$(5\times 2)+(m_1\times 3)+(m_2\times -2)=1\times 10$ ← Use $\sum m_i x_i = \bar{x}\sum m_i$, and $m_i = 10$.

$10+3m_1-2m_2=10$ ← Simplify.

$3m_1-2m_2=0$ **(2)**

Adding **(2)** $+2\times$ **(1)**, $2m_1+2m_2=10$ ← Eliminate m_2

$5m_1=10$ ← Solve for m_1

$m_1=2$

$m_2=3$ ← Use **(1)**.

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10 Let M be the total mass of the system, so we have

$M = (m-1)+(5-m)+m+(m+1)$

i.e. $M = 2m+5$

Given that the centre of mass is at $(0,1)$ taking moments gives

$-1\times(m-1)+(5-m)+2m = M$

i.e. $M = 6$

Hence $6 = 2m+5$ so $m = 0.5$

Challenge

Without loss of generality we can assume that $P=(0,0)$, $Q=(2,0)$ and $R=(5,0)$

Then the total mass is $M = 1+2+3 = 6$

Let $G=(x,0)$ be the centre of mass then taking moments gives

$6x = 2\times 2+3\times 5 = 19$ i.e. $x = \frac{19}{6}$

Hence the ratio

$PQ:PG = 2:\frac{19}{6} = 12:19$

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