Kinematics Mixed exercise 4

1 a $v = \dfrac{dx}{dt} = 4e^{0.5t}$

$x = \displaystyle\int 4e^{0.5t}\,dt = 8e^{0.5t} + A$

When $t = 0, x = 0$

$0 = 8 + A \Rightarrow A = -8$

$x = 8e^{0.5t} - 8 = 8(e^{0.5t} - 1)$

b $a = \dfrac{dv}{dt} = 2e^{0.5t}$

When $t = \ln 9$

$a = 2e^{0.5\ln 9} = 2e^{\ln 3} = 2 \times 3 = 6$

The acceleration of P when $t = \ln 9$ is $6\,\text{ms}^{-2}$

2 a $a = \dfrac{dv}{dt} = 20t\,e^{-t^2}$

$v = \displaystyle\int 20t\,e^{-t^2}\,dt = -10e^{-t^2} + A$

When $t = 0, v = 8$

$8 = -10 + A \Rightarrow A = 18$

$v = 18 - 10e^{-t^2}$

b As $t \to \infty, e^{-t^2} \to 0$ and $v \to 18$

The limiting velocity of P is $18\,\text{ms}^{-1}$

3 $a = \dfrac{dv}{dt} = \dfrac{18}{(2t+3)^3} = 18(2t+3)^{-3}$

$v = \displaystyle\int 18(2t+3)^{-3}\,dt = \dfrac{18}{-2 \times 2}(2t+3)^{-2} + A = A - \dfrac{9}{2(2t+3)^2}$

When $t = 0, v = 0$

$0 = A - \dfrac{9}{2 \times 3^2} \Rightarrow A = \dfrac{1}{2}$

$v = \dfrac{1}{2} - \dfrac{9}{2(2t+3)^2}$

When $v = 0.48$

$0.48 = \dfrac{1}{2} - \dfrac{9}{2(2t+3)^2} \Rightarrow \dfrac{9}{2(2t+3)^2} = 0.02$

So $(2t+3)^2 = \dfrac{9}{2 \times 0.02} = 225 \Rightarrow 2t+3 = \pm15 \Rightarrow t = 6$ or $t = -9$

As $t \geqslant 0$, the solution is $t = 6$

4 a $a = \dfrac{dv}{dt} = \dfrac{100}{(2t+5)^2} = 100(2t+5)^{-2}$

$v = \displaystyle\int 100(2t+5)^{-2}\,dt = \dfrac{100}{2\times -1}(2t+5)^{-1} + A = A - \dfrac{50}{2t+5}$

When $t = 0$, $v = 0$

$\quad 0 = A - \dfrac{50}{5} \Rightarrow A = 10$

$\quad v = 10 - \dfrac{50}{2t+5}$

b $v = \dfrac{dx}{dt} = 10 - \dfrac{50}{2t+5}$

$x = \displaystyle\int \left(10 - \dfrac{50}{2t+5}\right) dt = 10t - 25\ln(2t+5) + B$

When $t = 0$, $x = 0$

$\quad 0 = -25\ln 5 + B \Rightarrow B = 25\ln 5$

$\quad x = 10t - 25\ln(2t+5) + 25\ln 5$

When $t = 10$

$\quad x = 100 - 25\ln 25 + 25\ln 5 = 100 - 25\ln\dfrac{25}{5} = 100 - 25\ln 5 = 59.8\,\text{m (3 s.f.)}$

The distance moved by the car in the first 10 seconds of its motion is $(100 - 25\ln 5)\,\text{m}$.

5 a $a = \dfrac{dv}{dt} = \cos^2 t = \dfrac{1}{2} + \dfrac{1}{2}\cos 2t$

$v = \displaystyle\int \left(\dfrac{1}{2} + \dfrac{1}{2}\cos 2t\right) dt = \dfrac{1}{2}t + \dfrac{1}{4}\sin 2t + A$

When $t = 0$, $v = 0$

$\quad 0 = 0 + 0 + A \Rightarrow A = 0$

$\quad v = \dfrac{1}{2}t + \dfrac{1}{4}\sin 2t$

When $t = \pi$, $v = \dfrac{\pi}{2} + \dfrac{1}{4}\sin 2\pi = \dfrac{\pi}{2} + 0 = \dfrac{\pi}{2}\,\text{m s}^{-1}$

b $x = \displaystyle\int v\,dt = \displaystyle\int \dfrac{1}{2}t + \dfrac{1}{4}\sin 2t\,dt = \dfrac{1}{4}t^2 - \dfrac{1}{8}\cos 2t + B$

When $t = 0$, $x = 0$

$\quad 0 = 0 - \dfrac{1}{8} + B \Rightarrow B = \dfrac{1}{8}$

$\quad v = \dfrac{1}{4}t^2 - \dfrac{1}{8}\cos 2t + \dfrac{1}{8}$

When $t = \dfrac{\pi}{4}$, $x = \dfrac{\pi^2}{64} - \dfrac{1}{8}\cos\dfrac{\pi}{2} + \dfrac{1}{8} = \dfrac{\pi^2}{64} + \dfrac{1}{8} = \dfrac{1}{64}(\pi^2 + 8)\,\text{m}$ as required.

6 a When $t = 2.5, v = \dfrac{1}{2}t^2$

$$a = \dfrac{\mathrm{d}v}{\mathrm{d}t} = t$$

So when $t = 2.5, a = 2.5,$ i.e. the acceleration of P is $2.5\,\mathrm{m\,s^{-2}}$ in the direction of x-increasing

b When $t = 5, v = 8e^{4-t}$

$$a = \dfrac{\mathrm{d}v}{\mathrm{d}t} = -8e^{4-t}$$

So when $t = 5, a = -8e^{-1},$ i.e. the acceleration of P is $8e^{-1}\,\mathrm{m\,s^{-2}}$ in the direction of x-decreasing

c The distance of P from O when $t = 6$ is given by

$$x = \int_0^4 \frac{1}{2}t^2\,\mathrm{d}t + \int_4^6 8e^{4-t}\,\mathrm{d}t = \left[\frac{t^3}{6}\right]_0^4 + \left[-8e^{4-t}\right]_4^6$$

$$= \frac{64}{6} - 8e^{-2} + 8 = \frac{56}{3} - 8e^{-2}$$

The distance of P from O when t = e is $\left(\dfrac{56}{3} - 8e^{-2}\right)\mathrm{m} = 17.6\,\mathrm{m}$ (3 s.f.)

7 a $a = \dfrac{\mathrm{d}v}{\mathrm{d}t} = \dfrac{2t+3}{t+1} = 2 + \dfrac{1}{t+1}$

$v = \displaystyle\int a\,\mathrm{d}t = 2t + \ln(t+1) + A$

When $t = 0, v = 0$

$0 = 0 + A \Rightarrow A = 0$

So $v = 2t + \ln(t+1)$

b $x = \displaystyle\int v\,\mathrm{d}t = \int 2t + \ln(t+1)\,\mathrm{d}t$

Integrate $\ln(t + 1)$ using integration by parts

$$\int \ln(t+1)\,\mathrm{d}t = \int 1\ln(t+1)\,\mathrm{d}t = t\ln(t+1) - \int \frac{t}{t+1}\,\mathrm{d}t$$

$$= t\ln(t+1) - \int\left(1 - \frac{1}{t+1}\right)\mathrm{d}t = t\ln(t+1) - t + \ln(t+1) + C$$

$$= (t+1)\ln(t+1) - t + C$$

So $x = t^2 + (t+1)\ln(t+1) - t + C$

When $t = 0, x = 0$

$0 = 0 + 0 - 0 + C \Rightarrow C = 0$

So $x = t^2 + (t+1)\ln(t+1) - t$

When $t = 2$

$x = 4 + 3\ln 3 - 2 = (2 + 3\ln 3)\,\mathrm{m}$

8 **a** Solving $18 - \dfrac{t^2}{5} = 0$ gives

$18 = \dfrac{t^2}{5} \Rightarrow t^2 = 90 \Rightarrow t = \pm\sqrt{90} = \pm 3\sqrt{10}$

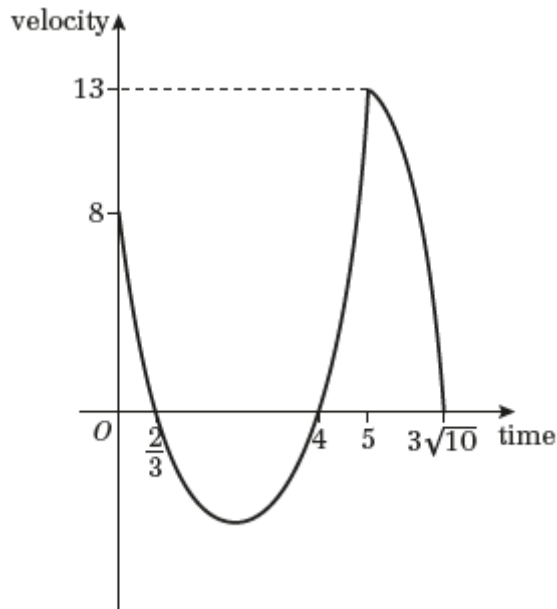As $T > 0, T = 3\sqrt{10}\,$s

b For $0 \leqslant t \leqslant 5$, the curve $y = 3t^2 - 14t + 8$ is a parabola with a positive t^2 coefficient
When $t = 5$, $y = 75 - 70 + 8 = 13$, so the curve connects (0, 8) and (5, 13)

$3t^2 - 14t + 8 = (3t - 2)(t - 4)$, so the curve cuts the x-axis at $\left(\dfrac{2}{3}, 0\right)$ and $(4, 0)$

For $5 < t \leqslant T$, the curve $y = 18 - \dfrac{t^2}{5}$ is a parabola with a negative t^2 coefficient

From part **a**, it connects (5, 13) and $(3\sqrt{10}, 0)$

c The graph has a minimum point where $\dfrac{\mathrm{d}}{\mathrm{d}t}(3t^2 - 14t + 8) = 0$

$6t - 14 = 0 \Rightarrow t = \dfrac{7}{3}$

The acceleration is positive for $\dfrac{\mathrm{d}v}{\mathrm{d}t} > 0$, so the solution is $\dfrac{7}{3} < t < 5$

8 d The total distance travelled is given by

$$\int_0^{\frac{2}{3}} \left(3t^2 - 14t + 8\right) \mathrm{d}t + \left|\int_{\frac{2}{3}}^4 \left(3t^2 - 14t + 8\right) \mathrm{d}t\right|$$

$$+ \int_4^5 \left(3t^2 - 14t + 8\right) \mathrm{d}t + \int_5^{3\sqrt{10}} \left(18 - \frac{t^2}{5}\right) \mathrm{d}t$$

$$= \left[t^3 - 7t^2 + 8t\right]_0^{\frac{2}{3}} + \left|\left[t^3 - 7t^2 + 8t\right]_{\frac{2}{3}}^4\right|$$

$$+ \left[t^3 - 7t^2 + 8t\right]_4^5 + \left[18t - \frac{t^3}{15}\right]_5^{3\sqrt{10}}$$

$$= \left(\frac{68}{27} - 0\right) + \left|\left(-16 - \frac{68}{27}\right)\right| + \left(-10 - (-16)\right)$$

$$+ \left(18\sqrt{90} - \frac{90\sqrt{90}}{15} - \left(90 - \frac{25}{3}\right)\right)$$

$$= \frac{68}{27} + \frac{500}{27} + \frac{162}{27} + 18\sqrt{90} - 6\sqrt{90} - \frac{245}{3}$$

$$= 12\sqrt{90} - \frac{1475}{27}$$

$$= 59.2$$

9 a $v = (t - 4)(3t - 8)$

$v = 3t^2 - 20t + 32$

$a = \dfrac{dv}{dt} = 6t - 20$

b Distance travelled

$$= \int_0^{\frac{8}{3}} \left(3t^2 - 20t + 32\right) dt$$

$$+ \left|\int_{\frac{8}{3}}^3 \left(3t^2 - 20t + 32\right) dt\right|$$

$$= \left[t^3 - 10t^2 + 32t\right]_0^{\frac{8}{3}}$$

$$+ \left|\left[t^3 - 10t^2 + 32t\right]_{\frac{8}{3}}^3\right|$$

$$= \left(\frac{512}{27} - \frac{640}{9} + \frac{256}{3}\right) + \left|\left(33 - \frac{896}{27}\right)\right|$$

$$= \frac{896}{27} + \frac{5}{27}$$

$$= \frac{901}{27}$$

9 c $x = \int v \, dt = \int \left(3t^2 - 20t + 32\right) dt$

$x = t^3 - 10t^2 + 32t + c$
At $t = 0$, $x = 0$, so $c = 0$
$x = t^3 - 10t^2 + 32t$
$x = t\left(t^2 - 10t + 32\right)$

If $t^2 - 10t + 32 = 0$ has a solution, then '$b^2 - 4ac$' ≥ 0.
However,
$b^2 - 4ac = 100 - 4 \times 1 \times 32 = -28 < 0$
So $t^2 - 10t + 32 = 0$ has no solutions.
Therefore P never returns to O.

10 a $\dfrac{dv}{dt} = -kv^2$

Separating the variables and integrating:

$$\int_{U}^{\frac{2U}{3}} \frac{dv}{v^2} = -k \int_{0}^{T} dt$$

$$\left[-\frac{1}{v}\right]_{U}^{\frac{2U}{3}} = -k\left[t\right]_{0}^{T}$$

$$-\frac{3}{2U} - \left(-\frac{1}{U}\right) = -kT$$

$$kT = \frac{3}{2U} - \frac{1}{U} = \frac{1}{2U}$$

$$T = \frac{1}{2kU}$$

11 a $a = 3t - 4$

$v = \int \left(3t - 4\right) dt = \dfrac{3t^2}{2} - 4t + c$

When $t = 0$, $v = 2$, so $c = 2$ $\dfrac{1}{2}\ln\left(\dfrac{5}{8}\right) = -kX$

$v = \dfrac{3t^2}{2} - 4t + 2$

Solving $\dfrac{3t^2}{2} - 4t + 2 = 0$

$3t^2 - 8t + 4 = 0$
$\left(3t - 2\right)\left(t - 2\right) = 0$

$t = \dfrac{2}{3}$ or $t = 2$

11 b Distance travelled

$$= \int_0^{\frac{2}{3}} \left(\frac{3t^2}{2} - 4t + 2 \right) dt + \left| \int_{\frac{2}{3}}^{2} \left(\frac{3t^2}{2} - 4t + 2 \right) dt \right| + \int_2^4 \left(\frac{3t^2}{2} - 4t + 2 \right) dt$$

$$= \left[\frac{t^3}{2} - 2t^2 + 2t \right]_0^{\frac{2}{3}} + \left| \left[\frac{t^3}{2} - 2t^2 + 2t \right]_{\frac{2}{3}}^{2} \right|$$

$$+ \left[\frac{t^3}{2} - 2t^2 + 2t \right]_2^4$$

$$= \left[\frac{\left(\frac{2}{3} \right)^3}{2} - 2 \left(\frac{2}{3} \right)^2 + 2 \left(\frac{2}{3} \right) \right]$$

$$- \left[\left(\frac{(2)^3}{2} - 2(2)^2 + 2(2) \right) \right.$$

$$\left. - \left(\frac{\left(\frac{2}{3} \right)^3}{2} - 2 \left(\frac{2}{3} \right)^2 + 2 \left(\frac{2}{3} \right) \right) \right]$$

$$+ \left(\frac{(4)^3}{2} - 2(4)^2 + 2(4) \right)$$

$$- \left(\frac{(2)^3}{2} - 2(2)^2 + 2(2) \right)$$

$$= \frac{248}{27}$$

12 $a = \dfrac{6}{x^2} = 6x^{-2}$

$\dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right) = 6x^{-2}$

$\dfrac{1}{2}v^2 = \int 6x^{-2}\,dx = \dfrac{6x^{-1}}{-1} + A$

$\dfrac{1}{2}v^2 = A - \dfrac{6}{x}$

At $x = 3$, $v = 4$

$\dfrac{1}{2}\times 16 = A - 2 \Rightarrow A = 10$

$\dfrac{1}{2}v^2 = 10 - \dfrac{6}{x}$

$v^2 = 20 - \dfrac{12}{x}$

$v = \sqrt{\left(20 - \dfrac{12}{x}\right)}$

13 $a = -\left(3 + \dfrac{1}{4}x\right)$

$\dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right) = -3 - \dfrac{1}{4}x$

$\dfrac{1}{2}v^2 = \int\left(-3 - \dfrac{1}{4}x\right)dx = -3x - \dfrac{1}{8}x^2 + A$

At $x = 0$, $v = 8$

$32 = -0 - 0 + A \Rightarrow A = 32$

$\dfrac{1}{2}v^2 = 32 - 3x - \dfrac{1}{8}x^2$

When $v = 0$

$0 = 32 - 3x - \dfrac{1}{8}x^2$

$x^2 + 24x - 256 = 0$

$(x + 32)(x - 8) = 0$

As $x > 0$

$x = 8$

14 $a = \dfrac{15}{4x^2} = \dfrac{15}{4}x^{-2}$

$\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{1}{2}v^2\right) = \dfrac{15}{4}x^{-2}$

$\dfrac{1}{2}v^2 = \int \dfrac{15}{4}x^{-2}\,\mathrm{d}x = -\dfrac{15}{4}x^{-1} + A$

$\dfrac{1}{2}v^2 = A - \dfrac{15}{4x}$

At $x = 5$, $v = -6$

$18 = A - \dfrac{15}{20} \Rightarrow A = 18\dfrac{3}{4} = \dfrac{75}{4}$

$\dfrac{1}{2}v^2 = \dfrac{75}{4} - \dfrac{15}{4x} = \dfrac{15}{4}\left(5 - \dfrac{1}{x}\right)$

When $v = 0$

$5 - \dfrac{1}{x} = 0 \Rightarrow x = \dfrac{1}{5}$

15 a $a = 3 - x$

$\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{1}{2}v^2\right) = 3 - x$

$\dfrac{1}{2}v^2 = \int (3 - x)\,\mathrm{d}x = 3x - \dfrac{x^2}{2} + A$

$v^2 = B + 6x - x^2$, where $B = 2A$

At $x = 0$, $v = 4$

$16 = B + 0 - 0 \Rightarrow B = 16$

$v^2 = 16 + 6x - x^2$

b $v^2 = 16 + 6x - x^2 = 25 - 9 + 6x - x^2$

$= 25 - (x - 3)^2$

As $(x - 3)^2 \geqslant 0$, $v^2 \leqslant 25$

The greatest value of v is 5.

16 a $a = \dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right) = \dfrac{x^2(5-x)}{2} = \dfrac{5x^2}{2} - \dfrac{x^3}{2}$

$\dfrac{1}{2}v^2 = \int\left(\dfrac{5x^2}{2} - \dfrac{x^3}{2}\right)dx = \dfrac{5x^3}{6} - \dfrac{x^4}{8} + A$

$v^2 = \dfrac{5x^3}{3} - \dfrac{x^4}{4} + B$, where $B = 2A$

At $x = 10$, $v = 0$

$0 = \dfrac{5000}{3} - \dfrac{10\,000}{4} + B \Rightarrow B = \dfrac{2500}{3}$

$v^2 = \dfrac{5x^3}{3} - \dfrac{x^4}{4} + \dfrac{2500}{3}$

b When $t = 0$, $x = 0$

$v^2 = \dfrac{2500}{3} \Rightarrow v = (\pm)\dfrac{50}{\sqrt{3}} = (\pm)\dfrac{50\sqrt{3}}{3}$

The speed of P when $t = 0$ is $\dfrac{50\sqrt{3}}{3}\,\mathrm{m\,s}^{-1}$.

17 a $a = \dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right) = \dfrac{20}{5x+2}$

$\dfrac{1}{2}v^2 = \int\dfrac{20}{5x+2}\,dx = 4\ln(5x+2) + A$

$v^2 = 8\ln(5x+2) + B$, where $B = 2A$

At $x = 0$, $v = 3$

$9 = 8\ln 2 + B \Rightarrow B = 9 - 8\ln 2$

$v^2 = 8\ln(5x+2) - 8\ln 2 + 9 = 8\ln\left(\dfrac{5x+2}{2}\right) + 9$

At $x = 12$

$v^2 = 8\ln 31 + 9 = 36.471\ldots$

$v = \sqrt{36.471\ldots} = 6.039\ldots$

The speed of P at $x = 12$ is 6.04 m s^{-1} (3 s.f.).

b When $v = 5$

$25 = 8\ln\left(\dfrac{5x+2}{2}\right) + 9$

$\ln\left(\dfrac{5x+2}{2}\right) = \dfrac{25-9}{8} = 2$

$\dfrac{5x+2}{2} = e^2$

$x = \dfrac{2e^2 - 2}{5} = 2.56$ (3 s.f.)

18 a $a = \dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{1}{2}v^2\right) = 4 + \dfrac{1}{2}x$

$\dfrac{1}{2}v^2 = 4x + \dfrac{x^2}{4} + A$

$v^2 = 8x + \dfrac{x^2}{2} + B$, where $B = 2A$

At $x = 0$, $v = 3$

$9 = 0 + 0 + B \Rightarrow B = 9$

$v^2 = 8x + \dfrac{x^2}{2} + 9$

At $x = 4$

$v^2 = 32 + 8 + 9 = 49 \Rightarrow v = 7$

The speed of P at $x = 4$ is 7 m s^{-1}.

b $a = \dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{1}{2}v^2\right) = -4 - \dfrac{1}{2}x$

$\dfrac{1}{2}v^2 = C - 4x - \dfrac{x^2}{4}$

$v^2 = D - 8x - \dfrac{x^2}{2}$, where $D = 2C$

At $x = 4$, $v = 7$

$49 = D - 32 - 8 \Rightarrow D = 89$

$v^2 = 89 - 8x - \dfrac{x^2}{2}$

When $v = 0$

$x^2 + 16x = 178 \Rightarrow x^2 + 16x + 64 = 242$

$(x + 8)^2 = 242 \Rightarrow x = 11\sqrt{2} - 8$, as $x > 0$

$x = 7.56$ (3 s.f.)

19 a $a = \dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{1}{2}v^2\right) = 4x + 6$

$\dfrac{1}{2}v^2 = 2x^2 + 6x + A$

$v^2 = 4x^2 + 12x + B$, where $B = 2A$

At $x = 0$, $v = 3$

$9 = 0 + 0 + B \Rightarrow B = 9$

$v^2 = 4x^2 + 12x + 9 = (2x + 3)^2$

As v is increasing as x increases

$v = 2x + 3$

19 b $v = \dfrac{dx}{dt} = 2x + 3$

Separating the variable and integrating

$$\int \frac{1}{2x+3} \, dx = \int 1 \, dt$$

$$\frac{1}{2} \ln(2x+3) = t + C$$

$$\ln(2x+3) = 2t + 2C$$

$$2x + 3 = e^{2t+2C} = De^{2t}, \text{ where } D = e^{2C}$$

When $t = 0$, $x = 0$

$$3 = De^0 \Rightarrow D = 3$$

$$2x + 3 = 3e^{2t}$$

$$x = \frac{3}{2}(e^{2t} - 1)$$

20 $v \dfrac{dv}{dx} = -k\left(U^2 + v^2\right)$

Separating the variables and integrating:

$$\int_{U}^{\frac{U}{2}} \frac{v \, dv}{v^2 + U^2} = -k \int_{0}^{X} dx$$

$$\left[\frac{1}{2} \ln\left(v^2 + U^2\right)\right]_{U}^{\frac{U}{2}} = -k\left[x\right]_{0}^{X}$$

$$\frac{1}{2} \ln\left(\frac{5U^2}{4}\right) - \frac{1}{2} \ln\left(2U^2\right) = -kX$$

$$X = -\frac{1}{2k} \ln\left(\frac{5}{8}\right)$$

$$X = \frac{1}{2k} \ln\left(\frac{8}{5}\right)$$

21 $v \dfrac{dv}{dx} = \dfrac{6000 - v^3}{8000v}$

Separating the variables and integrating:

$$\int_{4}^{8} \frac{8000v^2 \, dv}{6000 - v^3} = \int_{0}^{X} dx$$

$$\left[-\frac{8000}{3} \ln\left|6000 - v^3\right|\right]_{4}^{8} = \left[x\right]_{0}^{X}$$

$$-\frac{8000}{3} \ln 5488 - \left(-\frac{8000}{3} \ln 5936\right) = X$$

$$X = \frac{8000}{3} \ln\left(\frac{5936}{5488}\right)$$

$$X = 209 \text{ m}$$

Challenge

Initial velocity $= 32\,500\,\text{kmh}^{-1} = \dfrac{32\,500}{60^2}\,\text{kms}^{-1} = 9.0278\,\text{kms}^{-1}$

$$a = v\frac{\mathrm{d}v}{\mathrm{d}x} = -\frac{c}{x^2}$$

Separating the variables and integrating

$$\int v\,\mathrm{d}v = -\int cx^{-2}\,\mathrm{d}x$$

$$\frac{1}{2}v^2 = cx^{-1} + A$$

When $x = 6370$, $v = 9.028$

$$\frac{1}{2} \times 81.5008 = \frac{4}{6370} \times 10^5 + A$$

$$A = 40.7504 - 62.7943 = -22.0439$$

When $v = 0$

$$cx^{-1} = -A$$

$$x = -\frac{c}{A} = \frac{4}{22.0439} \times 10^5 = 1.81456 \times 10^4 = 18\,145.6$$

So distance above the Earth's surface $= 18\,145.6 - 6370 = 11\,776\,\text{km}$ (5 s.f.)