Dynamics 5B

1 $F = \frac{k}{d^2}$ where d = distance from centre distance (x - R) above surface \Rightarrow distance x from centre

$$\Rightarrow$$
 distance x from $\frac{k}{k}$

$$\therefore F = \frac{\pi}{x^2}$$

On surface F = mg, x = R

$$\therefore mg = \frac{k}{R^2}$$
$$k = mgR^2$$

:. Magnitude of the gravitational force is $\frac{mgR^2}{x^2}$.

The magnitude of the gravitational force on a particle on the surface of the earth is the magnitude of the weight of the particle.

2 For a particle of mass *m*, distance *x* from the centre of the earth:

$$F = ma$$

$$\frac{k}{x^2} = mA$$
Use the inverse square law.

On the surface of the earth, x = R, A = g

$$\therefore mg = \frac{k}{R^2}$$
$$k = mgR^2$$
$$\therefore mA = \frac{mgR^2}{x^2}$$
$$A = \frac{gR^2}{x^2}$$

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 $3 \quad F = ma$ $\frac{mgR^2}{mgR^2} = 0$

$$\frac{mgR^2}{x^2} = -m\ddot{x}$$

S is moving away from the earth, so theacceleration is in the direction of decreasing x.

Г

where x is the distance of S from the centre of the Earth.

$$v\frac{dv}{dx} = -g\frac{R^2}{x^2}$$
Use $\ddot{x} = v\frac{dv}{dx}$ as the acceleration is a function of x.

$$\int v \, dv = -g R^2 \int \frac{1}{x^2} \, dx$$

$$\frac{1}{2} v^2 = g\frac{R^2}{x} + C$$

$$x = 2R \quad v = \sqrt{gR}$$

$$\frac{1}{2} gR = \frac{gR^2}{2R} + C$$

$$C = 0$$

$$\frac{1}{2} v^2 = \frac{gR^2}{x}$$

$$x = R \quad \frac{1}{2} v^2 = \frac{gr^2}{R}$$

$$v^2 = 2g R$$

$$v = \sqrt{2gR}$$

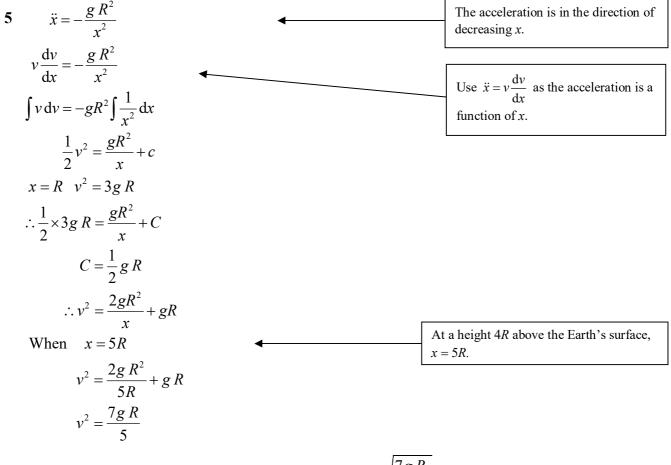
S was fired with speed $\sqrt{2gR}$.

4	F = ma			
	$\frac{mg R^2}{x^2} = -m\ddot{x}$	•		e acceleration is in the direction of reasing x .
	where <i>x</i> is the distance of the rocket from the centre of the Earth.			
	$v\frac{\mathrm{d}v}{\mathrm{d}x} = -\frac{gR^2}{x^2}$	•		e $\ddot{x} = v \frac{dv}{dx}$ as the acceleration is a action of <i>x</i> .
	$\int v \mathrm{d}v = -g R^2 \int \frac{1}{r^2} \mathrm{d}x$			
	$\frac{1}{2}v^2 = \frac{gR^2}{x} + C$			
	x = R, v = U	•		On the Earth's surface.
	$\frac{1}{2}U^2 = g\frac{R^2}{R} + C$			
	$C = \frac{1}{2}U^2 - gR$			
	x = (X + R)			ng a distance X, the rocket is a R) from the centre of the Earth.
	$\frac{1}{2}v^{2} = \frac{gR^{2}}{(X+R)} + \frac{1}{2}U^{2} - gR$	2		
	$v^{2} = \frac{2gR^{2} + U^{2}(X+R) - 2gR(X+R)}{(X+R)}$			
	$v = \sqrt{\left[\frac{U^2 X + U^2 R - 2g X}{\left(X + R\right)}\right]}$	\overline{RX}		
	XX 71 1 1 1 X 7		$\int U^2 X + U$	$\overline{R-2gRX}$

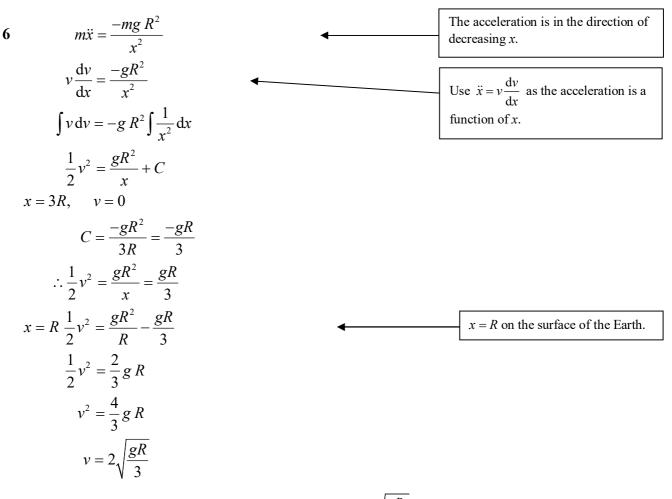
When it has travelled X meters, the speed of the rocket is $\sqrt{\left[\frac{U^2 X + U^2 R - 2g RX}{(X+R)}\right]}$

Further Mechanics 2

SolutionBank



 \therefore The speed at a height 4*R* above the Earth's surface is $\sqrt{\frac{7gR}{5}}$.



The particle hits the surface of the Earth's with speed $2\sqrt{\frac{gR}{3}}$.

7 a
$$F \propto \frac{1}{x^2}$$

 $F = \frac{k}{x^2}$
When $x = R$, $F = mg$
So $mg = \frac{k}{R^2}$
 $k = mgR^2$
 $F = \frac{mgR^2}{x^2}$

Further Mechanics 2

7 **b** Applying 'F = ma'

$$mv \frac{dv}{dx} = -\frac{mgR^2}{x^2}$$

$$v \frac{dv}{dx} = -\frac{gR^2}{x^2}$$
Separating the variables and integrating:
$$\int_{\sqrt{2gR}}^{V} v dv = -\int_{4R}^{R} \frac{gR^2}{x^2} dx$$

$$\left[\frac{v^2}{2}\right]_{\sqrt{2gR}}^{V} = \left[\frac{gR^2}{x}\right]_{4R}^{R}$$

$$\frac{V^2}{2} - gR = \frac{gR^2}{R} - \frac{gR^2}{4R}$$

$$\frac{V^2}{2} - gR = gR - \frac{gR}{4}$$

$$\frac{V^2}{2} - gR = \frac{3gR}{4}$$

$$\frac{V^2}{2} = \frac{7gR}{4}$$

$$V^2 = \frac{7gR}{2}$$

$$V = \sqrt{\frac{7gR}{2}}$$

Challenge

a Consider a mass *m* resting on the earth's surface. Suppose the earth has mass M_E .

Then by Newton's law of gravitation:

 $mg = \frac{GmM_E}{r^2}$ Rearranging gives:

$$M_{E} = \frac{gr^{2}}{G}$$
$$M_{E} = \frac{9.81 \times (6.3781 \times 10^{6})^{2}}{6.67 \times 10^{-11}}$$
$$M_{E} = 5.98 \times 10^{24} \text{ kg}$$

b density =
$$\frac{\text{mass}}{\text{volume}}$$

= $\frac{M_E}{\frac{4}{3}\pi r^3}$
= $\frac{5.983 \times 10^{24}}{\frac{4}{3}\pi \times (6.3781 \times 10^6)^3}$
= 5500 kg m⁻³ (3 s.f.)