Quality of tests 8B

1 a $H_0: \mu = 50 \quad H_1: \mu > 50$

Assume H_0, so that $\bar{X} \sim N\left(50, \frac{3^2}{20}\right)$

Standardise the $\bar{X}$ variable

$$Z = \frac{\bar{X} - 50}{\frac{3}{\sqrt{20}}} = \frac{\sqrt{20}(\bar{X} - 50)}{3}$$

Significance level 1%

From the tables, the 1% critical region for Z is $Z > 2.3263$

So the critical region for $\bar{X}$ is given by

$$\frac{\sqrt{20}(\bar{X} - 50)}{3} > 2.3263$$

$\Rightarrow \bar{X} > 51.5605\ldots$

b $\text{P(Type I error)} = \text{significance level} = 0.01$

c Using the normal cumulative distribution function on a calculator:

$\text{P(Type II error)} = \text{P}(\bar{X} \leqslant 51.5605... \mid \mu = 53) = 0.0159$ (4 d.p.)

2 a $H_0: \mu = 30 \quad H_1: \mu < 30$

Assume H_0, so that $\bar{X} \sim N\left(30, \frac{2^2}{16}\right)$

Standardise the $\bar{X}$ variable

$$Z = \frac{\bar{X} - 30}{\frac{2}{4}} = 2(\bar{X} - 30)$$

Significance level 5%

From the tables, the 5% critical region for Z is $Z < -1.6449$

So the critical region for $\bar{X}$ is given by

$2(\bar{X} - 30) < -1.6449$

$\Rightarrow \bar{X} > 29.17755\ldots$

b $\text{P(Type I error)} = \text{significance level} = 0.05$

c Using the normal cumulative distribution function on a calculator:

$\text{P(Type II error)} = \text{P}(\bar{X} \geqslant 29.17755.... \mid \mu = 28.5)$

$= 1 - \text{P}(\bar{X} < 29.17755.... \mid \mu = 28.5) = 1 - 0.9123 = 0.0877$ (4 d.p.)

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3 a $H_0: \mu = 40 \quad H_1: \mu \neq 40$

Assume H_0, so that $\bar{X} \sim N\left(40, \frac{4^2}{25}\right)$

Standardise the $\bar{X}$ variable

$$Z = \frac{\bar{X} - 40}{\frac{4}{5}} = 1.25(\bar{X} - 40)$$

Significance level 1%, so require 0.5% in each tail

From the tables, the critical region for Z is $Z > 2.5758$ or $Z < -2.5758$

So the critical values for $\bar{X}$ are given by

$1.25(\bar{X} - 40) = \pm 2.5758$

$\Rightarrow \bar{X} = 37.93936$ and $\bar{X} = 42.06064$

So the critical region for $\bar{X}$ is $\bar{X} < 37.9394$ or $\bar{X} > 42.0606$

b $\text{P(Type I error)} = \text{significance level} = 0.01$

c Using the normal cumulative distribution function on a calculator:

$$\begin{aligned}\text{P(Type II error)} &= \text{P}(37.9394 \leqslant \bar{X} \leqslant 42.0606 \mid \mu = 42) \\ &= \text{P}(\bar{X} \leqslant 42.0606 \mid \mu = 42) - \text{P}(\bar{X} \leqslant 37.9394 \mid \mu = 42) \\ &= 0.5302 - 0.000 = 0.5302 \text{ (4 d.p.)}\end{aligned}$$

4 a $H_0: \mu = 15 \quad H_1: \mu \neq 15$

Assume H_0, so that $\bar{X} \sim N\left(15, \frac{1}{25}\right)$

Standardise the $\bar{X}$ variable

$$Z = \frac{\bar{X} - 15}{\frac{1}{5}} = 5(\bar{X} - 15)$$

Significance level 5%, so require 2.5% in each tail

From the tables, the critical region for Z is $Z > 1.96$ or $Z < -1.96$

So the critical values for $\bar{X}$ are given by

$5(\bar{X} - 15) = \pm 1.96$

$\Rightarrow \bar{X} = 14.608$ and $\bar{X} = 15.392$

So the critical region for $\bar{X}$ is $\bar{X} < 14.608$ or $\bar{X} > 15.392$

b If the day's production is accepted although the mean diameter has changed, that is a Type II error. Using the normal cumulative distribution function on a calculator:

$$\begin{aligned}\text{P(Type II error)} &= \text{P}(14.608 \leqslant \bar{X} \leqslant 15.392 \mid \mu = 15.6) \\ &= \text{P}(\bar{X} \leqslant 15.392 \mid \mu = 15.6) - \text{P}(\bar{X} \leqslant 14.608 \mid \mu = 15.6) \\ &= 0.1492 - 0.000 = 0.1492 \text{ (4 d.p.)}\end{aligned}$$

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5 a $H_0 : \mu = 40 \quad H_1 : \mu > 40$

Assume H_0, so that $\bar{X} \sim N\left(40, \frac{8^2}{30}\right)$

Standardise the $\bar{X}$ variable

$$Z = \frac{\bar{X} - 40}{\frac{8}{\sqrt{30}}} = \frac{\sqrt{30}(\bar{X} - 40)}{8}$$

Significance level 5%

From the tables, the 5% critical region for Z is $Z > 1.6449$

So the critical value for $\bar{X}$ is given by

$$\frac{\sqrt{30}(\bar{X} - 40)}{8} = 1.6449$$

$$\Rightarrow \bar{X} = 42.4025$$

b Using the normal cumulative distribution function on a calculator:

$\text{P(Type II error)} = \text{P}(\bar{X} \leqslant 42.4205 \mid \mu = 42) = 0.6086$ (4 d.p.)

c The significance level should be increased above 5% because that will decrease the value of Z and therefore X giving a lower probability than that obtained in b.

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