Linear regression 1B

1 Substituting in the regression equation: $(x+2)+(y-3)=5$

$\Rightarrow x+y-1=5$

$\Rightarrow y=6-x$

2 Substituting in the regression equation: $p-10=s-100+2$

$\Rightarrow s=88+p$

3 Substituting in the regression equation: $\frac{y}{4}-2=6-4\left(\frac{x}{3}\right)$

$\Rightarrow 3y-24=72-16x$ (multiplying through by 12)

$\Rightarrow 3y=96-16x$

So $y=32-\frac{16}{3}x$

4 Substituting in the regression equation: $t-10=14+3(s-5)$

$\Rightarrow t=24+3s-15$

$\Rightarrow t=9+3s$

5 a $b=\frac{S_{xy}}{S_{xx}}=\frac{120}{240}=0.5$

$a=\bar{y}-b\bar{x}=6-0.5\times5=3.5$

So $y=3.5+0.5x$

b Substituting in the regression equation from part **a**: $\frac{d}{10}=3.5+0.5\times\frac{c}{2}$

$\Rightarrow d=35+2.5c$

6 a Calculating the summary statistics gives:

$\sum x=49 \quad \sum x^2=671 \quad \sum y=81 \quad \sum y^2=1451 \quad \sum xy=956$

$S_{xy}=\sum xy-\frac{\sum x\sum y}{n}=956-\frac{49\times81}{5}=162.2$

$S_{xx}=\sum x^2-\frac{\left(\sum x\right)^2}{n}=671-\frac{(49)^2}{5}=190.8$

$b=\frac{S_{xy}}{S_{xx}}=\frac{162.2}{190.8}=0.85010\ldots=0.850$ (3 s.f.)

$\bar{x}=\frac{\sum x}{n}=\frac{49}{5}=9.8 \qquad \bar{y}=\frac{\sum y}{n}=\frac{81}{5}=16.2$

$a=\bar{y}-b\bar{x}=16.2-\frac{162.2}{190.8}\times9.8=7.86897\ldots=7.87$ (3 s.f.)

Hence equation of regression line of y on x is: $y=7.87+0.850x$.

6 b Substituting in the regression equation from part **a**: $\frac{c}{5} = 7.869 + 0.8501 \times \left(\frac{a-8}{2}\right)$

$\Rightarrow 2c = 78.69 + 4.2505(a-8)$ (multiplying through by 10)

$\Rightarrow 2c = 78.69 + 4.2505a - 34.004$

$\Rightarrow 2c = 44.686 + 4.2505a$ (3 s.f.)

$\Rightarrow c = 22.3 + 2.13a$ (giving parameters to 3 s.f.)

Note that substituting into the equation from part **a** with the parameters rounded to 3 s.f., i.e. $y = 7.87 + 0.850x$, gives a slightly different result due to rounding:

$\frac{c}{5} = 7.87 + 0.85 \times \left(\frac{a-8}{2}\right)$

$\Rightarrow 2c = 78.7 + 4.25a - 34$

$\Rightarrow 2c = 44.7 + 4.25a - 34$

$\Rightarrow c = 22.35 + 2.125a$

$\Rightarrow c = 22.4 + 2.13a$ (giving parameters to 3 s.f.)

c Method 1

If $a = 32$, then $x = \frac{32-8}{2} = 12$

$y = 7.8689\ldots + 0.85010 \times 12 = 18.07017\ldots$

$c = 5y = 90.3508\ldots = £90.40$ (3 s.f)

Using part **b**,

Method 2

$c = 22.343\ldots + 2.12525\ldots \times 32 = 90.351\ldots = £90.40$ (3 s.f.)

7 a $b = \frac{S_{pv}}{S_{vv}} = \frac{15.26}{10.21} = 1.49461\ldots = 1.49$ (3 s.f.)

$a = \bar{p} - b\bar{v} = 9.88 - \frac{15.26}{10.21} \times 4.58 = 3.03467\ldots = 3.03$ (3 s.f.)

Hence equation of regression line of p on v is $p = 3.03 + 1.49v$

b When $x = 42$, $v = \frac{42-4}{8} = 4.75$

Hence $p = 3.03467\ldots + 1.49461 \times 4.75 = 10.134\ldots = 10.1$ tonnes (3 s.f.)