Continuous distributions 3F

1 a $\mathrm{E}(Y)=\mathrm{E}(X^2)$

$\mathrm{E}(X)=\dfrac{4.5+5.5}{2}=5$

$\mathrm{Var}(X)=\dfrac{(5.5-4.5)^2}{12}=\dfrac{1}{12}$

$\mathrm{Var}(X)=\mathrm{E}(X^2)-(\mathrm{E}(X))^2$

$\Rightarrow \mathrm{E}(X^2)=\mathrm{Var}(X)+(\mathrm{E}(X))^2=\dfrac{1}{12}+25=25\dfrac{1}{12}=\dfrac{301}{12}=25.083\text{ (3 d.p.)}$

Alternatively use $\mathrm{E}(\mathrm{g}(X))=\int \mathrm{g}(x)\mathrm{f}(x)\,\mathrm{d}x$ with

$\mathrm{f}(x)=\begin{cases}1 & 2\leqslant x\leqslant 3\\ 0 & \text{otherwise}\end{cases}$ and $\mathrm{g}(x)=x^2$

$\mathrm{E}(Y)=\mathrm{E}(X^2)=\int_{4.5}^{5.5}x^2\,\mathrm{d}x=\left[\dfrac{x^3}{3}\right]_{4.5}^{5.5}=\left(\dfrac{5.5^3}{3}-\dfrac{4.5^3}{3}\right)=25.083\text{ (3 d.p.)}$

2 a $\mathrm{f}(x)=\begin{cases}\dfrac{1}{6} & 5\leqslant x\leqslant 11\\ 0 & \text{otherwise}\end{cases}$

b $\mathrm{P}(7<R<10)=\dfrac{10-7}{11-5}=\dfrac{3}{6}=\dfrac{1}{2}=0.5$

c $\mathrm{E}(A)=\mathrm{E}(\pi R^2)=\pi(\mathrm{Var}(R)+(\mathrm{E}(R))^2$

$\mathrm{E}(A)=\pi\left(\dfrac{36}{12}+\left(\dfrac{5+11}{2}\right)^2\right)=\pi(3+64)=67\pi\text{ cm}^2$

3 a $\dfrac{1}{b-a}=\dfrac{1}{1-0}=1$

$\mathrm{P}(T<0.2)=(0.2-0)\times 1=0.2$

b $\mathrm{E}(T)=0.5$

c $\mathrm{Var}(T)=\int_0^1 t^2\,\mathrm{f}(t)\mathrm{d}t-\mu^2=\int_0^1 t^2\,\mathrm{d}t-0.5^2=\left[\dfrac{t^3}{3}\right]_0^1-0.25=\dfrac{1}{3}-\dfrac{1}{4}=\dfrac{1}{12}$

4 a $\dfrac{1}{b-a}=\dfrac{1}{10-2}=\dfrac{1}{8}$

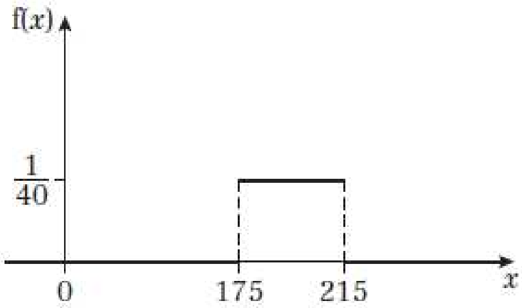
$\mathrm{P}(T>7)=(10-7)\times\dfrac{1}{8}=\dfrac{3}{8}$

4 b $P(T<5)=(5-2)\times\frac{1}{8}=\frac{3}{8}$

SO probability that Priya will take less than 5 minutes on three successive visits $=\left(\frac{3}{8}\right)^3=\frac{27}{512}$

c $P(T<8\mid T>5)=\frac{P(T<8\cap T>5)}{P(T>5)}=\frac{P(5<T<8)}{P(T>5)}=\frac{\frac{3}{8}}{\frac{5}{8}}=\frac{3}{5}$

5 a $f(x)=\begin{cases}\frac{1}{40} & 175\leqslant x\leqslant 215\\ 0 & \text{otherwise}\end{cases}$



b i $P(X<187)=\frac{187-175}{40}=\frac{12}{40}=\frac{3}{10}=0.3$

ii $P(X=187)=0$
(the probability of a continuous random variable taking a specific value is always 0)

c $Q_1=185$, as $P(X<185)=\frac{10}{40}=0.25$ and $Q_3=205$, as $P(X<205)=\frac{30}{40}=0.75$
Interquartile range $=205-185=20$

d $P(X\geqslant x)=\frac{215-x}{40}=0.65\Rightarrow 215-x=26\Rightarrow x=189$

e Let the random variable Y represent the number of cups that Brenda buys contains less than 187 ml, then using the result from part **b i**, $Y\sim B(5,0.3)$

$P(Y=3)=\binom{5}{3}0.3^3(1-0.3)^2=\frac{5\times4}{2}\times0.3^3\times0.7^2=0.1323$

6 a $P(X<-2.3)=\frac{-2.3-(-3.0)}{3.0-(-3.0)}=\frac{0.7}{6}=\frac{7}{60}$

b $P(|X|>2.0)=P(X>2.0)+P(X<2.0)=\frac{1}{6}+\frac{1}{6}=\frac{1}{3}$

6 c Let the random variable Y represent the number of the 10 rods that are cut within 2 mm of the target length, then using the result from part **b i**, $Y \sim \mathrm{B}\left(10, \frac{2}{3}\right)$

$$\mathrm{P}(Y=6)=\binom{10}{6}\times\left(\frac{2}{3}\right)^6\times\left(\frac{1}{3}\right)^4=0.2276 \text{ (4 d.p.)}$$

7 a $\mathrm{P}(Y>26)=\frac{28-26}{28-20}=\frac{2}{8}=\frac{1}{4}=0.25$

b Let the random variable Z represent the number of sweets in a bag of 20 sweets that have a length greater than 26 mm, then using the result from part **a**, $Z \sim \mathrm{B}(20, 0.25)$.

$\mathrm{P}(Z \geqslant 7)=1-\mathrm{P}(Z \leqslant 6)=1-0.7858=0.2142$ (4 d.p.)

(Obtaining the value for $\mathrm{P}(Z \leqslant 6)$ from a binomial cumulative distribution function table)

8 a $\mathrm{P}(X<5)=\frac{5-2}{7-2}=\frac{3}{5}=0.6$

b $\mathrm{P}(X>6)=\frac{7-6}{7-2}=\frac{1}{5}=0.2$

Let the random variable Y represent the number of flights in the next 10 flights that have a waiting time of more than 6 minutes, then $Y \sim \mathrm{B}(10, 0.2)$

$\mathrm{P}(Y \geqslant 3)=1-\mathrm{P}(Y \leqslant 2)=1-0.6778=0.3222$ (4 d.p.)

(Obtaining the value for $\mathrm{P}(Y \leqslant 2)$ from a binomial cumulative distribution function table)

9 The length of the shorter side is $(20 - X)$ cm, so

$$\mathrm{E}(A)=\mathrm{E}\big(X(20-X)\big)=\mathrm{E}(20X-X^2)=20\mathrm{E}(X)-\mathrm{E}(X^2)$$

$$\mathrm{E}(X)=\frac{20+10}{2}=15$$

$$\mathrm{Var}(X)=\frac{(20-10)^2}{12}=\frac{100}{12}=\frac{25}{3}$$

$$\mathrm{E}(X^2)=\mathrm{Var}(X)+(\mathrm{E}(X))^2=225+\frac{25}{3}$$

Hence $\mathrm{E}(A)=20\mathrm{E}(X)-\mathrm{E}(X^2)=20\times15-225-\frac{25}{3}$

$$=300-225-\frac{25}{3}=75-\frac{25}{3}=\frac{225-25}{3}=\frac{200}{3}\text{cm}^2$$