Estimation, confidence intervals and tests using a normal distribution 5B

1 $n = 9,\ \sigma^2 = 36,\ \bar{x} = 128$

a 95% C.I. for μ is $128 \pm 1.96 \times \frac{6}{\sqrt{9}} = (124.08,\ 131.92\ldots)$

$= (124,\ 132)$ (3 s.f.)

b 99 % C.I. for μ is $128 \pm 2.5758 \times \frac{6}{\sqrt{9}} = (122.84\ldots,\ 133.15\ldots)$

$= (123,\ 133)$ (3 s.f.)

2 $n = 25,\ \sigma = 4,\ \bar{x} = 85$

a 90% C.I for μ is $85 \pm 1.6449 \times \frac{4}{\sqrt{25}} = (83.684\ldots,\ 86.315\ldots)$

$= (83.7,\ 86.3)$ (3 s.f.)

b 95% C.I. for μ is $85 \pm 1.96 \times \frac{4}{\sqrt{125}} = (83.432,\ 86.568)$

$= (83.4,\ 86.6)$ (3 s.f.)

3 **a** Niall must assume that the population is normally distributed.

b First we find the mean and standard deviation of the sample

$$\bar{x} = \frac{23.1 + 21.8 + 24.6 + 22.5}{4} = 23$$

and we already know the variance to be $\sigma^2 = 4.41$,

This means that the standard deviation is $\sigma = \sqrt{4.41} = 2.1$.

Using the statistics tables, we find that the value that corresponds with a 98% confidence interval is 2.33.

This means our 98% confidence interval is found by substituting the values we have found into the equation:

$$\begin{aligned}\bar{x} \pm 2.33 \times \frac{\sigma}{\sqrt{n}} &= 23 \pm 2.33 \times \frac{2.1}{\sqrt{4}} \\ &= 23 \pm 2.4465 \\ &= (20.5535,\ 25.4465). \\ &= (20.6,\ 25.4)\ \text{(3 s.f.)}\end{aligned}$$

4 σ = 15

C.I. is $\bar{x} \pm z \times \frac{\sigma}{\sqrt{n}}$

width = $\frac{2z\sigma}{\sqrt{n}}$

a $90\% \Rightarrow z = 1.6449 \quad \therefore \frac{2 \times 1.6449 \times 15}{\sqrt{n}} < 2$

$\Rightarrow \sqrt{n} > 24.67\ldots \qquad \therefore n > 608.78\ldots$

So $n = 609$

b $95\% \Rightarrow z = 1.96 \therefore \frac{2 \times 1.96 \times 15}{\sqrt{n}} < 2$

$\Rightarrow \sqrt{n} > 1.96 \times 15 \therefore n > 864.36\ldots$

So $n = 865$

c $99\% \Rightarrow z = 2.5758 \therefore \frac{2 \times 2.5758 \times 15}{\sqrt{n}} < 2$

$\Rightarrow \sqrt{n} > 2.5758 \times 15 \therefore n > 1492.817\ldots$

So $n = 1493$

5 a A 95% confidence interval for a population parameter θ is an interval such that the probability the interval contains θ is 0.95.

b $\sigma = 0.70,\ n = 100,\ \bar{x} = \frac{190.2}{100} = 1.902$

95% C.I. for μ is $\bar{x} \pm 1.96 \times \frac{\sigma}{\sqrt{100}}$

$= 1.902 \pm 1.96 \times \frac{0.7}{10}$

$= (1.7648,\ 2.0392)$

$= (1.76,\ 2.04) \qquad (3 \text{ s.f.})$

6 a $\sigma = 50 \quad n = 200 \quad \bar{x} = 310$

90% C.I is $\bar{x} \pm 1.6449 \times \frac{\sigma}{\sqrt{200}}$

$= \left(310 \pm 1.6449 \times \frac{50}{\sqrt{200}}\right)$

$= (304.184\ldots,\ 315.815\ldots)$

$= (304,\ 316) \quad (3 \text{ s.f.})$

6 b First we calculate the probability that μ is contained in exactly 3 **specific** 90% confidence intervals out of the total 5.

The probability that this happens is:

$$90\% \times 90\% \times 90\% \times 10\% \times 10\%$$
$$= 0.9 \times 0.9 \times 0.9 \times 0.1 \times 0.1$$
$$= 0.00729.$$

Now we calculate that there are $5C3 = 10$ ways we may choose 3 out of 5 (using the binomial expansion or nCr button on a calculator). Therefore there are 10 specific examples of μ being contained in exactly 3 of the 5 90% confidence intervals and so we have a probability of 0.0729.

7 a Yes, Amy is correct. By the central limit theorem, for a large sample size, underlying population does not need to be normally distributed.

b $\sigma = 15\ 000 \quad n = 80 \quad \bar{x} = 75\ 872$

$$90\%\text{C.I is } \bar{x} \pm 1.6449 \times \frac{\sigma}{\sqrt{80}}$$
$$= \left(75\,872 \pm 1.6449 \times \frac{15\ 000}{\sqrt{200}}\right)$$
$$= (73\ 113.41\ldots,\ 78\ 630.58\ldots)$$
$$= (73\ 113,\ 78\ 631) \quad (\text{nearest integer})$$
$$\text{or } (73\ 100,\ 78\ 600) \quad (3 \text{ s.f.})$$

8 $\sigma = 13.5 \quad n = 250 \quad \bar{x} = 68.4$

a Must assume that these students form a random sample or that they are representative of the population.

b $95\%\ \text{C.I is } 68.4 \pm 1.96 \times \dfrac{13.5}{\sqrt{250}}$

$$= (66.726\ldots,\ 70.073\ldots)$$
$$= (66.7,\ 70.1)\ (3 \text{ s.f.})$$

c If $\mu = 65.3$ that is outside the C.I. so the examiner's sample was not representative. The examiner marked more 'better than average' candidates.

9 $\text{H} \sim \text{U}[\mu - 10,\ \mu + 10]$

a $\text{E}(H) = \mu \quad \text{Var}(H) = \dfrac{(20)^2}{12} = \dfrac{400}{12} = \dfrac{100}{3}$

9 b $n = 120 \quad \bar{h} = 78.7$

95% C.I. is $\bar{h} \pm 1.96 \times \frac{\sqrt{\frac{100}{3}}}{\sqrt{120}}$

$= \left(78.7 \pm 1.96 \times \sqrt{\frac{100}{360}}\right)$

$= (77.666\ldots, 79.733\ldots)$

$= (77.7, 79.7) \quad$ (3 s.f.)

10 a (23.2, 26.8) is 95% C.I. since it is the narrower interval.

b $\bar{x} = \frac{1}{2}(23.2 + 26.8) = 25$

$\therefore\ 1.96\frac{\sigma}{\sqrt{n}} = 25 - 23.2 = 1.8$

$\therefore\ \frac{\sigma}{\sqrt{n}} = 0.9183\ldots = 0.918 \quad$ (3 s.f.)

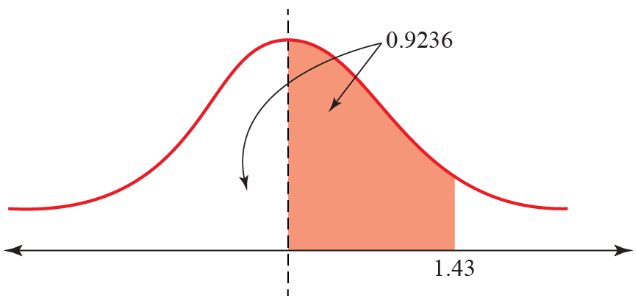
c $\hat{\mu} = \bar{x} = 25$ (mid-point of the intervals)

11 a $\bar{x} = \frac{1}{2}(128.14 + 141.86) = \frac{270}{2} = 135$

$\therefore$ C.I. will be (130, 140)

b $z \times \frac{\sigma}{\sqrt{n}} = 5 \quad$ but $\quad 1.96\frac{\sigma}{\sqrt{n}} = 6.86$

$\therefore\ z = \frac{5}{\left(\frac{6.86}{1.96}\right)} = 1.4285\ldots \qquad$ Use 1.43



$\therefore$ C.I. is $2 \times (0.9236 - 0.5)$

$= 0.8472 \qquad$ (tables)

or $= 0.846872\ldots \qquad$ (calculator)

$\therefore$ C.I. is 85%

11 c Now we know $1.96\frac{\sigma}{\sqrt{100}}=6.86$

$\therefore \sigma=\frac{6.86\times 10}{1.96}=35$

and require $z\times\frac{\sigma}{\sqrt{n}}=5$ where $z=1.96$

$\therefore \frac{1.96\times 35}{5}=\sqrt{n}$

$\Rightarrow n=188.23\ldots$

$\therefore$ Need n = 189 or more

12 $W\sim N(\mu, 2.4^2)$ $n=36$ $\overline{w}=31.4$

99% C.I. is $31.4\pm 2.5758\times\frac{2.4}{\sqrt{36}}$

$=(30.369\ldots, 32.430\ldots)$

$=(30.4, 32.4)$ (3 s.f.)

13 $\sigma=20$, $n=40$, $\overline{x}=266$

99% C.I. is $266\pm 2.5758\times\frac{20}{\sqrt{40}}$

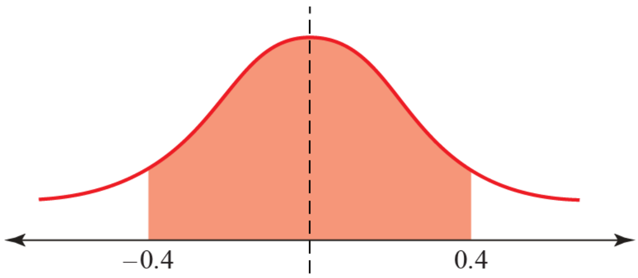
$=(257.854\ldots, 274.145\ldots)$

$=(258, 274)$ (3 s.f.)

14 $E\sim N(0, 1^2)$

a $P(|E|<0.4)=(0.6554-0.5)\times 2$

$=0.311$

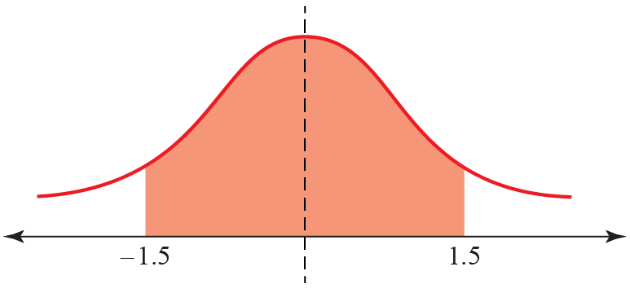


b $\overline{E}\sim N(0, \frac{1}{9})$

$P(|\overline{E}|<0.5)=P\left(|Z|<\frac{0.5}{\sqrt{\frac{1}{9}}}\right)$

$=(0.9332-0.5)\times 2$

$=0.866$



c 98% C.I. is $22.53\pm 2.3263\times\frac{1}{\sqrt{9}}$

$=(21.754\ldots, 23.305\ldots)$

$=(21.8, 23.3)$ (3 s.f.)