Estimation, confidence intervals and tests using a normal distribution 5D

1 $n_Q = 100 \quad \bar{x}_Q = 28.7 \quad s_Q = 7.32$

$n_S = 100 \quad \bar{x}_S = 30.6 \quad s_S = 3.51$

$H_0 : \mu_Q = \mu_S \quad H_1 : \mu_Q < \mu_S$

(i.e. Quickdry dries in a shorter time than Speedicover.)

$$\text{t.s. is } z = \frac{(30.6 - 28.7) - 0}{\sqrt{\frac{7.32^2}{100} + \frac{3.51^2}{100}}} = 2.34$$

Test $\mu_S > \mu_Q$ to get $z > 0$.

5% c.v. is $z = 1.6449$

t.s. is $2.34 > 1.6449$ so the result is significant.

There is evidence that Quickdry dries faster than Speedicover.

2 a $n_1 = 80 \quad \bar{x}_1 = 38.64 \quad s_1 = 6.59$

$n_2 = 120 \quad \bar{x}_2 = 40.13 \quad s_1 = 8.23$

$H_0 : \mu_1 = \mu_2 \quad H_1 : \mu_2 > \mu_1$

5% c.v. is $z = 1.6449$

$$\text{t.s. is } z = \frac{(40.13 - 38.64) - 0}{\sqrt{\frac{6.59^2}{80} + \frac{8.23^2}{120}}} = 1.42$$

t.s. is $1.42 < 1.6449$ so the result is not significant.

There is insufficient evidence to confirm that mean expenditure in the week is more than at weekends.

b We have assumed that $s_1 = \sigma_1$ and $s_2 = \sigma_2$.

3 $s = \sigma = 1.12,\ n = 250,\ \bar{x} = 9.88$

a $H_0 : \mu = 10 \quad H_1 : \mu \neq 10$

5% c.v. is $z = \pm 1.96$

$$\text{t.s. is } z = \frac{9.88 - 10}{\sqrt{\frac{1.12^2}{250}}} = -1.694\ldots > -1.96$$

Not significant

Insufficient evidence to support a change in mean mass.

b We have assumed that $s = \sigma$ since n is large.

4 a $H_0 : \mu_A = \mu_B \quad H_1 : \mu_A < \mu_B \qquad$ c.v. is $z = -2.3263$

$$\text{t.s. is } z = \frac{(84.1 - 87.9) - 0}{\sqrt{\frac{12.5^2}{90} + \frac{14.6^2}{110}}} = -1.9825\ldots > -2.3263$$

Not significant so accept H_0.

4 b $H_0 : \mu_A - \mu_B = 2 \quad H_1 : \mu_A - \mu_B > 2 \quad$ c.v. is $z = 1.6449$

$$\text{t.s. is } z = \frac{(125.1 - 119.3) - 2}{\sqrt{\frac{23.2^2}{150} + \frac{18.4^2}{200}}} = 1.6535\ldots > 1.6449$$

Significant so reject H_0.

c We have assumed $s_A = \sigma_A$ and $s_B = \sigma_B$ since the samples are both large.

5 $n = 100, \bar{x} = 83.6, s = 7.2$

$H_0 : \mu = 85 \quad H_1 : \mu < 85 \quad$ c.v. is $z = -1.6449$

$$\text{t.s. is } z = \frac{(83.6 - 85)}{\left(\frac{7.2}{\sqrt{100}}\right)} = -1.944\ldots < -1.6449$$

Significant

There is evidence that the weights of chocolate bars are less than the stated value.

6 $n_1 = 250 \quad \bar{x}_1 = 22.45 \quad S_1 = 2.9$

$n_2 = 280 \quad \bar{x}_2 = 22.96 \quad S_2 = 2.8$

Assume $S_i = \sigma_i$ since samples are large.

$H_0 : \mu_1 = \mu_2 \quad H_1 : \mu_1 \neq \mu_2$

Use 5% significance level
c.v. is $Z = \pm 1.96$

a $$\text{t.s.} = Z = \frac{(22.96 - 22.45) - 0}{\sqrt{\frac{2.9^2}{250} + \frac{2.8^2}{280}}}$$

t.s. $= Z = 2.054\ldots > 1.96$

Use $\bar{x}_2 - \bar{x}_1$ to make

Result is significant.

There is evidence of a difference in mean age of first-time mothers between these two dates.

b These is no need to have to assume that both populations were normally distributed since both samples were large so the Central Limit Theorem allows you to assume both sample means are normally distributed.

We have assumed that $S_1 = \sigma_1$ and $S_2 = \sigma_2$