Further hypothesis tests 6A

1 Confidence interval $= \left(\frac{(n-1)s^2}{\chi^2_{n-1}\left(\frac{\alpha}{2}\right)}, \frac{(n-1)s^2}{\chi^2_{n-1}\left(1-\frac{\alpha}{2}\right)} \right) = \left(\frac{14 \times 4.8}{26.119}, \frac{14 \times 4.8}{5.629} \right)$

$= (2.573, 11.938)$

2 First we find s^2 using the equation

$$s^2 = \frac{1}{n-1}\left(\sum x^2 - \frac{\left(\sum x\right)^2}{n} \right)$$

$$= \frac{1}{20-1}\left(884.3 - \frac{(132.4)^2}{20} \right)$$

$= 0.4112$ (4 s.f.).

The percentage points are $\chi^2_{19}(0.95) = 10.117$ and $\chi^2_{19}(0.05) = 30.144$.

The critical points are

$$\frac{(20-1)s^2}{\chi^2_{19}(0.95)} = \frac{19 \times 0.4112}{10.117} = 0.7722 \text{ and } \frac{(20-1)s^2}{\chi^2_{19}(0.05)} = \frac{19 \times 0.4112}{30.144} = 0.2592.$$

Thus, the 90% confidence interval for the variance is (0.2592, 0.7722).

3 $\bar{x} = 2.8785 \quad \ldots \quad s^2 = 0.45873\ldots$

Confidence interval $= \left(\frac{(n-1)s^2}{\chi^2_{n-1}\left(\frac{\alpha}{2}\right)}, \frac{(n-1)s^2}{\chi^2_{n-1}\left(1-\frac{\alpha}{2}\right)} \right) = \left(\frac{13 \times 0.458\ldots}{24.736}, \frac{13 \times 0.458\ldots}{5.009} \right)$

$= (0.241, 1.191)$

4 Confidence interval $= \left(\frac{(n-1)s^2}{\chi^2_{n-1}\left(\frac{\alpha}{2}\right)}, \frac{(n-1)s^2}{\chi^2_{n-1}\left(1-\frac{\alpha}{2}\right)} \right) = \left(\frac{4 \times 2.037}{11.143}, \frac{4 \times 2.037}{0.484} \right)$

$= (0.731, 16.835)$

5 a $\bar{x} = 62.1 \qquad s^2 = \frac{1}{9}\left(38\,938 \frac{621^2}{10} \right) = 41.544\ldots \quad s = 6.445\ldots$

Confidence interval $= \left(\frac{(n-1)s^2}{\chi^2_{n-1}\left(\frac{\alpha}{2}\right)}, \frac{(n-1)s^2}{\chi^2_{n-1}\left(1-\frac{\alpha}{2}\right)} \right) = \left(\frac{9 \times 41.544\ldots}{16.919}, \frac{9 \times 41.544}{3.325} \right)$

$= (22.099, 112.450)$

b Normal distribution

6 First we find s^2 using the equation

$$s^2 = \frac{1}{n-1}\left(\sum x^2 - \frac{\left(\sum x\right)^2}{n}\right)$$

$$= \frac{1}{10-1}\left(569258 - \frac{(2380)^2}{10}\right)$$

$$= \frac{1}{9}(569258 - 566440)$$

$$= \frac{2818}{9}$$

The percentage points are $\chi_9^2(0.975) = 2.700$ and $\chi_9^2(0.025) = 19.023$.

The critical points are:

$$\frac{(10-1)s^2}{\chi_9^2(0.975)} = \frac{9 \times \frac{2818}{9}}{2.700} = 1043.704 \text{ and } \frac{(10-1)s^2}{\chi_9^2(0.025)} = \frac{9 \times \frac{2818}{9}}{19.023} = 148.136.$$

The 95% confidence interval for the variance of the lengths is (148.136, 1043.704).

7 To obtain the lower limit of 7.9623 for a 95% confidence interval of 8 normally distributed samples, the formula $\frac{(8-1)s^2}{\chi_7^2(0.025)} = 7.9623$ may be rearranged in order to find $\sum x^2$.

Firstly, we can find the value $\chi_7^2(0.025) = 16.013$ in the chi-squared distribution tables.

So at this point we can rearrange to get the expression $s^2 = \frac{7.9623 \times 16.013}{7} = 18.21433$.

We know that:

$$s^2 = \frac{1}{n-1}\left(\sum x^2 - \frac{\left(\sum x\right)^2}{n}\right)$$

$$= \frac{1}{7}\left(\sum x^2 - \frac{(234)^2}{8}\right)$$

$$= \frac{1}{7}\left(\sum x^2 - 6844.5\right)$$

So we can solve these two expressions for s^2 simultaneously and find:

$$\frac{1}{7}\left(\sum x^2 - 6844.5\right) = 18.21433$$

$$\sum x^2 - 6844.5 = 127.5003$$

$$\sum x^2 = 6972.$$

8 The percentage points are $\chi^2_{24}(0.95) = 13.848$ and $\chi^2_{24}(0.05) = 36.415$.

We are given $s^2 = 6.21^2$ and so can calculate that the critical points are:

$$\frac{(25-1)s^2}{\chi^2_{24}(0.95)} = \frac{24 \times 6.21^2}{13.848} = 66.84 \text{ and } \frac{(25-1)s^2}{\chi^2_{24}(0.05)} = \frac{24 \times 6.21^2}{36.415} = 25.42.$$

So the 90% confidence interval for **variance** is (25.42, 66.84).

The 90% confidence interval for the **standard deviation** has the square root of the limits of this interval as its limits. i.e. (5.04, 8.18).

9 **a** First we find s^2 using the equation

$$s^2 = \frac{1}{n-1}\left(\sum x^2 - \frac{\left(\sum x\right)^2}{n}\right)$$
$$= \frac{1}{8-1}\left(341.18 - \frac{(52.2)^2}{8}\right)$$
$$= \frac{1}{7}\left(341.18 - \frac{2724.84}{8}\right)$$
$$= \frac{23}{280}$$

The percentage points are $\chi^2_7(0.995) = 0.989$ and $\chi^2_7(0.005) = 20.278$.

The critical points are:

$$\frac{(8-1)s^2}{\chi^2_7(0.995)} = \frac{7 \times \frac{23}{280}}{0.989} = 0.581 \text{ and } \frac{(8-1)s^2}{\chi^2_7(0.005)} = \frac{7 \times \frac{23}{280}}{20.278} = 0.028.$$

The 99% confidence interval for the **variance** of the diameters is (0.028, 0.581).

Hence the 99% confidence interval for the **standard deviation** of the diameters is (0.168, 0.763).

b We have assumed that this sample is from a normal distribution.

c Since the probability of the population standard deviation being between the values of 0.168 and 0.762 is 0.99, that means there is at most a 1% chance that the standard deviation is outside of this range.

Due to Francine requiring a standard deviation less than 0.15 mm (and this range lies entirely outside the 99% confidence interval), the machine should be recalibrated.