Further hypothesis tests 6B

1 a $\bar{x} = 16.605 \quad s^2 = \dfrac{5583.63 - 20(16.605)^2}{19} = 3.637\ldots$

b $H_0 : \sigma^2 = 1.5 \qquad H_1 : \sigma^2 > 1.5$

Critical region > 30.144

Test statistic $= \dfrac{(n-1)s^2}{\sigma^2} = \dfrac{19 \times 3.637..}{1.5} = 46.073$

The test statistic is in the critical region, so reject H_0

There is evidence to suggest $\sigma^2 > 1.5$

2 $\bar{x} = 0.337 \quad s^2 = 0.0028677\ldots$

$H_0 : \sigma^2 = 0.09 \qquad H_1 : \sigma^2 < 0.09$

Critical region ≤ 2.700

Test statistic $= \dfrac{(n-1)s^2}{\sigma^2} = \dfrac{9 \times 0.0028677\ldots}{0.09} = 0.287$

The test statistic is in the critical region, so reject H_0

There is evidence to suggest that variance is less than 0.09.

3 $H_0 : \sigma^2 = 4.1 \qquad H_1 : \sigma^2 \neq 4.1$

$\bar{x} = 5.74 \qquad s^2 = 6.940\ldots$

Critical region ≤ 2.7 and ≥ 19.023

Test statistic $= \dfrac{(n-1)s^2}{\sigma^2} = \dfrac{9 \times 6.940\ldots}{4.1} = 15.235$

The test statistic is not in the critical region, so accept . .

There is no evidence the variance does not equal 4.1.

4 $H_0 : \sigma^2 = 1.12^2 \qquad H_1 : \sigma^2 \neq 1.12^2$

Critical region ≤ 8.907 and ≥ 32.852

Test statistic $= \dfrac{(n-1)s^2}{\sigma^2} = \dfrac{19 \times 1.15}{1.12^2} = 17.419$

The test statistic is not in the critical region, so accept H_0

There is no evidence the variance does not equal 1.12.

5 **a** An unbiased estimation of μ is calculated as:

$$\bar{x} = \frac{\sum x}{n}$$
$$= \frac{149.941}{15}$$
$$= 9.996...$$

An unbiased estimation of σ^2 is calculated as:

$$s^2 = \frac{1}{n-1}\left(\sum x^2 - n\bar{x}^2\right)$$
$$= \frac{1}{15-1}\left(1498.83 - 15 \times 9.996...^2\right)$$
$$= \frac{1}{14}(0.00976...)$$
$$= 0.0006977....$$

b Our hypotheses are:

$H_0 : \sigma^2 = 0.04$
and
$H_1 : \sigma^2 \neq 0.04$

The significance level is 5% (2.5% at each tail) with $v = 14$ degrees of freedom. From the table, we find critical values of:

$\chi^2_{14}(0.975) = 5.629$
and
$\chi^2_{14}(0.025) = 26.119$

The critical regions are $\frac{(n-1)s^2}{\sigma^2} \geqslant 26.119$ and $\frac{(n-1)s^2}{\sigma^2} \leqslant 5.629$.
$s^2 = 0.0006977...$, and $\sigma^2 = 0.04$.

So our test statistic is:
$$\frac{(n-1)s^2}{\sigma^2} = \frac{(15-1)0.000697...}{0.04} = 0.244$$

0.244 < 5.629 and so 0.244 is in the critical region so we have sufficient evidence to reject H_0 and conclude that there has been a change in variance ($\sigma^2 \neq 0.04$).

6 a $s^2 = 0.06125$

b $H_0 : \sigma^2 = 0.19 \quad H_1 : \sigma^2 \neq 0.19$

Critical region ≤ 2.167 and ≥ 14.067

Test statistic $= \dfrac{(n-1)s^2}{\sigma^2} = \dfrac{7 \times 0.06125}{0.19} = 2.256$

The test statistic is not in the critical region, so we do not reject H_0.

There is no evidence that σ^2 does not equal 0.19.

7 a $H_0 : \sigma^2 = 110.25 \quad H_1 : \sigma^2 < 110.25$

$10.5^2 = 110.25$

Critical region ≤ 10.117

Test statistic $= \dfrac{(n-1)s^2}{\sigma^2} = \dfrac{19 \times 8.5^2}{110.25} = 12.451$

The test statistic is not in the critical region, so we do not reject H_0.

There is no evidence that the variance has reduced.

b Take a larger sample before committing to the new component.

8 a An unbiased estimate of μ is calculated as $\bar{x} = \dfrac{\sum x}{n} = \dfrac{32.12}{10} = 3.212$.

In order to calculate the standard error, we first calculate an unbiased estimate of the standard deviation.

$$s^2 = \frac{1}{n-1}\left(\sum x^2 - n\bar{x}^2\right)$$
$$= \frac{1}{10-1}\left(103.8592 - 10 \times 3.212^2\right)$$
$$= 0.07664.$$

Thus, the unbiased estimate of standard deviation is $s = 0.2768$.

Now we calculate the standard error to be:

$$\frac{s}{\sqrt{n}} = \frac{0.2768}{\sqrt{10}} = 0.0875 \text{ (3 s.f.)}$$

8 b Our hypotheses are:

$H_0 : \sigma = 0.25$
and
$H_1 : \sigma \neq 0.25$

The significance level is 5% (2.5% at each tail) with $v = 9$ degrees of freedom.
From the table, we find critical values of:

$\chi_9^2(0.975) = 2.700$
and
$\chi_9^2(0.025) = 19.023$

The critical regions are $\frac{(n-1)s^2}{\sigma^2} \geqslant 19.023$ and $\frac{(n-1)s^2}{\sigma^2} \leqslant 2.700$.
$s^2 = 0.07664$ and $\sigma^2 = 0.25^2$.

So our test statistic is:
$$\frac{(n-1)s^2}{\sigma^2} = \frac{(10-1)0.07664}{0.25^2} = 11.03616$$

$2.700 < 11.03616 < 19.023$ and so 11.03616 is not in the critical region so we do not have sufficient evidence to reject H_0. We therefore conclude that there has been no change in standard deviation.