Confidence intervals and tests using the *t*-distribution 7B

1 $H_0: \mu = 11 \qquad H_1: \mu > 11$

Significance level 5%

$\upsilon = 4$

The critical value is $t_4(0.05) = 2.132$, so the critical region is $t \geqslant 2.132$

$$\bar{x} = \frac{\sum x}{n} = \frac{57}{5} = 11.4$$

$$s^2 = \frac{1}{n-1}\left(\sum x^2 - n\bar{x}^2\right) = \frac{1}{4}(663 - 5 \times 11.4^2) = 3.3$$

$s = 1.8166$

$$\text{Test statistic } t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}} = \frac{11.4 - 11.0}{\frac{1.8166}{\sqrt{5}}} = 0.492$$

$0.492 < 2.132$, so the result is not significant. Accept H_0.

There is not enough evidence to suggest that μ is not 11.

2 $H_0: \mu = 19 \qquad H_1: \mu < 19$

Significance level 1%

$\upsilon = 27$

The critical value is $t_{27}(0.01) = 2.473$, so the critical region is $t \leqslant -2.473$.

$\bar{x} = 17.1 \qquad s^2 = 4 \qquad s = 2$

$$\text{Test statistic } t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}} = \frac{17.1 - 19}{\frac{2}{\sqrt{28}}} = -5.027$$

$-5.027 < -2.473$, so the result is significant and H_0 is rejected.

There is evidence to suggest that μ is less than 19.

3 $H_0: \mu = 3 \qquad H_1: \mu \neq 3$

Significance level 5%, probability in each tail = 0.025

$\upsilon = 12$

The critical value is $t_{12}(0.025) = 2.179$, so the critical region is $|t| \geqslant 2.179$

$\bar{x} = 3.26 \qquad s^2 = 0.64 \qquad s = 0.8$

$$\text{Test statistic } t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}} = \frac{3.26 - 3}{\frac{0.8}{\sqrt{13}}} = 1.172$$

$1.172 < 2.179$, so the result is not significant. Accept H_0.

There is not enough evidence to suggest that μ is not 3.

4 a The population variance is unknown and the sample size is not large.

4 b $H_0: \mu = 100$ $H_1: \mu \neq 100$

Significance level 5%, probability in each tail = 0.025

$\upsilon = 14$

The critical value is $t_{14}(0.025) = 2.145$, so the critical regions are $t \leqslant -2.145$ and $t \geqslant 2.145$

$$\bar{x} = \frac{\sum x}{n} = \frac{1473}{15} = 98.2$$

$$s^2 = \frac{1}{n-1}\left(\sum x^2 - n\bar{x}^2\right) = \frac{1}{14}(148\,119 - 15 \times 98.2^2) = 247.88571$$

$s = 15.74438$

$$\text{Test statistic } t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{98.2 - 100}{\frac{15.77438}{\sqrt{15}}} = -0.443$$

$-0.443 > -2.145$, so the result is not significant. Accept H_0.

There is not enough evidence to suggest that μ is not 100.

The manufacturer's claim is supported by the sample results.

5 $H_0: \mu = 1000$ $H_1: \mu > 1000$

Significance level 5%

$\upsilon = 7$

The critical value is $t_7(0.05) = 1.895$, so the critical region is $t \geqslant 1.895$

$$\bar{x} = \frac{\sum x}{n} = \frac{8390}{8} = 1048.75 \qquad \sum x^2 = 8862\,500$$

$$s^2 = \frac{1}{n-1}\left(\sum x^2 - n\bar{x}^2\right) = \frac{1}{7}(8862\,500 - 8 \times 1048.75^2) = 9069.6428$$

$s = 95.2347$

$$\text{Test statistic } t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{1048.75 - 1000}{\frac{95.2347}{\sqrt{8}}} = 1.448$$

$1.448 < 1.895$, so the result is not significant. Accept H_0.

There is not enough evidence to suggest that μ is greater than 1000.

6 $H_0: \mu = 6$ $H_1: \mu > 6$

Significance level 2.5%

$\upsilon = 13$

The critical value is $t_{13}(0.025) = 2.160$, so the critical region is $t \geqslant 2.160$

$$\bar{x} = \frac{\sum x}{n} = \frac{90.8}{14} = 6.48571\ldots$$

$$s^2 = \frac{1}{n-1}\left(\sum x^2 - n\bar{x}^2\right) = \frac{1}{13}(600 - 14 \times 6.48571^2) = 0.85362$$

$s = 0.92391$

$$\text{Test statistic } t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{6.48571 - 6}{\frac{0.92391}{\sqrt{14}}} = 1.967$$

$1.967 < 2.160$, so accept H_0. There is no evidence to support the manufacturer's claim.

7 a $H_0: \mu = 1.00 \qquad H_1: \mu > 1.00$

Choose a significance level of 10%

$\upsilon = 19$

The critical value is $t_{19}(0.10) = 1.328$, so the critical region is $t \geqslant 1.328$

$$\bar{x} = \frac{\sum x}{n} = \frac{21.7}{20} = 1.085$$

$$s^2 = \frac{1}{n-1}\left(\sum x^2 - n\bar{x}^2\right) = \frac{1}{19}(28.4 - 20 \times 1.085^2) = 0.25555\ldots$$

$s = 0.50552\ldots$

$$\text{Test statistic } t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{1.085 - 1}{\frac{0.50552}{\sqrt{20}}} = 0.752$$

$0.752 < 1.328$, so accept H_0. There is no evidence to suggest the radiation levels are greater than 1. This supports the company's claim that the amount of radiation has been reduced to an acceptable level.

b The amount of radiation is normally distributed.

8 a $H_0: \mu = 100 \qquad H_1: \mu > 100$

Significance level 5%

$\upsilon = 19$

The critical value is $t_{19}(0.05) = 1.729$, so the critical region is $t \geqslant 1.729$

$\bar{x} = 100 \qquad s = 15$

$$\text{Test statistic } t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{110 - 100}{\frac{15}{\sqrt{20}}} = 2.981$$

$2.981 > 1.729$, so the result is significant and H_0 is rejected.

There is evidence to support the company's claim that it can train people to improve their scores in the aptitude test.

b The percentage points are $\chi^2_{19}(0.05) = 30.144$ and $\chi^2_{19}(0.95) = 10.117$

The critical points for the variance are:

$$\frac{(n-1)s^2}{\chi^2_{19}(0.05)} = \frac{19(15^2)}{30.144} = 141.819 \text{ and } \frac{(n-1)s^2}{\chi^2_{19}(0.95)} = \frac{19(15^2)}{10.117} = 422.556$$

The 95% confidence interval for the standard deviation is $\left(\sqrt{141.819}, \sqrt{422.556}\right) = (11.91, 20.56)$.

12 lies in this interval, so there is insufficient evidence for rejecting H_0. There is no evidence that the standard deviation is different from 12.