Confidence intervals and tests using the *t*-distribution 7C

1 a i $H_0: \mu_D = 0 \qquad H_1: \mu_D \neq 0$

ii $H_0: \mu_D = 0 \qquad H_1: \mu_D > 0$

b $H_0: \mu_D = 0 \qquad H_1: \mu_D > 0$

Significance level 5%

$\upsilon = 5$

The critical value is $t_5(0.05) = 2.015$, so the critical region is $t \geqslant 2.015$

$\sum d = 7+5+0+0+10+8 = 30$, so $\bar{d} = \frac{30}{6} = 5$

$\sum d^2 = 49+25+100+64 = 238$

$s^2 = \frac{1}{n-1}\left(\sum d^2 - n\bar{d}^2\right) = \frac{1}{5}(238 - 6\times 5^2) = 17.6$

$s = 4.1952$

Test statistic $t = \dfrac{\bar{d} - \mu_D}{\frac{s}{\sqrt{n}}} = \dfrac{5-0}{\frac{4.1952}{\sqrt{6}}} = 2.919$

$2.919 > 2.015$, so the result is significant: reject H_0.

There is evidence to suggest that there has been an increase in shorthand speed.

2 $H_0: \mu_D = 0 \qquad H_1: \mu_D > 0$

Significance level 1%

$\upsilon = 9$

The critical value is $t_9(0.01) = 2.821$, so the critical region is $t \geqslant 2.821$

$\sum d = 2+2-1+2+2+3-5+0+2-2 = 5$, so $\bar{d} = \frac{5}{10} = 0.5$

$\sum d^2 = 4+4+1+4+4+9+25+4+4 = 59$

$s^2 = \frac{1}{n-1}\left(\sum d^2 - n\bar{d}^2\right) = \frac{1}{9}(59 - 10\times 0.5^2) = 6.27777$

$s = 2.5055$

Test statistic $t = \dfrac{\bar{d} - \mu_D}{\frac{s}{\sqrt{n}}} = \dfrac{0.5}{\frac{2.5055}{\sqrt{10}}} = 0.631$

$0.631 < 2.821$, so the result is not significant: accept H_0.

There is insufficient evidence to suggest that paper 2 is easier than paper 1 so the teacher is not correct.

3 a $H_0: \mu_D = 0$ $H_1: \mu_D > 0$

Significance level 5%

$\upsilon = 9$

The critical value is $t_9(0.05) = 1.833$, so the critical region is $t \geqslant 1.833$

$\sum d = 5+10+5+2+0+0+8+5+6+6 = 47$, so $\bar{d} = \frac{47}{10} = 4.7$

$\sum d^2 = 25+100+25+4+64+25+36+36 = 315$

$s^2 = \frac{1}{n-1}\left(\sum d^2 - n\bar{d}^2\right) = \frac{1}{9}(315 - 10\times 4.7^2) = 10.45555$

$s = 3.2335$

Test statistic $t = \frac{\bar{d}-\mu_D}{\frac{s}{\sqrt{n}}} = \frac{4.7-0}{\frac{3.2335}{\sqrt{10}}} = 4.596$

$4.596 > 1.833$, so the result is significant: reject H_0.

There is evidence to suggest that chewing the gum does reduce the craving for cigarettes.

b The differences are normally distributed.

4 $H_0: \mu_D = 0$ $H_1: \mu_D > 0$

Significance level 1%

$\upsilon = 9$

The critical value is $t_9(0.05) = 1.833$, so the critical region is $t \geqslant 1.833$

$\sum d = 5+2+4+0+5+12+2+6+9+1 = 46$, so $\bar{d} = \frac{46}{10} = 4.6$

$\sum d^2 = 25+4+16+25+144+4+36+81+1 = 336$

$s^2 = \frac{1}{n-1}\left(\sum d^2 - n\bar{d}^2\right) = \frac{1}{9}(336 - 10\times 4.6^2) = 13.82222$

$s = 3.7178$

Test statistic $t = \frac{\bar{d}-\mu_D}{\frac{s}{\sqrt{n}}} = \frac{4.6}{\frac{3.7178}{\sqrt{10}}} = 3.913$

$3.913 > 1.833$, so the result is significant: reject H_0.

There is evidence to suggest that the journey times have decreased.

5 a $H_0: \mu_D = 0 \qquad H_1: \mu_D \neq 0$

Significance level 10% – probability in each tail = 0.05

$\upsilon = 7$

The critical value is $t_7(0.05) = 1.895$, so the critical region is $|t| \geqslant 1.895$

$$\sum d = 10 - 9 + 11 - 5 + 8 + 7 - 10 + 8 = 20, \text{ so } \bar{d} = \frac{20}{8} = 2.5$$

$$\sum d^2 = 100 + 81 + 121 + 25 + 64 + 49 + 100 + 64 = 604$$

$$s^2 = \frac{1}{n-1}\left(\sum d^2 - n\bar{d}^2\right) = \frac{1}{7}(604 - 8 \times 2.5^2) = 79.14285$$

$s = 8.8962$

$$\text{Test statistic } t = \frac{\bar{d} - \mu_D}{\frac{s}{\sqrt{n}}} = \frac{2.5}{\frac{8.8962}{\sqrt{8}}} = 0.795$$

$0.795 < 1.895$, so the result is not significant: accept H_0.

The mock examination is a good predictor of results in the actual examination.

b The differences are normally distributed.

6 a Different people will have different productivity rates. There needs to be a common link to compare productivity before and after the introduction of the tea break. This reduces experimental error due to differences between individuals so that if a difference does exist, it is more likely to be detected.

b $H_0: \mu_D = 0 \qquad H_1: \mu_D > 0$

Significance level 5%

$\upsilon = 9$

The critical value is $t_9(0.05) = 1.833$, so the critical region is $t \geqslant 1.833$

$$\sum d = 5 + 11 + 4 + 3 + 11 + 11 + 1 + 3 + 5 + 11 = 65, \text{ so } \bar{d} = \frac{65}{10} = 6.5$$

$$\sum d^2 = 25 + 121 + 16 + 9 + 121 + 121 + 1 + 9 + 25 + 121 = 569$$

$$s^2 = \frac{1}{n-1}\left(\sum d^2 - n\bar{d}^2\right) = \frac{1}{9}(569 - 10 \times 6.5^2) = 16.2777$$

$s = 4.0346$

$$\text{Test statistic } t = \frac{\bar{d} - \mu_D}{\frac{s}{\sqrt{n}}} = \frac{6.5}{\frac{4.0346}{\sqrt{10}}} = 5.095$$

$5.095 > 1.833$, so the result is significant: reject H_0.

There is evidence to suggest a tea break increases the number of garments made.

7 H_0: $\mu_D = 0$ H_1: $\mu_D > 0$

Significance level 1%

$\upsilon = 7$

The critical value is $t_7(0.01) = 2.998$, so the critical region is $t \geqslant 2.998$

$\sum d = 0.2 + 0.2 - 0.1 + 1.4 + 0.5 + 3.4 + 2.6 + 0.4 = 8.6$, so $\bar{d} = \frac{8.6}{8} = 1.075$

$\sum d^2 = 0.04 + 0.04 + 0.01 + 1.96 + 0.25 + 11.56 + 6.76 + 0.16 = 20.78$

$s^2 = \frac{1}{n-1}\left(\sum d^2 - n\bar{d}^2\right) = \frac{1}{7}(20.78 - 8 \times 1.075^2) = 1.64785$

$s = 1.2837$

Test statistic $t = \frac{\bar{d} - \mu_D}{\frac{s}{\sqrt{n}}} = \frac{1.075}{\frac{1.2837}{\sqrt{8}}} = 2.369$

$2.369 < 2.998$, so the result is not significant: accept H_0.

There is not sufficient evidence to suggest that the drug increases the mean number of hours of sleep per night.