Confidence intervals and tests using the *t*-distribution 7D

1 a $s_p^2 = \frac{(9\times 4)+(14\times 5.3)}{10+15-2} = 4.7913$

$s_p = 2.189$

$t_{23}(0.025) = 2.069$

The confidence limits are:

$$(25-22)\pm 2.069\times 2.189\sqrt{\frac{1}{15}+\frac{1}{10}} = 3\pm 1.849 = 1.151 \text{ and } 4.849$$

The 95% confidence interval is (1.151, 4.849) or (1.51, 4.85) to 3 s.f.

b Three assumptions are made: the populations (width of the shells) are normally distributed; the two samples are independent; and the variances of the two populations (from sheltered shore and non-sheltered shore) are equal.

2 a For the soil-less compost:

$$\sum x_{sl} = 71.3 \Rightarrow \bar{x}_{sl} = \frac{71.3}{8} = 8.9125$$

$$\sum x_{sl}^2 = 639.53 \Rightarrow s_{sl}^2 = \frac{1}{7}\left(639.53 - 8\times 8.9125^2\right) = 0.58125$$

For the soil-based compost:

$$\sum x_{sb} = 120.4 \Rightarrow \bar{x}_{sb} = \frac{120.4}{10} = 12.04$$

$$\sum x_{sb}^2 = 1457.26 \Rightarrow s_{sb}^2 = \frac{1}{9}\left(1457.26 - 10\times 12.04^2\right) = 0.84933$$

So:

$$s_p^2 = \frac{(n_{sl}-1)s_{sl}^2 + (n_{sb}-1)s_{sb}^2}{n_{sl}+n_{sb}-2} = \frac{(7\times 0.58125)+(9\times 0.84933)}{8+10-2} = 0.732045$$

$s_p = 0.8556$

$t_{16}(0.05) = 1.746$

The confidence limits are:

$$(\bar{x}_{sb}-\bar{x}_{sl})\pm t_c s_p\sqrt{\frac{1}{n_{sb}}+\frac{1}{n_{sl}}} = (12.04-8.9125)\pm 1.746\times 0.8556\sqrt{\frac{1}{10}+\frac{1}{8}}$$

$$= 3.1275\pm 0.7086 = 2.4189 \text{ and } 3.8361$$

The 90% confidence interval is $(2.42, 3.84)$ to 3 s.f.

b This assumes that the variances of the two population (soil-less compost and soil-based compost) are equal. This is reasonable since compost is designed to increase the amount of growth, not the variability.

3 a $s_p^2 = \dfrac{(n_a - 1)s_a^2 + (n_b - 1)s_b^2}{n_a + n_b - 2} = \dfrac{(19 \times 6.12) + (19 \times 5.22)}{20 + 20 - 2} = 5.67$

$s_p = 2.38118$

$t_{38}(0.005) = 2.712$

The confidence limits are:

$$(\bar{x}_b - \bar{x}_a) \pm t_c s_p \sqrt{\frac{1}{n_b} + \frac{1}{n_a}} = (38.2 - 32.7) \pm 2.712 \times 2.38118\sqrt{\frac{1}{20} + \frac{1}{20}}$$

$$= 5.5 \pm 2.0421 = 3.4579 \text{ and } 7.5421$$

The 99% confidence interval is $(3.46, 7.54)$ to 3 s.f.

b Three assumptions are made: the populations (number of words spelled correctly) are normally distributed; the two samples are independent; and the variances of the two populations are equal.

c Zero is not in interval. This suggest that there is a significant difference between the two results and that method *B* seems better than method *A*.

4 Assume that the variances of the two populations are equal and that the populations are normally distributed.

$s_p^2 = \dfrac{(n_a - 1)s_a^2 + (n_b - 1)s_b^2}{n_a + n_b - 2} = \dfrac{(9 \times 32.488) + (9 \times 33.344)}{10 + 10 - 2} = 32.916$

$s_p = 5.73725$

$t_{18}(0.05) = 1.734$

The confidence limits are:

$$(\bar{x}_a - \bar{x}_b) \pm t_c s_p \sqrt{\frac{1}{n_a} + \frac{1}{n_b}} = (18.6 - 14.3) \pm 1.734 \times 5.73725\sqrt{\frac{1}{10} + \frac{1}{10}}$$

$$= 4.3 \pm 4.4491 = -0.1491 \text{ and } 8.7491$$

The 99% confidence interval is $(-0.149, 8.749)$ to 3 d.p.

5 a $\text{H}_0: \sigma_A^2 = \sigma_B^2 \qquad \text{H}_1: \sigma_A^2 \neq \sigma_B^2$

$\upsilon_l = 7 - 1 = 6 \qquad \upsilon_s = 8 - 1 = 7$

$s_l^2 = 1.6$ and $s_s^2 = 1.2$

The critical value is $F_{6,7}(0.05) = 3.87$

The test statistic is $\dfrac{s_l^2}{s_s^2} = \dfrac{1.6}{1.2} = 1.333\ldots$

$1.333 < 3.87$, so accept H_0.

There is no evidence that there is a difference in the variability of the yields.

The test assumes that the samples are taken from populations that are normally distributed.

b As the test in part **a** supports the assumption that the variances of the population are equal, the use of a *t*-distribution to find the confidence interval is justified providing that two other requirements are met: that the populations that are normally distributed and the two samples are independent.

5 c $s_p^2 = \frac{(n_a - 1)s_a^2 + (n_b - 1)s_b^2}{n_a + n_b - 2} = \frac{(7 \times 1.2) + (6 \times 1.6)}{8 + 7 - 2} = 1.38462$

$s_p = 1.17670$

$t_{13}(0.025) = 2.160$

The confidence limits are:

$$(\overline{x}_b - \overline{x}_a) \pm t_c s_p \sqrt{\frac{1}{n_b} + \frac{1}{n_a}} = (26.8 - 24.5) \pm 2.160 \times 1.1767\sqrt{\frac{1}{7} + \frac{1}{8}}$$

$$= 2.3 \pm 1.3154 = 0.9846 \text{ and } 3.6154$$

The 95% confidence interval is $(0.985, 3.62)$ to 3 s.f.