Numerical Methods 8A

1 $(x_0, y_0) = (1, 2)$

$h = 0.25$

$\left(\frac{dy}{dx}\right)_0 = 1^2 + 2^2 = 5$

$y_1 = y_0 + h\left(\frac{dy}{dx}\right)_0$

$y_1 = 2 + 0.25 \times 5$

$y_1 = 3.25$

$(x_1, y_1) = (1.25, 3.25)$

$\left(\frac{dy}{dx}\right)_1 = 1.25^2 + 3.25^2 = 12.125$

$y_2 = y_1 + h\left(\frac{dy}{dx}\right)_1$

$y_2 = 3.25 + 0.25 \times 12.125$

$y_2 = 6.28125$

$(x_2, y_2) = (1.5, 6.28125)$

$\left(\frac{dy}{dx}\right)_2 = 1.5^2 + 6.28125^2 = 41.70410\ldots$

$y_3 = y_2 + h\left(\frac{dy}{dx}\right)_2$

$y_3 = 6.28125 + 0.25 \times 41.70410\ldots$

$y_3 = 16.70728\ldots$

$\left(\frac{dy}{dx}\right)_3 = 1.75^2 + (16.70728\ldots)^2 = 282.19555\ldots$

$(x_3, y_3) = (1.75, 16.70728\ldots)$

$y_4 = y_3 + h\left(\frac{dy}{dx}\right)_3$

$y_4 = 16.70727\ldots + 0.25 \times 282.19555\ldots$

$y_4 = 87.25616\ldots$

Therefore, at $x = 2$, $y \approx 87.3$ (3 s.f.)

2 $h = \frac{3 - 2.5}{5} = 0.1$

$(x_0, y_0) = (2.5, 0)$

$\left(\frac{dy}{dx}\right)_0 = \sqrt{e^{2.5} + 2 \times e^0} = 3.76596\ldots$

$y_1 = y_0 + h\left(\frac{dy}{dx}\right)_0$

$y_1 = 0 + 0.1 \times 3.76596\ldots$

$y_1 = 0.37660\ldots$

$(x_1, y_1) = (2.6, 0.37660\ldots)$

$\left(\frac{dy}{dx}\right)_1 = \sqrt{e^{2.6} + 2 \times e^{0.37660\ldots}} = 4.04702\ldots$

$y_2 = y_1 + h\left(\frac{dy}{dx}\right)_1$

$y_2 = 0.37660\ldots + 0.1 \times 4.04702\ldots$

$y_2 = 0.78130\ldots$

$(x_2, y_2) = (2.7, 0.78130\ldots)$

$\left(\frac{dy}{dx}\right)_2 = \sqrt{e^{2.7} + 2 \times e^{0.78130\ldots}} = 4.38729\ldots$

$y_3 = y_2 + h\left(\frac{dy}{dx}\right)_2$

$y_3 = 0.78130\ldots + 0.1 \times 4.38729\ldots$

$y_3 = 1.22003\ldots$

$(x_3, y_3) = (2.8, 1.22003\ldots)$

$\left(\frac{dy}{dx}\right)_3 = \sqrt{e^{2.8} + 2 \times e^{1.22003\ldots}} = 4.81863\ldots$

$y_4 = y_3 + h\left(\frac{dy}{dx}\right)_3$

$y_4 = 1.22003\ldots + 0.1 \times 4.81863\ldots$

$y_4 = 1.70189\ldots$

$(x_4, y_4) = (2.9, 1.70189\ldots)$

$\left(\frac{dy}{dx}\right)_4 = \sqrt{e^{2.9} + 2 \times e^{1.70189\ldots}} = 5.39840\ldots$

$y_5 = y_4 + h\left(\frac{dy}{dx}\right)_4$

$y_5 = 1.70189\ldots + 0.1 \times 5.39840\ldots$

$y_5 = 2.24173\ldots$

So $f(3) \approx 2.24$ (3 s.f.)

3 a $(x_0, y_0) = (e, 2)$

$y_1 = 2.4$

$\left(\frac{dy}{dx}\right)_0 = \ln\left(e + 2^2\right) = 1.90483\ldots$

$\frac{y_1 - y_0}{h} = \left(\frac{dy}{dx}\right)_0 = 1.90483\ldots$

$\frac{2.4 - 2}{h} = 1.90483\ldots$

$h = \frac{0.4}{1.90483\ldots} = 0.20999\ldots$

So $h \approx 0.21$ (2 d.p.)

b $\left(\frac{dy}{dx}\right)_1 = \ln\left((e + 0.21) + 2.4^2\right) = 2.16198\ldots$

$y_2 = y_1 + h\left(\frac{dy}{dx}\right)_1$

$y_2 = 2.4 + 0.21 \times 2.16198\ldots$

$y_2 = 2.85401\ldots$

So $y_2 \approx 2.854$ (3 d.p.)

$\left(\frac{dy}{dx}\right)_2 = \ln\left((e + 0.42) + (2.85401\ldots)^2\right)$

$= 2.42336\ldots$

$y_3 = y_2 + h\left(\frac{dy}{dx}\right)_2$

$y_3 = 2.85401\ldots + 0.21 \times 2.42336\ldots$

$y_3 = 3.36292\ldots$

So $y_3 \approx 3.363$ (3 d.p.)

4 $(t_0, v_0) = (2, 10)$

$h = \frac{5 - 2}{2} = 1.5$

$\left(\frac{dv}{dt}\right)_0 = \frac{10 - 2}{10 \times 2 - 2^3} = 0.66667\ldots$

$v_1 = v_0 + h\left(\frac{dv}{dt}\right)_0$

$v_1 = 10 + 1.5 \times 0.66667\ldots$

$v_1 = 11$

$(t_1, v_1) = (3.5, 11)$

$\left(\frac{dv}{dt}\right)_1 = \frac{11 - 3.5}{11 \times 3.5 - 3.5^3} = -1.71428\ldots$

$v_2 = v_1 + h\left(\frac{dv}{dt}\right)_1$

$v_2 = 11 + 1.5 \times -1.71428\ldots$

$v_2 = 8.42857\ldots$

So $v_2 \approx 8.4$ (1 d.p.)

Therefore, the value of the asset is £8400 five days after it is purchased.

5 $(t_0, \theta_0) = (0, 0)$

$h = \frac{0.3 - 0}{2} = 0.15$

$\left(\frac{d\theta}{dt}\right)_0 = \sqrt{9.8(2\cos 0 - 1)} = 3.13050\ldots$

$\theta_1 = \theta_0 + h\left(\frac{d\theta}{dt}\right)_0$

$\theta_1 = 0 + 0.15 \times 3.13050\ldots$

$\theta_1 = 0.46957\ldots$

$(t_1, \theta_1) = (0.15, 0.46957\ldots)$

$\left(\frac{d\theta}{dt}\right)_1 = \sqrt{9.8(2\cos 0.46957\ldots - 1)} = 2.7710\ldots$

$\theta_2 = \theta_1 + h\left(\frac{d\theta}{dt}\right)_1$

$\theta_2 = 0.46957\ldots + 0.15 \times 2.7710\ldots$

$\theta_2 = 0.88523\ldots$

So $\theta \approx 0.885$ (3 s.f.) when $t = 0.3$