Vectors 11B

1 $\mathbf{v}_1$ **i** $8\begin{pmatrix}1\\0\end{pmatrix}+0\begin{pmatrix}0\\1\end{pmatrix}=8\mathbf{i}$

ii $8\begin{pmatrix}1\\0\end{pmatrix}+0\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}8\\0\end{pmatrix}$

$\mathbf{v}_2$ **i** $9\begin{pmatrix}1\\0\end{pmatrix}+3\begin{pmatrix}0\\1\end{pmatrix}=9\mathbf{i}+3\mathbf{j}$

ii $9\begin{pmatrix}1\\0\end{pmatrix}+3\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}9\\3\end{pmatrix}$

$\mathbf{v}_3$ **i** $-4\begin{pmatrix}1\\0\end{pmatrix}+2\begin{pmatrix}0\\1\end{pmatrix}=-4\mathbf{i}+2\mathbf{j}$

ii $-4\begin{pmatrix}1\\0\end{pmatrix}+2\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}-4\\2\end{pmatrix}$

$\mathbf{v}_4$ **i** $3\begin{pmatrix}1\\0\end{pmatrix}+5\begin{pmatrix}0\\1\end{pmatrix}=3\mathbf{i}+5\mathbf{j}$

ii $3\begin{pmatrix}1\\0\end{pmatrix}+5\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}3\\5\end{pmatrix}$

$\mathbf{v}_5$ **i** $-3\begin{pmatrix}1\\0\end{pmatrix}-2\begin{pmatrix}0\\1\end{pmatrix}=-3\mathbf{i}-2\mathbf{j}$

ii $-3\begin{pmatrix}1\\0\end{pmatrix}-2\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}-3\\-2\end{pmatrix}$

$\mathbf{v}_6$ **i** $0\begin{pmatrix}1\\0\end{pmatrix}-5\begin{pmatrix}0\\1\end{pmatrix}=-5\mathbf{j}$

ii $0\begin{pmatrix}1\\0\end{pmatrix}-5\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}0\\-5\end{pmatrix}$

2 **a** $4\mathbf{a}=4(2\mathbf{i}+3\mathbf{j})$
$=8\mathbf{i}+12\mathbf{j}$

b $\frac{1}{2}\mathbf{a}=\frac{1}{2}(2\mathbf{i}+3\mathbf{j})$
$=\mathbf{i}+\frac{1}{2}\mathbf{j}$

2 **c** $-\mathbf{b}=-(4\mathbf{i}-\mathbf{j})$
$=-4\mathbf{i}+\mathbf{j}$

d $2\mathbf{b}+\mathbf{a}=2(4\mathbf{i}-\mathbf{j})+(2\mathbf{i}+3\mathbf{j})$
$=(8\mathbf{i}-2\mathbf{j})+(2\mathbf{i}+3\mathbf{j})$
$=(8+2)\mathbf{i}+(-2+3)\mathbf{j}$
$=10\mathbf{i}+\mathbf{j}$

e $3\mathbf{a}-2\mathbf{b}=3(2\mathbf{i}+3\mathbf{j})-2(4\mathbf{i}-\mathbf{j})$
$=(6\mathbf{i}+9\mathbf{j})-(8\mathbf{i}-2\mathbf{j})$
$=(6-8)\mathbf{i}+(9+2)\mathbf{j}$
$=-2\mathbf{i}+11\mathbf{j}$

f $\mathbf{b}-3\mathbf{a}=(4\mathbf{i}-\mathbf{j})-3(2\mathbf{i}+3\mathbf{j})$
$=(4\mathbf{i}-\mathbf{j})-(6\mathbf{i}+9\mathbf{j})$
$=(4-6)\mathbf{i}+(-1-9)\mathbf{j}$
$=-2\mathbf{i}-10\mathbf{j}$

g $4\mathbf{b}-\mathbf{a}=4(4\mathbf{i}-\mathbf{j})-(2\mathbf{i}+3\mathbf{j})$
$=(16\mathbf{i}-4\mathbf{j})-(2\mathbf{i}+3\mathbf{j})$
$=(16-2)\mathbf{i}+(-4-3)\mathbf{j}$
$=14\mathbf{i}-7\mathbf{j}$

h $2\mathbf{a}-3\mathbf{b}=2(2\mathbf{i}+3\mathbf{j})-3(4\mathbf{i}-\mathbf{j})$
$=(4\mathbf{i}+6\mathbf{j})-(12\mathbf{i}-3\mathbf{j})$
$=(4-12)\mathbf{i}+(6+3)\mathbf{j}$
$=-8\mathbf{i}+9\mathbf{j}$

3 **a** $5\mathbf{a}=5\begin{pmatrix}9\\7\end{pmatrix}$
$=\begin{pmatrix}45\\35\end{pmatrix}$

b $-\frac{1}{2}\mathbf{c}=-\frac{1}{2}\begin{pmatrix}-8\\-1\end{pmatrix}$
$=\begin{pmatrix}4\\0.5\end{pmatrix}$

c $\mathbf{a}+\mathbf{b}+\mathbf{c}=\begin{pmatrix}9\\7\end{pmatrix}+\begin{pmatrix}11\\-3\end{pmatrix}+\begin{pmatrix}-8\\-1\end{pmatrix}$
$=\begin{pmatrix}12\\3\end{pmatrix}$

d $2\mathbf{a}-\mathbf{b}+\mathbf{c}=2\begin{pmatrix}9\\7\end{pmatrix}-\begin{pmatrix}11\\-3\end{pmatrix}+\begin{pmatrix}-8\\-1\end{pmatrix}$
$=\begin{pmatrix}-1\\16\end{pmatrix}$

3 e $2\mathbf{b} + 2\mathbf{c} - 3\mathbf{a} = 2\begin{pmatrix}11\\-3\end{pmatrix} + 2\begin{pmatrix}-8\\-1\end{pmatrix} - 3\begin{pmatrix}9\\7\end{pmatrix}$

$= \begin{pmatrix}-21\\-29\end{pmatrix}$

f $\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b} = \frac{1}{2}\begin{pmatrix}9\\7\end{pmatrix} + \frac{1}{2}\begin{pmatrix}11\\-3\end{pmatrix}$

$= \begin{pmatrix}10\\2\end{pmatrix}$

4 a $\mathbf{a} + \lambda\mathbf{b} = (2\mathbf{i} + 5\mathbf{j}) + \lambda(3\mathbf{i} - \mathbf{j})$

$= (2 + 3\lambda)\mathbf{i} + (5 - \lambda)\mathbf{j}$

Parallel to **i**, so $5 - \lambda = 0$, $\lambda = 5$.

b $\mu\mathbf{a} + \mathbf{b} = \mu(2\mathbf{i} + 5\mathbf{j}) + (3\mathbf{i} - \mathbf{j})$

$= (2\mu + 3)\mathbf{i} + (5\mu - 1)\mathbf{j}$

Parallel to **j**, so $2\mu + 3 = 0$, $\mu = -\frac{3}{2}$

5 a $\mathbf{c} + \lambda\mathbf{d} = (3\mathbf{i} + 4\mathbf{j}) + \lambda(\mathbf{i} - 2\mathbf{j})$

$= (3 + \lambda)\mathbf{i} + (4 - 2\lambda)\mathbf{j}$

Parallel to **i** + **j**, so $3 + \lambda = 4 - 2\lambda$

$3\lambda = 1$, $\lambda = \frac{1}{3}$

b $\mu\mathbf{c} + \mathbf{d} = \mu(3\mathbf{i} + 4\mathbf{j}) + (\mathbf{i} - 2\mathbf{j})$

$= (3\mu + 1)\mathbf{i} + (4\mu - 2)\mathbf{j}$

Parallel to **i** + 3**j**, so $4\mu - 2 = 3(3\mu + 1)$

$4\mu - 2 = 9\mu + 3$

$5\mu = -5$, $\mu = -1$

c $\mathbf{c} - s\mathbf{d} = (3\mathbf{i} + 4\mathbf{j}) - s(\mathbf{i} - 2\mathbf{j})$

$= (3 - s)\mathbf{i} + (4 + 2s)\mathbf{j}$

Parallel to 2**i** + **j**, so

$3 - s = 2(4 + 2s)$

$3 - \mathbf{s} = 8 + 4\mathrm{i}$

$-5 = 5s$, $s = -1$

d $\mathbf{d} - t\mathbf{c} = (\mathbf{i} - 2\mathbf{j}) - t(3\mathbf{i} + 4\mathbf{j})$

$= (1 - 3t)\mathbf{i} + (-2 - 4t)\mathbf{j}$

Parallel to −2**i** + 3**j**, so

$-2(-2 - 4t) = 3(1 - 3t)$

$4 + 8t = 3 - 9t$

$1 = -17t$, $t = -\frac{1}{17}$

6 $\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$

$= -(4\mathbf{i} + 3\mathbf{j}) + 5\mathbf{i} + 2\mathbf{j}$

$= \mathbf{i} - \mathbf{j}$

7 a $\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC}$

$= -(2\mathbf{i} + 4\mathbf{j}) + 7\mathbf{i}$

$= 5\mathbf{i} - 4\mathbf{j}$

$= 5\begin{pmatrix}1\\0\end{pmatrix} - 4\begin{pmatrix}0\\1\end{pmatrix}$

$= \begin{pmatrix}5\\-4\end{pmatrix}$

b $\overrightarrow{AP} = \frac{3}{5}\overrightarrow{AC}$

$= \frac{3}{5}(5\mathbf{i} - 4\mathbf{j})$

$= 3\mathbf{i} - \frac{12}{5}\mathbf{j}$

$= 3\begin{pmatrix}1\\0\end{pmatrix} - \frac{12}{5}\begin{pmatrix}0\\1\end{pmatrix}$

$= \begin{pmatrix}3\\-\frac{12}{5}\end{pmatrix}$

c $\overrightarrow{OP} = \overrightarrow{OA} + \frac{3}{5}\overrightarrow{AC}$

$= 2\mathbf{i} + 4\mathbf{j} + \frac{3}{5}(5\mathbf{i} - 4\mathbf{j})$

$= 5\mathbf{i} + \frac{8}{5}\mathbf{j}$

$= 5\begin{pmatrix}1\\0\end{pmatrix} + \frac{8}{5}\begin{pmatrix}0\\1\end{pmatrix}$

$= \begin{pmatrix}5\\\frac{8}{5}\end{pmatrix}$

8 $\begin{pmatrix}10\\k\end{pmatrix} - 2\begin{pmatrix}j\\3\end{pmatrix} = \begin{pmatrix}2\\5\end{pmatrix}$

So $10 - 2j = 2$ and $k - 6 = 5$

$j = 4$ and $k = 11$

9 $\begin{pmatrix}p\\-q\end{pmatrix} + 2\begin{pmatrix}q\\p\end{pmatrix} = \begin{pmatrix}7\\4\end{pmatrix}$

So $p + 2q = 7$ **(1)**

and $-q + 2p = 4$ **(2)**

Solve the two equations simultaneously.

Multiply equation **(2)** by 2 to give

$-2q + 4p = 8$ **(3)**

Add equations **(1)** and **(3)**

$5p = 15$

$p = 3$ and $q = 2$

10 a The resultant vector

$\mathbf{a} + \mathbf{b} = 3\mathbf{i} - 2\mathbf{j} + p\mathbf{i} - 2p\mathbf{j}$

$= (3 + p)\mathbf{i} - (2 + 2p)\mathbf{j}$

$(3 + p)\mathbf{i} - (2 + 2p)\mathbf{j}$ is parallel to $2\mathbf{i} - 3\mathbf{j}$

so $(3 + p)\mathbf{i} - (2 + 2p)\mathbf{j} = \lambda(2\mathbf{i} - 3\mathbf{j})$

so $3 + p = 2\lambda$ **(1)**

and $2 + 2p = 3\lambda$ **(2)**

Solve the two equations simultaneously.

Multiply equation **(1)** by 2 to give

$6 + 2p = 4\lambda$

$\lambda = 4$

$p = 5$

b When $p = 5$

$(3 + p)\mathbf{i} - (2 + 2p)\mathbf{j} = 8\mathbf{i} - 12\mathbf{j}$

