Vectors 11F

1 a Speed $= |3\mathbf{i}+4\mathbf{j}|$
$= \sqrt{3^2+4^2}$
$= \sqrt{9+16}$
$= \sqrt{25}$
$= 5 \text{ m s}^{-1}$

b Speed $= |24\mathbf{i}+7\mathbf{j}|$
$= \sqrt{24^2+(-7)^2}$
$= \sqrt{576+49}$
$= \sqrt{625}$
$= 25 \text{ km h}^{-1}$

c Speed $= |5\mathbf{i}+2\mathbf{j}|$
$= \sqrt{5^2+2^2}$
$= \sqrt{25+4}$
$= \sqrt{29}$
$= 5.39 \text{ m s}^{-1}$ (3 s.f.)

d Speed $= |-7\mathbf{i}+4\mathbf{j}|$
$= \sqrt{(-7)^2+4^2}$
$= \sqrt{49+16}$
$= \sqrt{65}$
$= 8.06 \text{ cm s}^{-1}$ (3 s.f.)

2 a Distance = speed × time
$= \sqrt{8^2+6^2} \times 5$
$= 5 \times \sqrt{64+36}$
$= 5 \times \sqrt{100}$
$= 50 \text{ km}$

b Distance = speed × time
$= \sqrt{5^2+(-1)^2} \times 10$
$= 10 \times \sqrt{25+1}$
$= 10 \times \sqrt{26}$
$= 51.0 \text{ m}$ (3 s.f.)

c Distance = speed × time
$= \sqrt{6^2+2^2} \times 0.75$
$= 0.75 \times \sqrt{36+4}$
$= 0.75 \times \sqrt{40}$
$= 4.74 \text{ km}$ (3 s.f.)

d Distance = speed × time
$= \sqrt{(-4)^2+(-7)^2} \times 120$
$= 120 \times \sqrt{16+49}$
$= 120 \times \sqrt{65}$
$= 967 \text{ cm}$ (3 s.f.)

3 a Speed $= \sqrt{(-3)^2+4^2}$
$= \sqrt{9+16}$
$= \sqrt{25}$
$= 5 \text{ m s}^{-1}$

Distance $= 5 \times 15 = 75$ m

b Speed $= \sqrt{2^2+5^2}$
$= \sqrt{4+25}$
$= \sqrt{29}$
$= 5.39 \text{ m s}^{-1}$ (3 s.f.)

Distance $= 3 \times 5.39 = 16.2$ m (3 s.f.)

c Speed $= \sqrt{5^2+(-2)^2}$
$= \sqrt{25+4}$
$= \sqrt{29}$
$= 5.39 \text{ km h}^{-1}$ (3 s.f.)

Distance $= 3 \times 5.39 = 16.2$ km (3 s.f.)

3 d Speed $= \sqrt{12^2 + (-5)^2}$

$= \sqrt{144 + 25}$

$= \sqrt{169}$

$= 13 \text{ km h}^{-1}$

Distance $= 0.5 \times 13 = 6.5$ km

4 a $= \dfrac{(16\mathbf{i} - 5\mathbf{j}) - (2\mathbf{i} + 3\mathbf{j})}{5}$

$= \dfrac{14\mathbf{i} - 8\mathbf{j}}{5}$

$= 2.8\mathbf{i} - 1.6\mathbf{j} \text{ m s}^{-2}$

5 a $\tan\theta = \frac{7}{5}$

$\theta = \tan^{-1}\frac{7}{5}$

$= 54.5°$ (3 s.f.)

b magnitude of $\mathbf{F} = |m\mathbf{a}|$

$= 0.3\sqrt{5^2 + 7^2}$

$= 0.3\sqrt{74}$

6 a $\tan\theta = \frac{1}{2}$

$\theta = \tan^{-1}\frac{1}{2}$

$= 26.6°$ below

b $3\mathbf{i} - 4\mathbf{j} + p\mathbf{i} + q\mathbf{j} = \lambda(2\mathbf{i} - \mathbf{j})$

$(3 + p)\mathbf{i} + (q - 4)\mathbf{j} = 2\lambda\mathbf{i} - \lambda\mathbf{j}$

$3 + p = 2\lambda$ and $q - 4 = -\lambda$

Multiplying the second equation by 2:

$2q - 8 = -2\lambda$

Solving simultaneously:

$3 + p = -2q + 8$

$p + 2q = 5$

c When $p = 1, \lambda = 2$

$\mathbf{R} = 2(2\mathbf{i} - \mathbf{j})$

$= 4\mathbf{i} - 2\mathbf{j}$

$|\mathbf{R}| = \sqrt{4^2 + 2^2}$

$= \sqrt{20}$

$= 2\sqrt{5}$ N

7 a $\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$

$= -(30\mathbf{i} + 40\mathbf{j}) + 40\mathbf{i} - 60\mathbf{j}$

$= 10\mathbf{i} - 100\mathbf{j}$

7 b $AB = \sqrt{30^2 + 40^2} = \sqrt{2500} = 50$

$AC = \sqrt{40^2 + (-60)^2} = \sqrt{5200}$

$BC = \sqrt{10^2 + (-100)^2} = \sqrt{10\,100}$

Using the cosine rule:

$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

$\cos A = \dfrac{\sqrt{5200}^2 + 50^2 - \sqrt{10\,100}^2}{2(\sqrt{5200})(50)}$

$\cos A = \dfrac{5200 + 2500 - 10\,100}{1000\sqrt{52}}$

$\cos A = \dfrac{-2400}{1000\sqrt{52}}$

$A = 109.440\,03...$

So $\angle BAC = 109.4°$

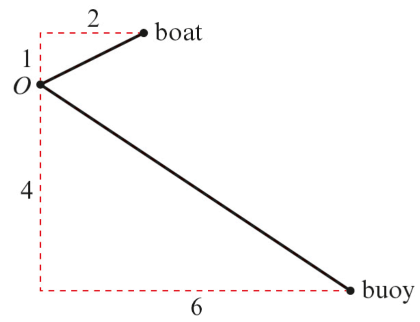
c Area $= \frac{1}{2}bc\sin A$

$= \frac{1}{2} \times \sqrt{5200} \times 50 \times \sin 109.4°$

$= 1700.418...$

$= 1700 \text{ m}^2$ (3 s.f.)

8 a



Distance $= \sqrt{(6-2)^2 + (-4-1)^2}$

$= \sqrt{16 + 25}$

$= \sqrt{41}$ km

b Bearing $= 270° + \tan^{-1}\frac{5}{4}$

$= 321.3°$

c The vector of the boat to the buoy

$= -(2\mathbf{i} + \mathbf{j}) + 6\mathbf{i} - 4\mathbf{j} = 4\mathbf{i} - 5\mathbf{j}$

Velocity $= 8\mathbf{i} - 10\mathbf{j}$

so $\lambda(8\mathbf{i} - 10\mathbf{j}) = 4\mathbf{i} - 5\mathbf{j}$

$\lambda = \frac{1}{2}$

Therefore, the boat is travelling directly towards the buoy.

8 d Speed $= \sqrt{8^2 + 10^2}$

$= \sqrt{164}$

$= 2\sqrt{41}\ \text{km h}^{-1}$

e Time $= \dfrac{\text{distance}}{\text{speed}}$

$= \dfrac{\sqrt{41}}{2\sqrt{41}}$

$= \dfrac{1}{2}$ hour

$= 30$ minutes