Differentiation 12H

1 a $y = 12x^2 + 3x + 8$

$$\frac{dy}{dx} = 24x + 3$$

$$\frac{d^2y}{dx^2} = 24$$

b $y = 15x + 6 + \dfrac{3}{x}$

$$= 15x + 6 + 3x^{-1}$$

$$\frac{dy}{dx} = 15 - 3x^{-2}$$

$$\frac{d^2y}{dx^2} = 6x^{-3}$$

c $y = 9\sqrt{x} - \dfrac{3}{x^2}$

$$= 9x^{\frac{1}{2}} - 3x^{-2}$$

$$\frac{dy}{dx} = \frac{9}{2}x^{-\frac{1}{2}} + 6x^{-3}$$

$$\frac{d^2y}{dx^2} = -\frac{9}{4}x^{-\frac{3}{2}} - 18x^{-4}$$

$$\frac{d^2y}{dx^2} = -\frac{9}{4\left(\sqrt{x}\right)^3} - \frac{18}{x^4}$$

d $y = (5x + 4)(3x - 2)$

$$= 15x^2 + 2x - 8$$

$$\frac{dy}{dx} = 30x + 2$$

$$\frac{d^2y}{dx^2} = 30$$

e $y = \dfrac{3x + 8}{x^2}$

$$= \frac{3x}{x^2} + \frac{8}{x^2}$$

$$= \frac{3}{x} + 8x^{-2}$$

$$= 3x^{-1} + 8x^{-2}$$

$$\frac{dy}{dx} = -3x^{-2} - 16x^{-3}$$

$$\frac{d^2y}{dx^2} = 6x^{-3} + 48x^{-4}$$

2 $\text{f}(t) = \dfrac{t^2 + 2}{\sqrt{t}} = t^{\frac{3}{2}} + 2t^{-\frac{1}{2}}$

$$\text{f}'(t) = \frac{3}{2}t^{\frac{1}{2}} - t^{-\frac{3}{2}}$$

$$\text{Acceleration} = \text{f}''(t) = \frac{3}{4\sqrt{t}} + \frac{3}{2\left(\sqrt{t}\right)^5}$$

3 $y = (2x - 3)^3$

$$= 8x^3 - 36x^2 + 54x - 27$$

$$\frac{dy}{dx} = 24x^2 - 72x + 54$$

$$\frac{d^2y}{dx^2} = 48x - 72$$

$$48x - 72 = 0$$

$$x = \frac{3}{2}$$

4 $\text{f}(x) = px^3 - 3px^2 + x^2 - 4$

$$\text{f}'(x) = 3px^2 - 6px + 2x$$

$$\text{f}''(x) = 6px - 6p + 2$$

$$\text{f}''(2) = -1$$

$$12p - 6p + 2 = -1$$

$$6p = -3$$

$$p = -\frac{1}{2}$$