

Integration 13B

$$1 \quad a \quad \int x^3 dx = \frac{x^4}{4} + c$$

$$b \quad \int x^7 dx = \frac{x^8}{8} + c$$

$$c \quad \int 3x^{-4} dx = \frac{3x^{-3}}{-3} + c = -x^{-3} + c$$

$$d \quad \int 5x^2 dx = \frac{5x^3}{3} + c$$

$$2 \quad a \quad \int (x^4 + 2x^3) dx = \frac{x^5}{5} + \frac{2x^4}{4} + c \\ = \frac{x^5}{5} + \frac{x^4}{2} + c$$

$$b \quad \int (2x^3 - x^2 + 5x) dx = \frac{2x^4}{4} - \frac{x^3}{3} + \frac{5x^2}{2} + c \\ = \frac{x^4}{2} - \frac{x^3}{3} + \frac{5x^2}{2} + c$$

$$c \quad \int (5x^{\frac{3}{2}} - 3x^2) dx = \frac{5x^{\frac{5}{2}}}{\frac{5}{2}} - \frac{3x^3}{3} + c \\ = 2x^{\frac{5}{2}} - x^3 + c$$

$$3 \quad a \quad \int (4x^{-2} + 3x^{\frac{1}{2}}) dx = \frac{4x^{-1}}{-1} + \frac{3x^{\frac{3}{2}}}{\frac{3}{2}} + c \\ = -4x^{-1} + 6x^{\frac{3}{2}} + c$$

$$b \quad \int (6x^{-2} - x^{\frac{1}{2}}) dx = \frac{6x^{-1}}{-1} - \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c \\ = -6x^{-1} - \frac{2}{3}x^{\frac{3}{2}} + c$$

$$c \quad \int (2x^{\frac{3}{2}} + x^2 - x^{\frac{1}{2}}) dx$$

$$= \frac{2x^{\frac{5}{2}}}{-\frac{1}{2}} + \frac{x^3}{3} - \frac{x^{\frac{3}{2}}}{\frac{1}{2}} + c \\ = -4x^{\frac{5}{2}} + \frac{x^3}{3} - 2x^{\frac{3}{2}} + c$$

$$4 \quad a \quad \int (4x^3 - 3x^{-4} + r) dx = \frac{4x^4}{4} - \frac{3x^{-3}}{-3} + rx + c \\ = x^4 + x^{-3} + rx + c$$

$$b \quad \int (x + x^{\frac{1}{2}} + x^{\frac{3}{2}}) dx = \frac{x^2}{2} + \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + c \\ = \frac{x^2}{2} + 2x^{\frac{3}{2}} - 2x^{\frac{5}{2}} + c$$

$$c \quad \int (px^4 + 2q + 3x^{-2}) dx \\ = \frac{px^5}{5} + 2qx + \frac{3x^{-1}}{-1} + c \\ = \frac{px^5}{5} + 2qx - 3x^{-1} + c$$

$$5 \quad a \quad \int (3t^2 - t^{-2}) dt = \frac{3t^3}{3} - \frac{t^{-1}}{-1} + c \\ = t^3 + t^{-1} + c$$

$$b \quad \int (2t^2 - 3t^{\frac{3}{2}} + 1) dt = \frac{2t^3}{3} - \frac{3t^{\frac{5}{2}}}{\frac{5}{2}} + t + c \\ = \frac{2t^3}{3} + 6t^{\frac{5}{2}} + t + c$$

$$c \quad \int (pt^3 + q^2 + pr^3) dt \\ = \frac{pt^4}{4} + q^2t + pr^3t + c$$

$$6 \quad a \quad \int \frac{(2x^3 + 3)}{x^2} dx = \int \left(\frac{2x^3}{x^2} + \frac{3}{x^2} \right) dx \\ = \int (2x + 3x^{-2}) dx \\ = \frac{2x^2}{2} + \frac{3x^{-1}}{-1} + c \\ = x^2 - \frac{3}{x} + c$$

$$\begin{aligned} \text{b } \int (2x+3)^2 dx &= \int (4x^2 + 12x + 9) dx \\ &= \frac{4x^3}{3} + 6x^2 + 9x + c \end{aligned}$$

$$\begin{aligned} \text{6 c } \int (2x+3)\sqrt{x} dx &= \int (2x+3)x^{\frac{1}{2}} dx \\ &= \int \left(2x^{\frac{3}{2}} + 3x^{\frac{1}{2}} \right) dx \\ &= \frac{2x^{\frac{5}{2}}}{\frac{5}{2}} + \frac{3x^{\frac{3}{2}}}{\frac{3}{2}} + c \\ &= \frac{4x^{\frac{5}{2}}}{5} + 2x^{\frac{3}{2}} + c \end{aligned}$$

$$\begin{aligned} \text{7 a } \int \left(x + \frac{1}{x} \right)^2 dx &= \int \left(x^2 + 2 + \frac{1}{x^2} \right) dx \\ &= \int (x^2 + 2 + x^{-2}) dx \\ &= \frac{x^3}{3} + 2x + \frac{x^{-1}}{-1} + c \\ &= \frac{x^3}{3} + 2x - \frac{1}{x} + c \end{aligned}$$

$$\begin{aligned} \text{b } \int (\sqrt{x} + 2)^2 dx &= \int (x + 4\sqrt{x} + 4) dx \\ &= \int \left(x + 4x^{\frac{1}{2}} + 4 \right) dx \\ &= \frac{x^2}{2} + \frac{4x^{\frac{3}{2}}}{\frac{3}{2}} + 4x + c \\ &= \frac{x^2}{2} + \frac{8x^{\frac{3}{2}}}{3} + 4x + c \end{aligned}$$

$$\begin{aligned} \text{c } \int \left(\frac{1}{\sqrt{x}} + 2\sqrt{x} \right) dx &= \int (x^{-\frac{1}{2}} + 2x^{\frac{1}{2}}) dx \\ &= \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{2x^{\frac{3}{2}}}{\frac{3}{2}} + c \\ &= 2x^{\frac{1}{2}} + \frac{4x^{\frac{3}{2}}}{3} + c \end{aligned}$$

$$\text{8 a } \int \left(x^{\frac{2}{3}} + \frac{4}{x^3} \right) dx = \int \left(x^{\frac{2}{3}} + 4x^{-3} \right) dx$$

$$\begin{aligned} &= \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + \frac{4x^{-2}}{-2} + c \\ &= \frac{3x^{\frac{5}{3}}}{5} - \frac{2}{x^2} + c \end{aligned}$$

$$\begin{aligned} \text{8 b } \int \left(\frac{2+x}{x^3} + 3 \right) dx &= \int (2x^{-3} + x^{-2} + 3) dx \\ &= \frac{2x^{-2}}{-2} + \frac{x^{-1}}{-1} + 3x + c \\ &= -\frac{1}{x^2} - \frac{1}{x} + 3x + c \end{aligned}$$

$$\begin{aligned} \text{c } \int (x^2 + 3)(x-1) dx &= \int (x^3 - x^2 + 3x - 3) dx \\ &= \frac{x^4}{4} - \frac{x^3}{3} + \frac{3x^2}{2} - 3x + c \end{aligned}$$

$$\begin{aligned} \text{d } \int \frac{(2x+1)^2}{\sqrt{x}} dx &= \int \left(\frac{4x^2 + 4x + 1}{x^{\frac{1}{2}}} \right) dx \\ &= \int (4x^{\frac{3}{2}} + 4x^{\frac{1}{2}} + x^{-\frac{1}{2}}) dx \\ &= \frac{8x^{\frac{5}{2}}}{5} + \frac{8x^{\frac{3}{2}}}{3} + 2x^{\frac{1}{2}} + c \end{aligned}$$

$$\begin{aligned} \text{e } \int \left(3 + \frac{\sqrt{x} + 6x^3}{x} \right) dx &= \int (3 + x^{-\frac{1}{2}} + 6x^2) dx \\ &= 3x + 2x^{\frac{1}{2}} + 2x^3 + c \end{aligned}$$

$$\begin{aligned} \text{f } \int \sqrt{x}(\sqrt{x} + 3)^2 dx &= \int x^{\frac{1}{2}} \left(x + 6x^{\frac{1}{2}} + 9 \right) dx \\ &= \int (x^{\frac{3}{2}} + 6x + 9x^{\frac{1}{2}}) dx \\ &= \frac{2x^{\frac{5}{2}}}{5} + 3x^2 + 6x^{\frac{3}{2}} + c \end{aligned}$$

$$\begin{aligned} \text{9 a } \int \left(\frac{A}{x^2} - 3 \right) dx &= \int (Ax^{-2} - 3) dx \\ &= \frac{Ax^{-1}}{-1} - 3x + c \\ &= -\frac{A}{x} - 3x + c \end{aligned}$$

$$\begin{aligned}
 9 \text{ b } \int \left(\sqrt{Px} + \frac{2}{x^3} \right) dx &= \int \left(\sqrt{Px^{\frac{1}{2}}} + 2x^{-3} \right) dx \\
 &= \frac{\sqrt{Px^{\frac{3}{2}}}}{\frac{3}{2}} + \frac{2x^{-2}}{-2} + c \\
 &= \frac{2\sqrt{Px^{\frac{3}{2}}}}{3} - \frac{1}{x^2} + c
 \end{aligned}$$

$$\begin{aligned}
 \text{c } \int \left(\frac{p}{x^2} + q\sqrt{x} + r \right) dx \\
 &= \frac{px^{-1}}{-1} + \frac{qx^{\frac{3}{2}}}{\frac{3}{2}} + rx + c \\
 &= -\frac{p}{x} + \frac{2qx^{\frac{3}{2}}}{3} + rx + c
 \end{aligned}$$

$$\begin{aligned}
 10 \int \left(\frac{6}{x^2} + 4\sqrt{x} - 3x + 2 \right) dx \\
 &= \int (6x^{-2} + 4x^{\frac{1}{2}} - 3x + 2) dx \\
 &= \frac{6x^{-1}}{-1} + \frac{4x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{3x^2}{2} + 2x + c \\
 &= -\frac{6}{x} + \frac{8x^{\frac{3}{2}}}{3} - \frac{3x^2}{2} + 2x + c
 \end{aligned}$$

$$\begin{aligned}
 11 \int \left(8x^3 + 6x - \frac{3}{\sqrt{x}} \right) dx \\
 &= \int (8x^3 + 6x - 3x^{-\frac{1}{2}}) dx \\
 &= \frac{8x^4}{4} + \frac{6x^2}{2} - \frac{3x^{\frac{1}{2}}}{\frac{1}{2}} + c \\
 &= 2x^4 + 3x^2 - 6\sqrt{x} + c
 \end{aligned}$$

$$\begin{aligned}
 12 \text{ a } (2+5\sqrt{x})^2 &= (2+5\sqrt{x})(2+5\sqrt{x}) \\
 &= 4 + 10\sqrt{x} + 10\sqrt{x} + 25x \\
 &= 4 + 20\sqrt{x} + 25x
 \end{aligned}$$

So $k = 20$

$$\begin{aligned}
 \text{b } \int (4 + 20\sqrt{x} + 25x) dx \\
 &= \int (4 + 20x^{\frac{1}{2}} + 25x) dx
 \end{aligned}$$

$$\begin{aligned}
 &= 4x + \frac{20x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{25x^2}{2} + c \\
 &= 4x + \frac{40x^{\frac{3}{2}}}{3} + \frac{25x^2}{2} + c
 \end{aligned}$$

$$13 \int \left(3x^5 - \frac{4}{\sqrt{x}} \right) dx = \int (3x^5 - 4x^{-\frac{1}{2}}) dx$$

$$\begin{aligned}
 &= \frac{3x^6}{6} - \frac{4x^{\frac{1}{2}}}{\frac{1}{2}} + c \\
 &= \frac{x^6}{2} - 8\sqrt{x} + c
 \end{aligned}$$

$$14 \int \left(\frac{p}{2x^2} + pq \right) dx = \int \left(\frac{px^{-2}}{2} + pq \right) dx$$

$$\begin{aligned}
 &= \frac{px^{-1}}{2 \times -1} + pqx + c \\
 &= -\frac{p}{2x} + pqx + c \\
 &= \frac{2}{x} + 10x + c
 \end{aligned}$$

$$-\frac{p}{2} = 2 \text{ and } pq = 10$$

$$p = -4 \text{ and } q = -2.5$$

15 a Using the binomial expansion:

$$\begin{aligned}
 f(x) &= 2^{10} + \binom{10}{1} 2^9 (-x)^1 + \binom{10}{2} 2^8 (-x)^2 \\
 &= 1024 - 5120x + 11520x^2
 \end{aligned}$$

$$\text{b } \int (1024 - 5120x + 11520x^2) dx$$

$$= 1024x - \frac{5120x^2}{2} + \frac{11520x^3}{3} + c$$

$$= 1024x - 2560x^2 + 3840x^3 + c$$

$$\text{So } A = 1024, B = -2560 \text{ and } C = 3840$$