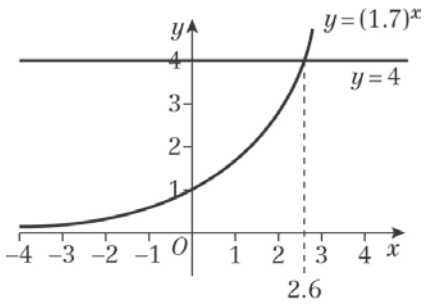
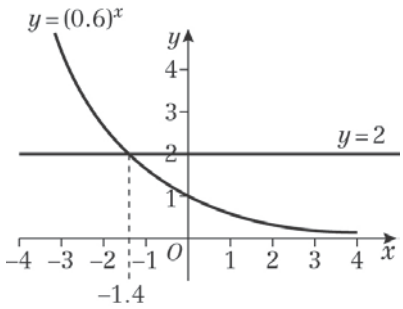
Exponentials and logarithms 14A

1 a



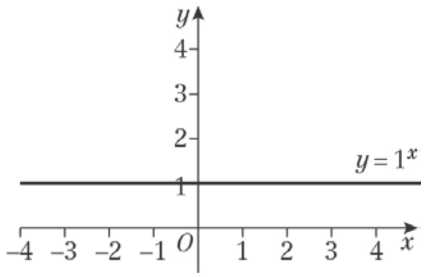
b Where $y = 4$, $x \approx 2.6$

2 a



b Where $y = 2$, $x \approx -1.4$

3

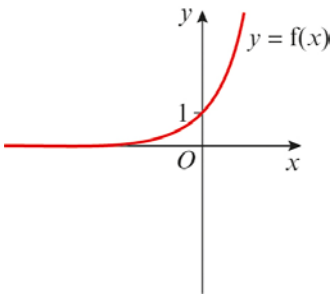


4 a True because, when $x = 0$, $a^0 = 1$ when a is positive

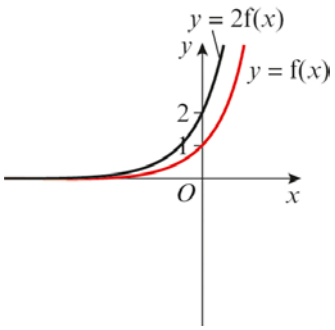
b False. For example, when $a = \frac{1}{2}$, the function $f(x) = a^x$ is not an increasing function.

c True because, when a is positive, $a^x > 0$ for all values of x.

5 a The graph crosses the y-axis when $x = 0$.
$y = 3^0$
So $y = 1$
The graph crosses the y-axis at (0, 1).
Asymptote is at $y = 0$.

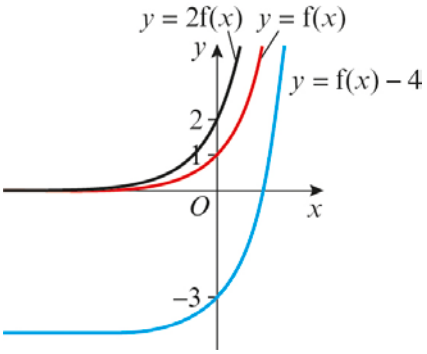


b The graph is a vertical stretch by scale factor 2.
The graph crosses the y-axis when $x = 0$.
$y = 2 \times 3^0$
So $y = 2$
The graph crosses the y-axis at (0, 2).
Asymptote is at $y = 0$.

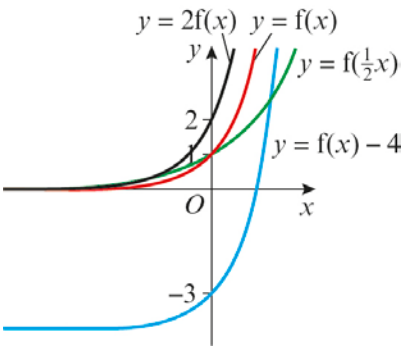


c The graph is a translation by the vector $\begin{pmatrix} 0 \\ -4 \end{pmatrix}$.
The graph crosses the y-axis when $x = 0$.
$y = 3^0 - 4$
So $y = -3$
The graph crosses the y-axis at (0, −3).
Asymptote is at $y = -4$.

5 c



d The graph is a horizontal stretch by scale factor 2.
The graph crosses the y-axis when $x = 0$.
$y = 3^{\frac{1}{2} \times 0}$
So $y = 1$
The graph crosses the y-axis at (0, 1).
Asymptote is at $y = 0$.

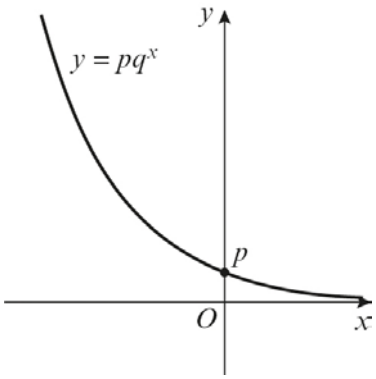


6 Substitute the coordinates into $y = ka^x$.
$6 = ka^1$ (equation 1)
$48 = ka^4$ (equation 2)
Solve simultaneously: divide equation 2 by equation 1,

$$48 \div 6 = \frac{ka^4}{ka}$$

$a^3 = 8$
$a = 2$, $k = 3$

7 a As x increases, y decreases



b Substitute the coordinates into $y = pq^x$.
$150 = pq^{-3}$ (equation 1)
$0.048 = pq^2$ (equation 2)
Solve simultaneously, divide equation 2 by equation 1.

$$0.048 \div 150 = \frac{pq^2}{pq^{-3}}$$

$q^5 = 0.00032$
$q = 0.2$
$p = 0.048 \div 0.2^2 = 1.2$
$p = 1.2$, $q = 0.2$

Challenge

To draw the graph, note that it is a translation of the graph $y = 2^x$ by the vector $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$.

The graph crosses the y-axis when $x = 0$,
so $y = 2^{0-2} + 5$
$y = 5.25$
The graph crosses the y-axis at (0, 5.25).
Asymptote is at $y = 5$.