Algebraic expressions 1D

1 a $x^3 \div x^{-2} = x^{3-(-2)}$
$= x^5$

b $x^5 \div x^7 = x^{5-7}$
$= x^{-2}$

c $x^{\frac{3}{2}} \times x^{\frac{5}{2}} = x^{\frac{3}{2}+\frac{5}{2}}$
$= x^4$

d $\left(x^2\right)^{\frac{3}{2}} = x^{2\times\frac{3}{2}}$
$= x^3$

e $\left(x^3\right)^{\frac{5}{3}} = x^{3\times\frac{5}{3}}$
$= x^5$

f $3x^{0.5} \times 4x^{-0.5} = 12x^{0.5+(-0.5)}$
$= 12x^0$
$= 12$

g $9x^{\frac{2}{3}} \div 3x^{\frac{1}{6}} = 3x^{\frac{2}{3}-\frac{1}{6}}$
$= 3x^{\frac{1}{2}}$

h $5x^{\frac{7}{5}} \div x^{\frac{2}{5}} = 5x^{\frac{7}{5}-\frac{2}{5}}$
$= 5x$

i $3x^4 \times 2x^{-5} = 6x^{4+(-5)}$
$= 6x^{-1}$

j $\sqrt{x} \times \sqrt[3]{x} = x^{\frac{1}{2}} \times x^{\frac{1}{3}}$
$= x^{\frac{1}{2}+\frac{1}{3}}$
$= x^{\frac{5}{6}}$
$= (\sqrt[6]{x})^5$

k $\left(\sqrt{x}\right)^3 \times \left(\sqrt[3]{x}\right)^4 = x^{\frac{3}{2}} \times x^{\frac{4}{3}}$
$= x^{\frac{3}{2}+\frac{4}{3}}$
$= x^{\frac{17}{6}}$
$= (\sqrt[6]{x})^{17}$

l $\dfrac{\left(\sqrt[3]{x}\right)^2}{\sqrt{x}} = x^{\frac{2}{3}} \div x^{\frac{1}{2}}$
$= x^{\frac{2}{3}-\frac{1}{2}}$
$= x^{\frac{1}{6}}$
$= \sqrt[6]{x}$

2 a $25^{\frac{1}{2}} = \sqrt{25}$
$= 5$

b $81^{\frac{3}{2}} = \left(\sqrt{81}\right)^3$
$= 9^3$
$= 729$

c $27^{\frac{1}{3}} = \sqrt[3]{27}$
$= 3$

d $4^{-2} = \dfrac{1}{4^2}$
$= \dfrac{1}{16}$

e $9^{-\frac{1}{2}} = \dfrac{1}{9^{\frac{1}{2}}}$
$= \dfrac{1}{\sqrt{9}}$
$= \dfrac{1}{3}$

f $(-5)^{-3} = \dfrac{1}{(-5)^3}$
$= -\dfrac{1}{125}$

g $\left(\dfrac{3}{4}\right)^0 = 1$

h $1296^{\frac{3}{4}} = \left(\sqrt[4]{1296}\right)^3$
$= 6^3$
$= 216$

2 i $\left(\frac{25}{16}\right)^{\frac{3}{2}} = \frac{\left(\sqrt{25}\right)^3}{\left(\sqrt{16}\right)^3}$

$= \pm\frac{5^3}{4^3}$

$= \pm\frac{125}{64}$

j $\left(\frac{27}{8}\right)^{\frac{2}{3}} = \frac{\left(\sqrt[3]{27}\right)^2}{\left(\sqrt[3]{8}\right)^2}$

$= \frac{3^2}{2^2}$

$= \frac{9}{4}$

k $\left(\frac{6}{5}\right)^{-1} = \left(\frac{5}{6}\right)^1$

$= \frac{5}{6}$

l $\left(\frac{343}{512}\right)^{-\frac{2}{3}} = \frac{\left(\sqrt[3]{512}\right)^2}{\left(\sqrt[3]{343}\right)^2}$

$= \frac{8^2}{7^2}$

$= \frac{64}{49}$

3 a $\left(64x^{10}\right)^{\frac{1}{2}} = \sqrt{64}x^{10\times\frac{1}{2}}$

$= 8x^5$

b $\frac{5x^3 - 2x^2}{x^5} = \frac{5x^3}{x^5} - \frac{2x^2}{x^5}$

$= 5\times x^{3-5} - 2\times x^{2-5}$

$= 5x^{-2} - 2x^{-3}$

$= \frac{5}{x^2} - \frac{2}{x^3}$

c $\left(125x^{12}\right)^{\frac{1}{3}} = \sqrt[3]{125}x^{12\times\frac{1}{3}}$

$= 5x^4$

3 d $\frac{x+4x^3}{x^3} = \frac{x}{x^3} + \frac{4x^3}{x^3}$

$= x^{1-3} + 4\times x^{3-3}$

$= x^{-2} + 4x^0$

$= \frac{1}{x^2} + 4$

e $\frac{2x+x^2}{x^4} = \frac{2x}{x^4} + \frac{x^2}{x^4}$

$= 2\times x^{1-4} + x^{2-4}$

$= 2x^{-3} + x^{-2}$

$= \frac{2}{x^3} + \frac{1}{x^2}$

f $\left(\frac{4}{9}x^4\right)^{\frac{3}{2}} = \left(\sqrt{\frac{4}{9}}\right)^3 x^{4\times\frac{3}{2}}$

$= \frac{8}{27}x^6$

g $\frac{9x^2 - 15x^5}{3x^3} = \frac{9x^2}{3x^3} - \frac{15x^5}{3x^3}$

$= 3\times x^{2-3} - 5\times x^{5-3}$

$= 3x^{-1} - 5x^2$

$= \frac{3}{x} - 5x^2$

h $\frac{5x+3x^2}{15x^3} = \frac{5x}{15x^3} + \frac{3x^2}{15x^3}$

$= \frac{1}{3}\times x^{1-3} + \frac{1}{5}\times x^{2-3}$

$= \frac{1}{3}x^{-2} + \frac{1}{5}x^{-1}$

$= \frac{1}{3x^2} + \frac{1}{5x}$

4 a $81^{\frac{1}{4}} = \sqrt[4]{81}$

$= 3$

b $x(2x^{-\frac{1}{3}})^4 = x\times 2^4 x^{-\frac{1}{3}\times 4}$

$= 2^4 x^{-\frac{4}{3}+1}$

$= 16x^{-\frac{1}{3}}$

5 a $y^{\frac{1}{3}} = \left(\frac{1}{8}x^3\right)^{\frac{1}{3}}$

$= \frac{1}{\sqrt[3]{8}}x^{3\times\frac{1}{3}}$

$= \frac{x}{2}$

b $\frac{1}{2}y^{-2} = \frac{1}{2}\left(\frac{1}{8}x^3\right)^{-2}$

$= \frac{1}{2}\times\frac{1}{\left(\frac{1}{8}x^3\right)^2}$

$= \frac{1}{2}\times\frac{64}{x^{3\times2}}$

$= \frac{32}{x^6}$

$= 32x^{-6}$

6 a $x^{\frac{1}{2}} = 7$

$\left(x^{\frac{1}{2}}\right)^2 = 7^2$

$x = 49$

b $y^{\frac{4}{3}} = 81$

$\left(y^{\frac{4}{3}}\right)^{\frac{3}{4}} = 81^{\frac{3}{4}}$

$y = 27$

c $x^{-\frac{3}{2}} = 8$

$\left(x^{-\frac{3}{2}}\right)^{-\frac{2}{3}} = 8^{\frac{-2}{3}}$

$x = \frac{1}{4}$

d $z^{-\frac{3}{4}} = 1000$

$\left(z^{-\frac{3}{4}}\right)^{-\frac{4}{3}} = 1000^{-\frac{4}{3}}$

$z = \frac{1}{10000}$

7 $27\sqrt{x} = \frac{1}{x}$

$x^{\frac{3}{2}} = \frac{1}{27}$

$\left(x^{\frac{3}{2}}\right)^{\frac{2}{3}} = \left(\frac{1}{27}\right)^{\frac{2}{3}}$

$x = \frac{1}{9}$