

Algebraic expressions 1F

1 a
$$\frac{1}{\sqrt{5}} = \frac{1 \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}}$$

$$= \frac{\sqrt{5}}{5}$$

b
$$\frac{1}{\sqrt{11}} = \frac{1 \times \sqrt{11}}{\sqrt{11} \times \sqrt{11}}$$

$$= \frac{\sqrt{11}}{11}$$

c
$$\frac{1}{\sqrt{2}} = \frac{1 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}}$$

$$= \frac{\sqrt{2}}{2}$$

d
$$\frac{\sqrt{3}}{\sqrt{15}} = \frac{\sqrt{3} \times \sqrt{15}}{\sqrt{15} \times \sqrt{15}}$$

$$= \frac{\sqrt{3 \times 15}}{15}$$

$$= \frac{\sqrt{45}}{15}$$

$$= \frac{\sqrt{9 \times 5}}{15}$$

$$= \frac{\sqrt{9} \times \sqrt{5}}{15}$$

$$= \frac{\sqrt{5}}{5}$$

e
$$\frac{\sqrt{12}}{\sqrt{48}} = \frac{\sqrt{12}}{\sqrt{12} \times \sqrt{4}}$$

$$= \frac{1}{\sqrt{4}}$$

$$= \frac{1}{2}$$

f
$$\frac{\sqrt{5}}{\sqrt{80}} = \frac{\sqrt{5}}{\sqrt{5} \times \sqrt{16}}$$

$$= \frac{1}{\sqrt{16}}$$

$$= \frac{1}{4}$$

g
$$\frac{\sqrt{12}}{\sqrt{156}} = \frac{\sqrt{12}}{\sqrt{12} \times \sqrt{13}}$$

$$= \frac{1}{\sqrt{13}}$$

$$= \frac{1 \times \sqrt{13}}{\sqrt{13} \times \sqrt{13}}$$

$$= \frac{\sqrt{13}}{13}$$

h
$$\frac{\sqrt{7}}{\sqrt{63}} = \frac{\sqrt{7}}{\sqrt{7} \times \sqrt{9}}$$

$$= \frac{1}{\sqrt{9}}$$

$$= \frac{1}{3}$$

2 a
$$\frac{1}{1+\sqrt{3}} = \frac{1 \times (1-\sqrt{3})}{(1+\sqrt{3})(1-\sqrt{3})}$$

$$= \frac{1-\sqrt{3}}{1-\sqrt{3}+\sqrt{3}-\sqrt{9}}$$

$$= \frac{1-\sqrt{3}}{-2} \text{ or } \frac{-1+\sqrt{3}}{2}$$

b
$$\frac{1}{2+\sqrt{5}} = \frac{1 \times (2-\sqrt{5})}{(2+\sqrt{5})(2-\sqrt{5})}$$

$$= \frac{2-\sqrt{5}}{4-5}$$

$$= \frac{2-\sqrt{5}}{-1}$$

$$= -2+\sqrt{5}$$

c
$$\frac{1}{3-\sqrt{7}} = \frac{3+\sqrt{7}}{(3-\sqrt{7})(3+\sqrt{7})}$$

$$= \frac{3+\sqrt{7}}{9-7}$$

$$= \frac{3+\sqrt{7}}{2}$$

2 d

$$\begin{aligned}\frac{4}{3-\sqrt{5}} &= \frac{4 \times (3+\sqrt{5})}{(3-\sqrt{5})(3+\sqrt{5})} \\&= \frac{12+4\sqrt{5}}{9-5} \\&= \frac{12+4\sqrt{5}}{4} \\&= 3+\sqrt{5}\end{aligned}$$

e

$$\begin{aligned}\frac{1}{\sqrt{5}-\sqrt{3}} &= \frac{\sqrt{5}+\sqrt{3}}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})} \\&= \frac{\sqrt{5}+\sqrt{3}}{5-3} \\&= \frac{\sqrt{5}+\sqrt{3}}{2}\end{aligned}$$

f

$$\begin{aligned}\frac{3-\sqrt{2}}{4-\sqrt{5}} &= \frac{(3-\sqrt{2})(4+\sqrt{5})}{(4-\sqrt{5})(4+\sqrt{5})} \\&= \frac{(3-\sqrt{2})(4+\sqrt{5})}{16-5} \\&= \frac{(3-\sqrt{2})(4+\sqrt{5})}{11} \\&= \frac{12+3\sqrt{5}-4\sqrt{2}-\sqrt{10}}{11}\end{aligned}$$

g

$$\begin{aligned}\frac{5}{2+\sqrt{5}} &= \frac{5 \times (2-\sqrt{5})}{(2+\sqrt{5})(2-\sqrt{5})} \\&= \frac{5(2-\sqrt{5})}{4-5} \\&= \frac{5(2-\sqrt{5})}{-1} \\&= 5(\sqrt{5}-2)\end{aligned}$$

h

$$\begin{aligned}\frac{5\sqrt{2}}{\sqrt{8}-\sqrt{7}} &= \frac{5\sqrt{2}(\sqrt{8}+\sqrt{7})}{(\sqrt{8}-\sqrt{7})(\sqrt{8}+\sqrt{7})} \\&= \frac{5(\sqrt{8\times 2}+\sqrt{2\times 7})}{8-7}\end{aligned}$$

h

$$\begin{aligned}\frac{5\sqrt{2}}{\sqrt{8}-\sqrt{7}} &= \frac{5(\sqrt{16}+\sqrt{14})}{1} \\&= 5(4+\sqrt{14})\end{aligned}$$

i

$$\begin{aligned}\frac{11}{3+\sqrt{11}} &= \frac{11(3-\sqrt{11})}{(3+\sqrt{11})(3-\sqrt{11})} \\&= \frac{11(3-\sqrt{11})}{9-11} \\&= \frac{11(3-\sqrt{11})}{-2} \\&= \frac{-11(3-\sqrt{11})}{2}\end{aligned}$$

j

$$\begin{aligned}\frac{\sqrt{3}-\sqrt{7}}{\sqrt{3}+\sqrt{7}} &= \frac{(\sqrt{3}-\sqrt{7})(\sqrt{3}-\sqrt{7})}{(\sqrt{3}+\sqrt{7})(\sqrt{3}-\sqrt{7})} \\&= \frac{3-\sqrt{21}-\sqrt{21}+7}{3-7} \\&= \frac{10-2\sqrt{21}}{-4} \\&= \frac{5-\sqrt{21}}{-2} \text{ or } \frac{\sqrt{21}-5}{2}\end{aligned}$$

k

$$\begin{aligned}\frac{\sqrt{17}-\sqrt{11}}{\sqrt{17}+\sqrt{11}} &= \frac{(\sqrt{17}-\sqrt{11})(\sqrt{17}-\sqrt{11})}{(\sqrt{17}+\sqrt{11})(\sqrt{17}-\sqrt{11})} \\&= \frac{17-\sqrt{187}-\sqrt{187}+11}{17-11} \\&= \frac{28-2\sqrt{187}}{6} \\&= \frac{14-\sqrt{187}}{3}\end{aligned}$$

l

$$\begin{aligned}\frac{\sqrt{41}+\sqrt{29}}{\sqrt{41}-\sqrt{29}} &= \frac{(\sqrt{41}+\sqrt{29})(\sqrt{41}+\sqrt{29})}{(\sqrt{41}-\sqrt{29})(\sqrt{41}+\sqrt{29})} \\&= \frac{41+2\sqrt{41}\sqrt{29}+29}{41-29} \\&= \frac{70+2\sqrt{1189}}{12} \\&= \frac{35+\sqrt{1189}}{6}\end{aligned}$$

2 m

$$\begin{aligned} \frac{\sqrt{2}-\sqrt{3}}{\sqrt{3}-\sqrt{2}} &= \frac{(\sqrt{2}-\sqrt{3})(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})} \\ &= \frac{\sqrt{6}+2-3-\sqrt{6}}{3-2} \\ &= \frac{-1}{1} \\ &= -1 \end{aligned}$$

3 a

$$\begin{aligned} \frac{1}{(3-\sqrt{2})^2} &= \frac{1}{(3-\sqrt{2})(3-\sqrt{2})} \\ &= \frac{1}{9-3\sqrt{2}-3\sqrt{2}+\sqrt{4}} \\ &= \frac{1}{11-6\sqrt{2}} \\ &= \frac{1 \times (11+6\sqrt{2})}{(11-6\sqrt{2})(11+6\sqrt{2})} \\ &= \frac{11+6\sqrt{2}}{121+66\sqrt{2}-66\sqrt{2}-72} \\ &= \frac{11+6\sqrt{2}}{49} \end{aligned}$$

b

$$\begin{aligned} \frac{1}{(2+\sqrt{5})^2} &= \frac{1}{(2+\sqrt{5})(2+\sqrt{5})} \\ &= \frac{1}{4+2\sqrt{5}+2\sqrt{5}+\sqrt{25}} \\ &= \frac{1}{9+4\sqrt{5}} \\ &= \frac{1 \times (9-4\sqrt{5})}{(9+4\sqrt{5})(9-4\sqrt{5})} \\ &= \frac{9-4\sqrt{5}}{81-36\sqrt{5}+36\sqrt{5}-80} \\ &= \frac{9-4\sqrt{5}}{1} \\ &= 9-4\sqrt{5} \end{aligned}$$

c

$$\begin{aligned} \frac{4}{(3-\sqrt{2})^2} &= \frac{4}{(3-\sqrt{2})(3-\sqrt{2})} \\ &= \frac{4}{9-3\sqrt{2}-3\sqrt{2}+\sqrt{4}} \end{aligned}$$

3 c

$$\begin{aligned} \frac{4}{(3-\sqrt{2})^2} &= \frac{4}{11-6\sqrt{2}} \\ &= \frac{4 \times (11+6\sqrt{2})}{(11-6\sqrt{2})(11+6\sqrt{2})} \\ &= \frac{44+24\sqrt{2}}{121+66\sqrt{2}-66\sqrt{2}-72} \\ &= \frac{44+24\sqrt{2}}{49} \end{aligned}$$

d

$$\begin{aligned} \frac{3}{(5+\sqrt{2})^2} &= \frac{3}{(5+\sqrt{2})(5+\sqrt{2})} \\ &= \frac{3}{25+5\sqrt{2}+5\sqrt{2}+\sqrt{4}} \\ &= \frac{3}{27+10\sqrt{2}} \\ &= \frac{3 \times (27-10\sqrt{2})}{(27+10\sqrt{2})(27-10\sqrt{2})} \\ &= \frac{3 \times (27-10\sqrt{2})}{729-270\sqrt{2}+270\sqrt{2}-200} \\ &= \frac{81-30\sqrt{2}}{529} \end{aligned}$$

e

$$\begin{aligned} \frac{1}{(5+\sqrt{2})(3-\sqrt{2})} &= \frac{1}{15-5\sqrt{2}+3\sqrt{2}-\sqrt{4}} \\ &= \frac{1}{13-2\sqrt{2}} \\ &= \frac{1 \times (13+2\sqrt{2})}{(13-2\sqrt{2})(13+2\sqrt{2})} \\ &= \frac{13+2\sqrt{2}}{169+26\sqrt{2}-26\sqrt{2}-8} \\ &= \frac{13+2\sqrt{2}}{161} \end{aligned}$$

f

$$\begin{aligned} \frac{2}{(5-\sqrt{3})(2+\sqrt{3})} &= \frac{2}{10+5\sqrt{3}-2\sqrt{3}-\sqrt{9}} \\ &= \frac{2}{7+3\sqrt{3}} \\ &= \frac{2 \times (7-3\sqrt{3})}{(7+3\sqrt{3})(7-3\sqrt{3})} \end{aligned}$$

3 f

$$\begin{aligned} \frac{2}{(5-\sqrt{3})(2+\sqrt{3})} &= \frac{14-6\sqrt{3}}{49-21\sqrt{3}+21\sqrt{3}-27} \\ &= \frac{14-6\sqrt{3}}{22} \\ &= \frac{7-3\sqrt{3}}{11} \end{aligned}$$

4

$$\begin{aligned} \frac{3-2\sqrt{5}}{\sqrt{5}-1} &= \frac{(3-2\sqrt{5})(\sqrt{5}+1)}{(\sqrt{5}-1)(\sqrt{5}+1)} \\ &= \frac{3\sqrt{5}+3-10-2\sqrt{5}}{5+\sqrt{5}-\sqrt{5}-1} \\ &= \frac{\sqrt{5}-7}{4} \\ &= \frac{-7}{4} + \frac{\sqrt{5}}{4} \end{aligned}$$

$$p = -\frac{7}{4}, q = \frac{\sqrt{5}}{4}$$