Algebraic expressions 1F

1 a $\frac{1}{\sqrt{5}} = \frac{1\times\sqrt{5}}{\sqrt{5}\times\sqrt{5}}$

$= \frac{\sqrt{5}}{5}$

b $\frac{1}{\sqrt{11}} = \frac{1\times\sqrt{11}}{\sqrt{11}\times\sqrt{11}}$

$= \frac{\sqrt{11}}{11}$

c $\frac{1}{\sqrt{2}} = \frac{1\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}}$

$= \frac{\sqrt{2}}{2}$

d $\frac{\sqrt{3}}{\sqrt{15}} = \frac{\sqrt{3}\times\sqrt{15}}{\sqrt{15}\times\sqrt{15}}$

$= \frac{\sqrt{3\times15}}{15}$

$= \frac{\sqrt{45}}{15}$

$= \frac{\sqrt{9\times5}}{15}$

$= \frac{\sqrt{9}\times\sqrt{5}}{15}$

$= \frac{\sqrt{5}}{5}$

e $\frac{\sqrt{12}}{\sqrt{48}} = \frac{\sqrt{12}}{\sqrt{12}\times\sqrt{4}}$

$= \frac{1}{\sqrt{4}}$

$= \frac{1}{2}$

f $\frac{\sqrt{5}}{\sqrt{80}} = \frac{\sqrt{5}}{\sqrt{5}\times\sqrt{16}}$

$= \frac{1}{\sqrt{16}}$

$= \frac{1}{4}$

g $\frac{\sqrt{12}}{\sqrt{156}} = \frac{\sqrt{12}}{\sqrt{12}\times\sqrt{13}}$

$= \frac{1}{\sqrt{13}}$

$= \frac{1\times\sqrt{13}}{\sqrt{13}\times\sqrt{13}}$

$= \frac{\sqrt{13}}{13}$

h $\frac{\sqrt{7}}{\sqrt{63}} = \frac{\sqrt{7}}{\sqrt{7}\times\sqrt{9}}$

$= \frac{1}{\sqrt{9}}$

$= \frac{1}{3}$

2 a $\frac{1}{1+\sqrt{3}} = \frac{1\times\left(1-\sqrt{3}\right)}{\left(1+\sqrt{3}\right)\left(1-\sqrt{3}\right)}$

$= \frac{1-\sqrt{3}}{1-\sqrt{3}+\sqrt{3}-\sqrt{9}}$

$= \frac{1-\sqrt{3}}{-2}$ or $\frac{-1+\sqrt{3}}{2}$

b $\frac{1}{2+\sqrt{5}} = \frac{1\times\left(2-\sqrt{5}\right)}{\left(2+\sqrt{5}\right)\left(2-\sqrt{5}\right)}$

$= \frac{2-\sqrt{5}}{4-5}$

$= \frac{2-\sqrt{5}}{-1}$

$= -2+\sqrt{5}$

c $\frac{1}{3-\sqrt{7}} = \frac{3+\sqrt{7}}{\left(3-\sqrt{7}\right)\left(3+\sqrt{7}\right)}$

$= \frac{3+\sqrt{7}}{9-7}$

$= \frac{3+\sqrt{7}}{2}$

2 d
$$\frac{4}{3-\sqrt{5}}=\frac{4\times\left(3+\sqrt{5}\right)}{\left(3-\sqrt{5}\right)\left(3+\sqrt{5}\right)}$$
$$=\frac{12+4\sqrt{5}}{9-5}$$
$$=\frac{12+4\sqrt{5}}{4}$$
$$=3+\sqrt{5}$$

e
$$\frac{1}{\sqrt{5}-\sqrt{3}}=\frac{\sqrt{5}+\sqrt{3}}{\left(\sqrt{5}-\sqrt{3}\right)\left(\sqrt{5}+\sqrt{3}\right)}$$
$$=\frac{\sqrt{5}+\sqrt{3}}{5-3}$$
$$=\frac{\sqrt{5}+\sqrt{3}}{2}$$

f
$$\frac{3-\sqrt{2}}{4-\sqrt{5}}=\frac{\left(3-\sqrt{2}\right)\left(4+\sqrt{5}\right)}{\left(4-\sqrt{5}\right)\left(4+\sqrt{5}\right)}$$
$$=\frac{\left(3-\sqrt{2}\right)\left(4+\sqrt{5}\right)}{16-5}$$
$$=\frac{\left(3-\sqrt{2}\right)\left(4+\sqrt{5}\right)}{11}$$
$$=\frac{12+3\sqrt{5}-4\sqrt{2}-\sqrt{10}}{11}$$

g
$$\frac{5}{2+\sqrt{5}}=\frac{5\times\left(2-\sqrt{5}\right)}{\left(2+\sqrt{5}\right)\left(2-\sqrt{5}\right)}$$
$$=\frac{5\left(2-\sqrt{5}\right)}{4-5}$$
$$=\frac{5\left(2-\sqrt{5}\right)}{-1}$$
$$=5\left(\sqrt{5}-2\right)$$

h
$$\frac{5\sqrt{2}}{\sqrt{8}-\sqrt{7}}=\frac{5\sqrt{2}\left(\sqrt{8}+\sqrt{7}\right)}{\left(\sqrt{8}-\sqrt{7}\right)\left(\sqrt{8}+\sqrt{7}\right)}$$
$$=\frac{5\left(\sqrt{8\times2}+\sqrt{2\times7}\right)}{8-7}$$

h
$$\frac{5\sqrt{2}}{\sqrt{8}-\sqrt{7}}=\frac{5\left(\sqrt{16}+\sqrt{14}\right)}{1}$$
$$=5\left(4+\sqrt{14}\right)$$

i
$$\frac{11}{3+\sqrt{11}}=\frac{11\left(3-\sqrt{11}\right)}{\left(3+\sqrt{11}\right)\left(3-\sqrt{11}\right)}$$
$$=\frac{11\left(3-\sqrt{11}\right)}{9-11}$$
$$=\frac{11\left(3-\sqrt{11}\right)}{-2}$$
$$=\frac{-11(3-\sqrt{11})}{2}$$

j
$$\frac{\sqrt{3}-\sqrt{7}}{\sqrt{3}+\sqrt{7}}=\frac{\left(\sqrt{3}-\sqrt{7}\right)\left(\sqrt{3}-\sqrt{7}\right)}{\left(\sqrt{3}+\sqrt{7}\right)\left(\sqrt{3}-\sqrt{7}\right)}$$
$$=\frac{3-\sqrt{21}-\sqrt{21}+7}{3-7}$$
$$=\frac{10-2\sqrt{21}}{-4}$$
$$=\frac{5-\sqrt{21}}{-2}\text{ or }\frac{\sqrt{21}-5}{2}$$

k
$$\frac{\sqrt{17}-\sqrt{11}}{\sqrt{17}+\sqrt{11}}=\frac{\left(\sqrt{17}-\sqrt{11}\right)\left(\sqrt{17}-\sqrt{11}\right)}{\left(\sqrt{17}+\sqrt{11}\right)\left(\sqrt{17}-\sqrt{11}\right)}$$
$$=\frac{17-\sqrt{187}-\sqrt{187}+11}{17-11}$$
$$=\frac{28-2\sqrt{187}}{6}$$
$$=\frac{14-\sqrt{187}}{3}$$

l
$$\frac{\sqrt{41}+\sqrt{29}}{\sqrt{41}-\sqrt{29}}=\frac{\left(\sqrt{41}+\sqrt{29}\right)\left(\sqrt{41}+\sqrt{29}\right)}{\left(\sqrt{41}-\sqrt{29}\right)\left(\sqrt{41}+\sqrt{29}\right)}$$
$$=\frac{41+2\sqrt{41}\sqrt{29}+29}{41-29}$$
$$=\frac{70+2\sqrt{1189}}{12}$$
$$=\frac{35+\sqrt{1189}}{6}$$

2 m $\dfrac{\sqrt{2}-\sqrt{3}}{\sqrt{3}-\sqrt{2}}=\dfrac{\left(\sqrt{2}-\sqrt{3}\right)\left(\sqrt{3}+\sqrt{2}\right)}{\left(\sqrt{3}-\sqrt{2}\right)\left(\sqrt{3}+\sqrt{2}\right)}$

$=\dfrac{\sqrt{6}+2-3-\sqrt{6}}{3-2}$

$=\dfrac{-1}{1}$

$=-1$

3 a $\dfrac{1}{(3-\sqrt{2})^2}=\dfrac{1}{\left(3-\sqrt{2}\right)\left(3-\sqrt{2}\right)}$

$=\dfrac{1}{9-3\sqrt{2}-3\sqrt{2}+\sqrt{4}}$

$=\dfrac{1}{11-6\sqrt{2}}$

$=\dfrac{1\times(11+6\sqrt{2})}{(11-6\sqrt{2})(11+6\sqrt{2})}$

$=\dfrac{11+6\sqrt{2}}{121+66\sqrt{2}-66\sqrt{2}-72}$

$=\dfrac{11+6\sqrt{2}}{49}$

b $\dfrac{1}{\left(2+\sqrt{5}\right)^2}=\dfrac{1}{\left(2+\sqrt{5}\right)\left(2+\sqrt{5}\right)}$

$=\dfrac{1}{4+2\sqrt{5}+2\sqrt{5}+\sqrt{25}}$

$=\dfrac{1}{9+4\sqrt{5}}$

$=\dfrac{1\times(9-4\sqrt{5})}{(9+4\sqrt{5})(9-4\sqrt{5})}$

$=\dfrac{9-4\sqrt{5}}{81-36\sqrt{5}+36\sqrt{5}-80}$

$=\dfrac{9-4\sqrt{5}}{1}$

$=9-4\sqrt{5}$

c $\dfrac{4}{\left(3-\sqrt{2}\right)^2}=\dfrac{4}{\left(3-\sqrt{2}\right)\left(3-\sqrt{2}\right)}$

$=\dfrac{4}{9-3\sqrt{2}-3\sqrt{2}+\sqrt{4}}$

3 c $\dfrac{4}{\left(3-\sqrt{2}\right)^2}=\dfrac{4}{11-6\sqrt{2}}$

$=\dfrac{4\times(11+6\sqrt{2})}{(11-6\sqrt{2})(11+6\sqrt{2})}$

$=\dfrac{44+24\sqrt{2}}{121+66\sqrt{2}-66\sqrt{2}-72}$

$=\dfrac{44+24\sqrt{2}}{49}$

d $\dfrac{3}{\left(5+\sqrt{2}\right)^2}=\dfrac{3}{\left(5+\sqrt{2}\right)\left(5+\sqrt{2}\right)}$

$=\dfrac{3}{25+5\sqrt{2}+5\sqrt{2}+\sqrt{4}}$

$=\dfrac{3}{27+10\sqrt{2}}$

$=\dfrac{3\times\left(27-10\sqrt{2}\right)}{\left(27+10\sqrt{2}\right)\left(27-10\sqrt{2}\right)}$

$=\dfrac{3\times\left(27-10\sqrt{2}\right)}{729-270\sqrt{2}+270\sqrt{2}-200}$

$=\dfrac{81-30\sqrt{2}}{529}$

e $\dfrac{1}{\left(5+\sqrt{2}\right)\left(3-\sqrt{2}\right)}=\dfrac{1}{15-5\sqrt{2}+3\sqrt{2}-\sqrt{4}}$

$=\dfrac{1}{13-2\sqrt{2}}$

$=\dfrac{1\times\left(13+2\sqrt{2}\right)}{\left(13-2\sqrt{2}\right)\left(13+2\sqrt{2}\right)}$

$=\dfrac{13+2\sqrt{2}}{169+26\sqrt{2}-26\sqrt{2}-8}$

$=\dfrac{13+2\sqrt{2}}{161}$

f $\dfrac{2}{\left(5-\sqrt{3}\right)\left(2+\sqrt{3}\right)}=\dfrac{2}{10+5\sqrt{3}-2\sqrt{3}-\sqrt{9}}$

$=\dfrac{2}{7+3\sqrt{3}}$

$=\dfrac{2\times\left(7-3\sqrt{3}\right)}{\left(7+3\sqrt{3}\right)\left(7-3\sqrt{3}\right)}$

3 f $$\frac{2}{\left(5-\sqrt{3}\right)\left(2+\sqrt{3}\right)}=\frac{14-6\sqrt{3}}{49-21\sqrt{3}+21\sqrt{3}-27}$$

$$=\frac{14-6\sqrt{3}}{22}$$

$$=\frac{7-3\sqrt{3}}{11}$$

4 $$\frac{3-2\sqrt{5}}{\sqrt{5}-1}=\frac{\left(3-2\sqrt{5}\right)\left(\sqrt{5}+1\right)}{\left(\sqrt{5}-1\right)\left(\sqrt{5}+1\right)}$$

$$=\frac{3\sqrt{5}+3-10-2\sqrt{5}}{5+\sqrt{5}-\sqrt{5}-1}$$

$$=\frac{\sqrt{5}-7}{4}$$

$$=\frac{-7}{4}+\frac{\sqrt{5}}{4}$$

$$p=-\frac{7}{4}, q=\frac{\sqrt{5}}{4}$$