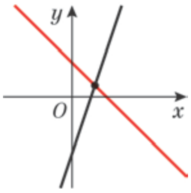
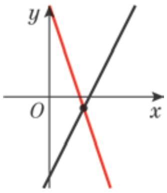
Equations and inequalities 3C

1 a i



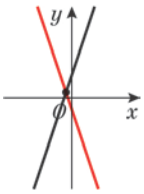
ii (2, 1)

b i



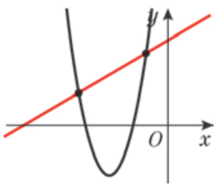
ii (3, −1)

c i Rearrange $3x + y + 1 = 0$ to give
$y = -3x - 1$



ii (−0.5, 0.5)

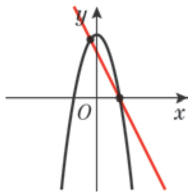
2 a Rearrange $2y = 2x + 11$ to give $y = x + \frac{11}{2}$



b (−1.5, 4) and (3.5, 9)

c Substitute values for x into each equation.
When $x = -1.5$:
$2y = 2(-1\tfrac{1}{2}) + 11 = 8,\ y = 4$
When $x = 3.5$:
$2y = 2(3\tfrac{1}{2}) + 11 = 18,\ y = 9$
When $x = -1.5$:
$y = 2(-1\tfrac{1}{2})^2 - 3(-1\tfrac{1}{2}) - 5 = \frac{9}{2} + \frac{9}{2} - 5 = 4$
When $x = 3.5$:
$y = 2(3\tfrac{1}{2})^2 - 3(3\tfrac{1}{2}) - 5 = \frac{49}{2} - \frac{21}{2} - 5 = 9$

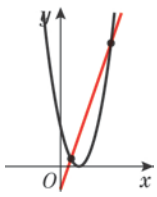
3 a $y = 9 - x^2$
$y = -2x + 6$



b (−1, 8) and (3, 0)

c Substitute each value of x and y into each equation:
$(-1)^2 + 8 = 1 + 8 = 9$
$2(-1) + 8 = -2 + 8 = 6$
$(3)^2 + 0 = 9$
$2(3) + 0 = 6$

4 a $y = (x - 2)^2$
$0 = (x - 2)^2$
$x = 2$
When $x = 0$, $y = 4$



b $(x - 2)^2 = 3x - 2$
$x^2 - 4x + 4 - 3x + 2 = 0$
$x^2 - 7x + 6 = 0$
$(x - 6)(x - 1) = 0$
$x = 6$ or $x = 1$
When $x = 1$, $y = 1$
When $x = 6$, $y = 16$
(1, 1) and (6, 16) are the points of intersection.

5 $y = x - 4$
Substitute into $y^2 = 2x^2 - 17$:
$(x-4)^2 = 2x^2 - 17$
$x^2 - 8x + 16 = 2x^2 - 17$
$0 = x^2 + 8x - 33$
$0 = (x+11)(x-3)$
$x = -11$ or $x = 3$

5 Substitute into $y = x - 4$:
When $x = -11$, $y = -11 - 4 = -15$
When $x = 3$, $y = 3 - 4 = -1$
Intersection points:
$(-11, -15)$ and $(3, -1)$

6 $y = 3x - 1$
Substitute into $y^2 - xy = 15$:
$$(3x-1)^2 - x(3x-1) = 15$$
$$9x^2 - 6x + 1 - 3x^2 + x = 15$$
$$6x^2 - 5x - 14 = 0$$
$$(6x+7)(x-2) = 0$$
$x = -\frac{7}{6}$ or $x = 2$
Substitute into $y = 3x - 1$:
When $x = -\frac{7}{6}$, $y = -\frac{21}{6} - 1 = -\frac{9}{2}$
When $x = 2$, $y = 6 - 1 = 5$
Intersection points:
$\left(-1\frac{1}{6}, -4\frac{1}{2}\right)$ and $(2, 5)$

7 a $6x^2 + 3x - 7 = 2x + 8$
$6x^2 + x - 15 = 0$
Using the discriminant:
$b^2 - 4ac = 1 + 360 = 361$
$361 > 0$
Therefore, there are 2 points of intersection.

b $4x^2 - 18x + 40 = 10x - 9$
$4x^2 - 28x + 49 = 0$
Using the discriminant:
$b^2 - 4ac = 784 - 784 = 0$
Therefore, there is 1 point of intersection.

c Rearrange $7x + y + 3 = 0$ to give:
$y = -7x - 3$
$3x^2 - 2x + 4 = -7x - 3$
$3x^2 + 5x + 7 = 0$
Using the discriminant:
$b^2 - 4ac = 25 - 84 = -59$
$-59 < 0$
Therefore, there are 0 points of intersection.

8 a Rearrange $2x - y = 1$ and then substitute into $x^2 + 4ky + 5k = 0$:
$y = 2x - 1$
$x^2 + 4k(2x - 1) + 5k = 0$
$x^2 + 8kx - 4k + 5k = 0$
$x^2 + 8kx + k = 0$

b Using the discriminant,
$b^2 - 4ac = 0$
$(8k)^2 - 4(1)(k) = 0$
$64k^2 - 4k = 0$
$4k(16k - 1) = 0$
$k = 0$ or $k = \frac{1}{16}$
As k is a non-zero constant, $k = \frac{1}{16}$

c $x^2 + 8(\frac{1}{16})x + \frac{1}{16} = 0$
$16x^2 + 8x + 1 = 0$
$(4x + 1)^2 = 0$
$x = -\frac{1}{4}$, $y = -\frac{3}{2}$

9 $p = 0.3x - 6$
If the swimmer touches the bottom of the pool, then
$0.5x^2 - 3x = 0.3x - 6$
$0.5x^2 - 3.3x + 6 = 0$
Using the discriminant:
$b^2 - 4ac = (-3.3)^2 - 4 \times 0.5 \times 6$
$= -1.11$
As -1.11 is negative, there are no solutions, so the swimmer does not reach the bottom of the pool.