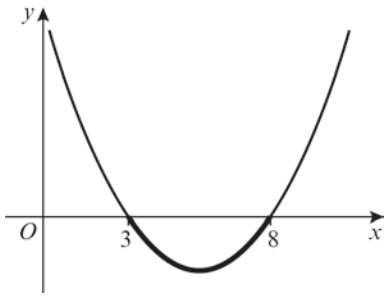
Equations and inequalities 3E

1 a $x^2 - 11x + 24 = 0$

$(x-3)(x-8) = 0$

$x = 3,\ x = 8$

Sketch of $y = x^2 - 11x + 24$:



$x^2 - 11 = 24 < 0$ when $3 < x < 8$

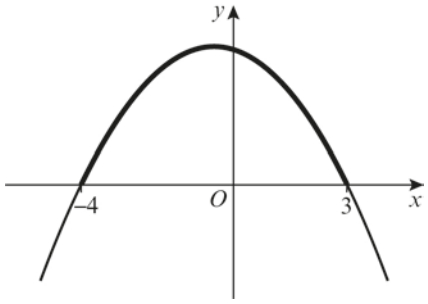
b $12 - x - x^2 = 0$

$0 = x^2 + x - 12$

$0 = (x+4)(x-3)$

$x = -4,\ x = 3$

Sketch of $y = 12 - x - x^2$:



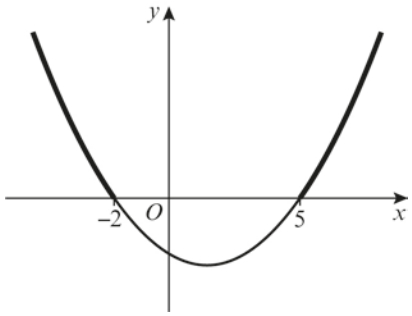
$12 - x - x^2 > 0$ when $-4 < x < 3$

c $x^2 - 3x - 10 = 0$

$(x+2)(x-5) = 0$

$x = -2,\ x = 5$

Sketch of $y = x^2 - 3x - 10$:



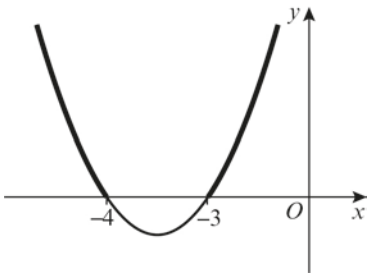
$x^2 - 3x - 10 > 0$ when $x < -2$ or $x > 5$

d $x^2 + 7x + 12 = 0$

$(x+4)(x+3) = 0$

$x = -4,\ x = -3$

Sketch of $y = x^2 + 7x + 12$:



$x^2 + 7x + 12 \geq 0$ when $x \leq -4$ or $x \geq -3$

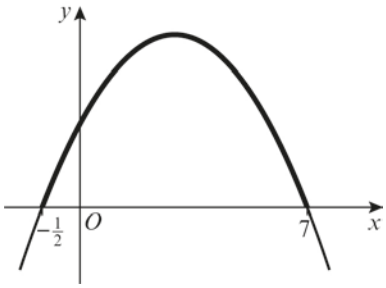
e $7 + 13x - 2x^2 = 0$

$2x^2 - 13x - 7 = 0$

$(2x+1)(x-7) = 0$

$x = -\frac{1}{2},\ x = 7$

Sketch of $y = 7 + 13x - 2x^2$:



$7 + 13x - 2x^2 > 0$ when $-\frac{1}{2} < x < 7$

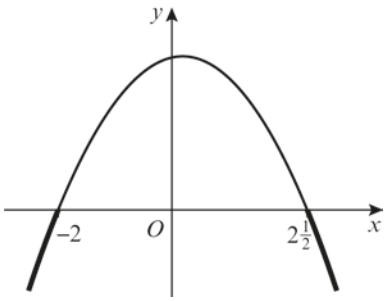
f $10 + x - 2x^2 = 0$

$2x^2 - x - 10 = 0$

$(2x-5)(x+2) = 0$

$x = \frac{5}{2},\ x = -2$

Sketch of $y = 10 + x - 2x^2$:



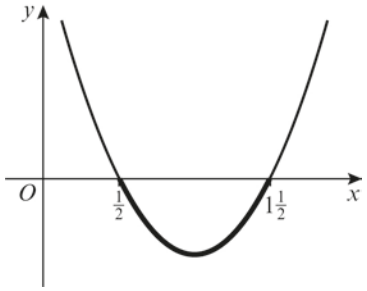
$10 + x - 2x^2 < 0$ when $x < -2$ or $x > \frac{5}{2}$

1 g $4x^2 - 8x + 3 = 0$

$(2x-1)(2x-3) = 0$

$x = \frac{1}{2},\ x = \frac{3}{2}$

Sketch of $y = 4x^2 - 8x = 3$:



$4x^2 - 8x + 3 \le 0$ when $\frac{1}{2} \le x \le \frac{3}{2}$

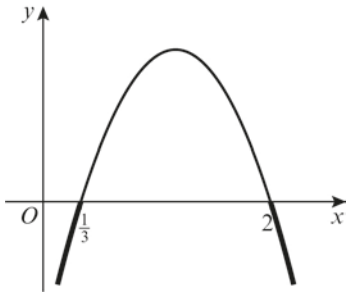
h $-2 + 7x - 3x^2 = 0$

$3x^2 - 7x + 2 = 0$

$(3x-1)(x-2) = 0$

$x = \frac{1}{3},\ x = 2$

Sketch of $y = -2 + 7x - 3x^2$:



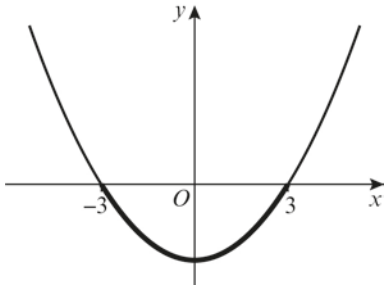
$-2 + 7x - 3x^2 < 0$ when $x < \frac{1}{3}$ or $x > 2$

i $x^2 - 9 = 0$

$(x+3)(x-3) = 0$

$x = -3,\ x = 3$

Sketch of $y = x^2 - 9$:



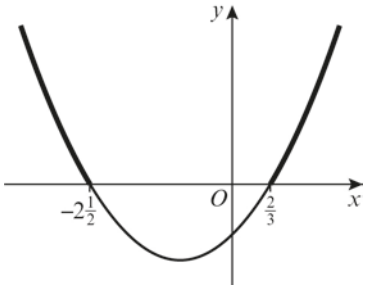
$x^2 - 9 < 0$ when $-3 < x < 3$

j $6x^2 + 11x - 10 = 0$

$(3x-2)(2x+5) = 0$

$x = \frac{2}{3},\ x = -\frac{5}{2}$

Sketch of $y = 6x^2 + 11x - 10$:



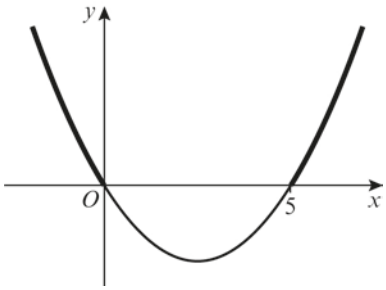
$6x^2 + 11x - 10 > 0$ when $x < -\frac{5}{2}$ or $x > \frac{2}{3}$

k $x^2 - 5x = 0$

$x(x-5) = 0$

$x = 0,\ x = 5$

Sketch of $y = x^2 - 5x$:



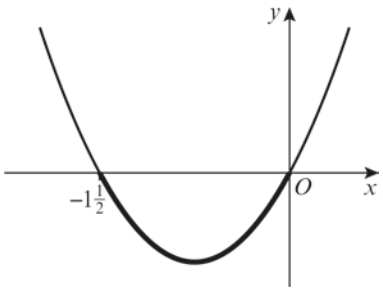
$x^2 - 5x > 0$ when $x < 0$ or $x > 5$

l $2x^2 + 3x = 0$

$x(2x+3) = 0$

$x = 0,\ x = -\frac{3}{2}$

Sketch of $y = 2x^2 + 3x$:



$2x^2 + 3x \le 0$ when $-\frac{3}{2} \le x \le 0$

2 a $x^2 = 10 - 3x$

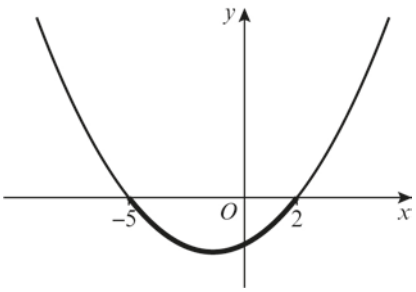
$x^2 + 3x - 10 = 0$

$(x+5)(x-2) = 0$

$x = -5,\ x = 2$

$x^2 < 10 - 3x \Rightarrow x^2 + 3x - 10 < 0$

Sketch of $y = x^2 + 3x - 10$:



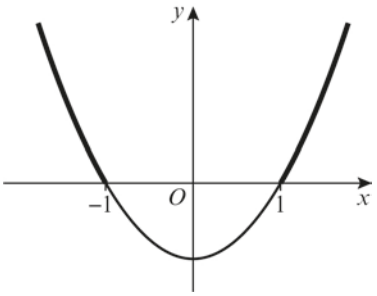
$x^2 + 3x - 10 < 0$ when $-5 < x < 2$

b $11 < x^2 + 10$

$0 < x^2 + 10 - 11$

$x^2 - 1 > 0$

Sketch of $y = x^2 - 1$:



$x^2 - 1 > 0$ when $x < -1$ or $x > 1$

c $x(3-2x) = 1$

$3x - 2x^2 = 1$

$0 = 2x^2 - 3x + 1$

$0 = (2x-1)(x-1)$

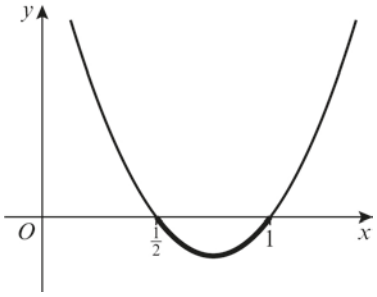
$x = \frac{1}{2},\ x = 1$

$x(3-2x) > 1$

$\Rightarrow -2x^2 + 3x - 1 > 0$

$\Rightarrow 2x^2 - 3x + 1 < 0$

c Sketch of $y = 2x^2 - 3x + 1$:



$2x^2 - 3x + 1 < 0$ when $\frac{1}{2} < x < 1$

d $x(x+11) = 3(1-x^2)$

$x^2 + 11x = 3 - 3x^2$

$x^2 + 3x^2 + 11x - 3 = 0$

$4x^2 + 11x - 3 = 0$

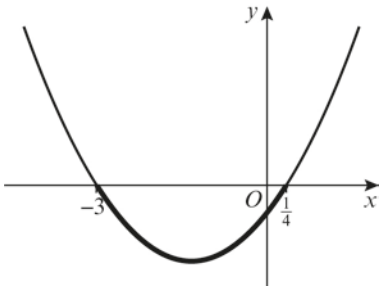
$(4x-1)(x+3) = 0$

$x = \frac{1}{4},\ x = -3$

$x(x+11) < 3(1-x^2)$

$\Rightarrow 4x^2 + 11x - 3 < 0$

Sketch of $y = 4x^2 + 11x - 3$:



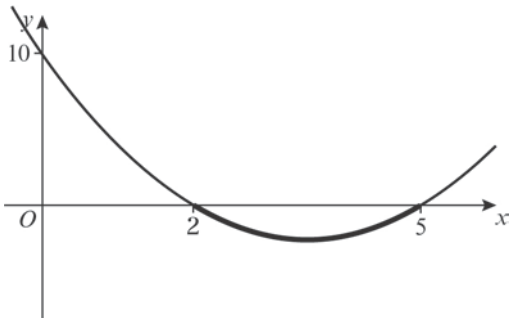
$4x^2 + 11x - 3 < 0$ when $-3 < x < \frac{1}{4}$

3 a $x^2 - 7x + 10 < 0$

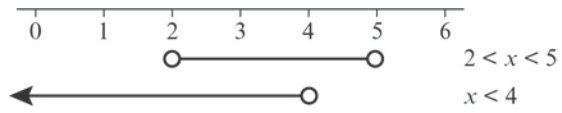
$x^2 - 7x + 10 = 0$

$(x - 2)(x - 5) = 0$

$x = 2$ or $x = 5$

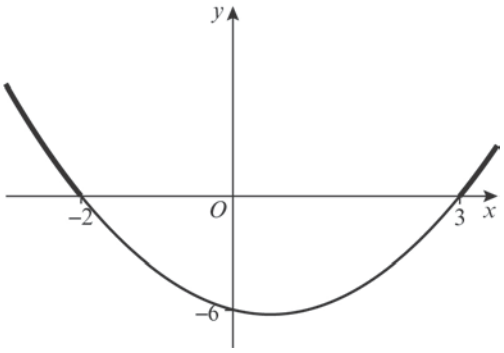


3 a So $2 < x < 5$
$3x + 5 < 17$
$3x < 12$
$x < 4$

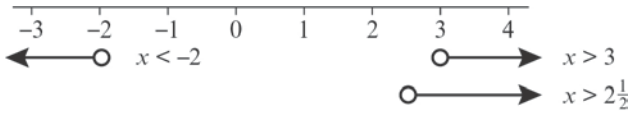


So the required values are $2 < x < 4$
$\{x: 2 < x < 4\}$

b $x^2 - x - 6 > 0$
$x^2 - x - 6 = 0$
$(x - 3)(x + 2) = 0$
$x = 3$ or $x = -2$

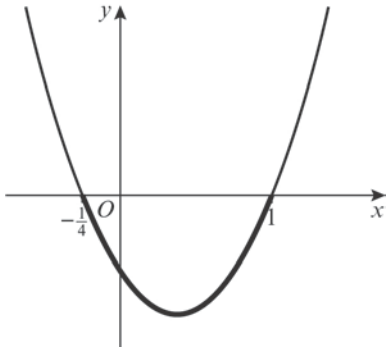


So $x < -2$ or $x > 3$
$10 - 2x < 5$
$-2x < -5$
$x > 2\frac{1}{2}$

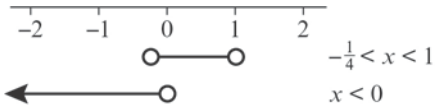


So the required values are $x > 3$
$\{x: x > 3\}$

c $4x^2 - 3x - 1 < 0$
$4x^2 - 3x - 1 = 0$
$(x - 1)(4x + 1) = 0$
$x = 1$ or $x = -\frac{1}{4}$

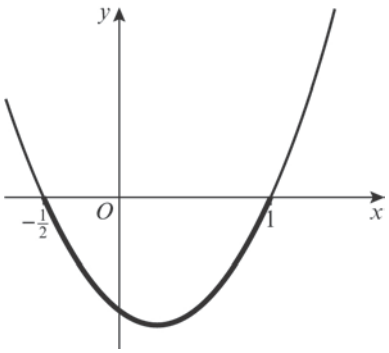


c So $-\frac{1}{4} < x < 1$
$4x + 8 < 15 - x - 7$
$5x < 0$
$x < 0$

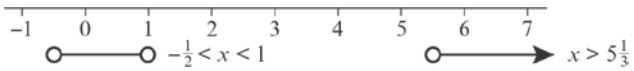


So the required values are $-\frac{1}{4} < x < 0$
$\{x: -\frac{1}{4} < x < 0\}$

d $2x^2 - x - 1 < 0$
$2x^2 - x - 1 = 0$
$(2x + 1)(x - 1) = 0$
$x = -\frac{1}{2}$ or $x = 1$

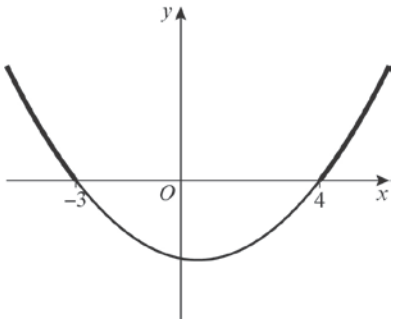


So $-\frac{1}{2} < x < 1$
$14 < 3x - 2$
$3x > 16$
$x > 5\frac{1}{3}$



So there are no values.

e $x^2 - x - 12 > 0$
$x^2 - x - 12 = 0$
$(x - 4)(x + 3) = 0$
$x = 4$ or $x = -3$

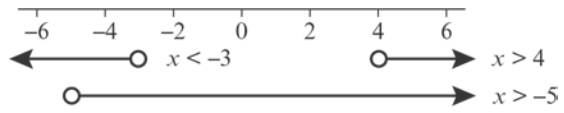


3 e So $x < -3$ or $x > 4$

$3x + 17 > 2$

$3x > -15$

$x > -5$



So the required values are $-5 < x < -3$ and $x > 4$

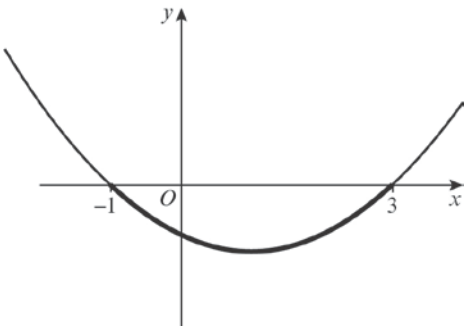
$\{x: -5 < x < -3\} \cup \{x: x > 4\}$

f $x^2 - 2x - 3 < 0$

$x^2 - 2x - 3 = 0$

$(x - 3)(x + 1) = 0$

$x = 3$ or $x = -1$



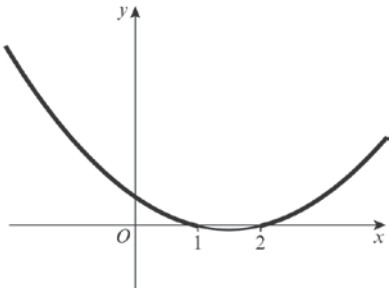
So $-1 < x < 3$

$x^2 - 3x + 2 > 0$

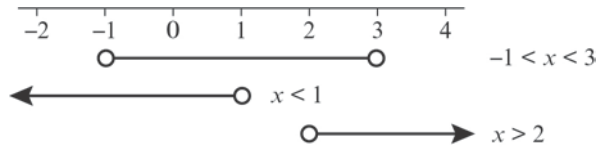
$x^2 - 3x + 2 = 0$

$(x - 2)(x - 1) = 0$

$x = 2$ or $x = 1$



f So $x < 1$ or $x > 2$



So the required values are $-1 < x < 1$ and $2 < x < 3$

$\{x: -1 < x < 1\} \cup \{x: 2 < x < 3\}$

4 a $\frac{2}{x} < 1$

Multiply both sides by x^2:

$2x < x^2$

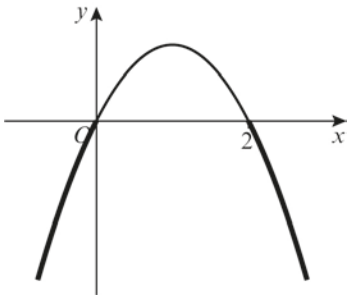
$2x - x^2 < 0$

Solve the quadratic to find the critical values:

$2x - x^2 = 0$

$x(2 - x) = 0$

$x = 0$ or $x = 2$



The solution is $x < 0$ or $x > 2$

b $5 > \frac{4}{x}$

Multiply both sides by x^2:

$5x^2 > 4x$

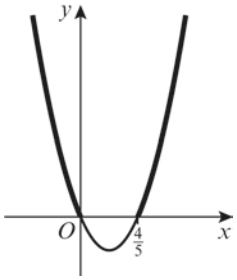
$5x^2 - 4x > 0$

Solve the quadratic to find the critical values:

$5x^2 - 4x = 0$

$x(5x - 4) = 0$

$x = 0$ or $x = \frac{4}{5}$



The solution is $x < 0$ or $x > \frac{4}{5}$.

4 c $\frac{1}{x}+3>2$

$\frac{1}{x}>-1$

Multiply both sides by x^2:

$x>-x^2$

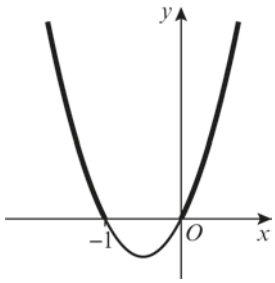
$x^2+x>0$

Solve the quadratic to find the critical values:

$x^2+x=0$

$x(x+1)=0$

$x=0$ or $x=-1$



The solution is $x<-1$ or $x>0$.

d $6+\frac{5}{x}>\frac{8}{x}$

$6>\frac{3}{x}$

Multiply both sides by x^2:

$6x^2>3x$

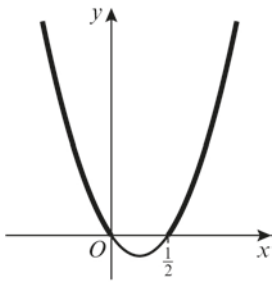
$6x^2-3x>0$

Solve the quadratic to find the critical values:

$6x^2-3x=0$

$3x(2x-1)=0$

$x=0$ or $x=\frac{1}{2}$



The solution is $x<0$ or $x>\frac{1}{2}$.

e $25>\frac{1}{x^2}$

$25x^2>1$

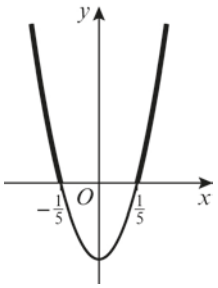
$25x^2-1>0$

Solve the quadratic to find the critical values:

$25x^2-1=0$

$(5x-1)(5x+1)=0$

$x=\frac{1}{5}$ or $x=-\frac{1}{5}$



The solution is $x<-\frac{1}{5}$ or $x>\frac{1}{5}$.

f $\frac{6}{x^2}+\frac{7}{x}\leq 3$

Multiply both sides by x^2:

$6+7x\leq 3x^2$

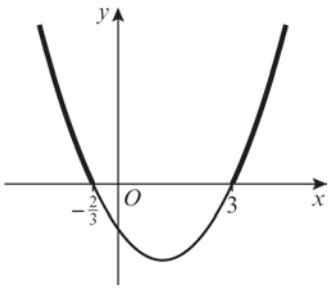
$3x^2-7x-6\geq 0$

Solve the quadratic to find the critical values:

$3x^2-7x-6=0$

$(3x+2)(x-3)=0$

$x=-\frac{2}{3}$ or $x=3$



The solution is $x\leq -\frac{2}{3}$ or $x\geq 3$.

5 a Using the quadratic formula:

$a=1,\ b=-k,\ c=k+3$

$b^2-4ac<0$ for no real roots, so

$k^2-4(k+3)<0$

$k^2-4k-12<0$

$(k-6)(k+2)<0$

$-2<k<6$

5 b Using the quadratic formula:

$a = p,\ b = p,\ c = -2$

$b^2 - 4ac > 0$ for real roots, so

$p^2 + 8p > 0$

$p(p+8) > 0$

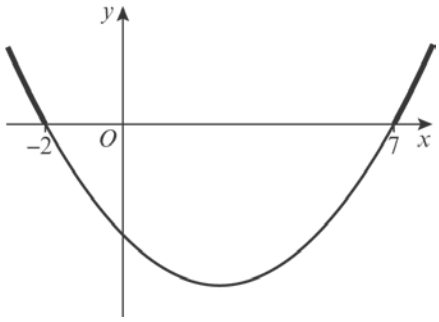
$p > 0$ or $p < -8$

6 $x^2 - 5x - 14 > 0$

$x^2 - 5x - 14 = 0$

$(x - 7)(x + 2) = 0$

$x = 7$ or $x = -2$



So the required values are $x < -2$ or $x > 7$

$\{x:\ x < -2\} \cup \{x: x > 7\}$

7 a $2(3x - 1) < 4 - 3x$

$6x - 2 < 4 - 3x$

$9x < 6$

$x < \frac{2}{3}$

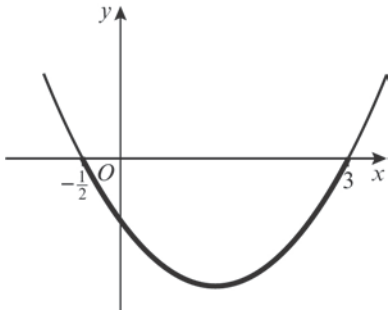
$\{x:\ x < \frac{2}{3}\}$

b $2x^2 - 5x - 3 < 0$

$2x^2 - 5x - 3 = 0$

$(2x + 1)(x - 3) = 0$

$x = -\frac{1}{2}$ or $x = 3$



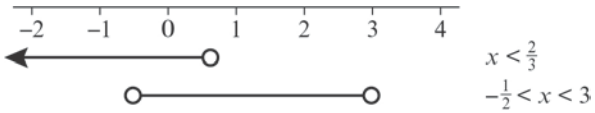
So $-\frac{1}{2} < x < 3$

$\{x: -\frac{1}{2} < x < 3\}$

c $6x - 2 < 4 - 3x \Rightarrow x < \frac{2}{3}$

$2x^2 - 5x - 3 < 0 \Rightarrow -\frac{1}{2} < x < 3$

7 c



So the required values are $-\frac{1}{2} < x < \frac{2}{3}$

$\{x: -\frac{1}{2} < x < \frac{2}{3}\}$

8 $\dfrac{5}{x-3} < 2$

Multiply both sides by $(x - 3)^2$:

$5(x - 3) < 2(x - 3)^2$

$5x - 15 < 2x^2 - 12x + 18$

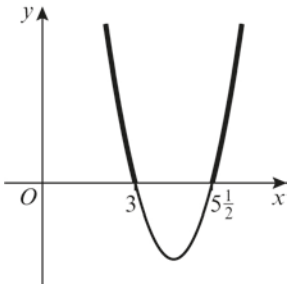
$2x^2 - 17x + 33 > 0$

Solve the quadratic to find the critical values:

$2x^2 - 17x + 33 = 0$

$(2x - 11)(x - 3) = 0$

$x = \frac{11}{2}$ or $x = 3$



The solution is $x < 3$ or $x > 5\frac{1}{2}$

9 $kx^2 - 2kx + 3 = 0$

For no real roots, using the discriminant:

$b^2 - 4ac < 0$

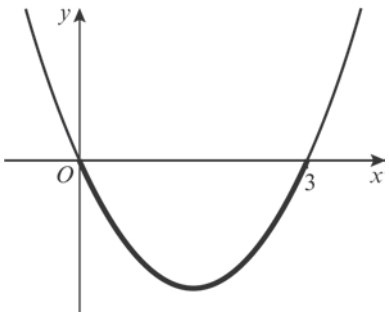
$(-2k)^2 - 4(k)(3) < 0$

$4k^2 - 12k < 0$

$4k^2 - 12k = 0$

$4k(k - 3) = 0$

$k = 0$ or $k = 3$



So $0 < k < 3$

When $k = 0$, the equation gives $3 = 0$

Therefore, $0 \leq k < 3$.