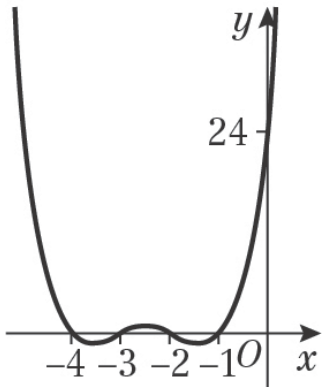
Graphs and transformations 4B

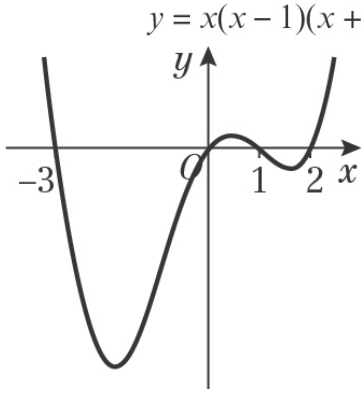
1 a $y = (x + 1)(x + 2)(x + 3)(x + 4)$
$0 = (x + 1)(x + 2)(x + 3)(x + 4)$
So $x = -1$, $x = -2$, $x = -3$ or $x = -4$
The curve crosses the x-axis at $(-1, 0)$, $(-2, 0)$, $(-3, 0)$ and $(-4, 0)$.
When $x = 0$, $y = 1 \times 2 \times 3 \times 4 = 24$
So the curve crosses the y-axis at $(0, 24)$.
$x \to \infty$, $y \to \infty$
$x \to -\infty$, $y \to \infty$

$y = (x + 1)(x + 2)(x + 3)(x + 4)$

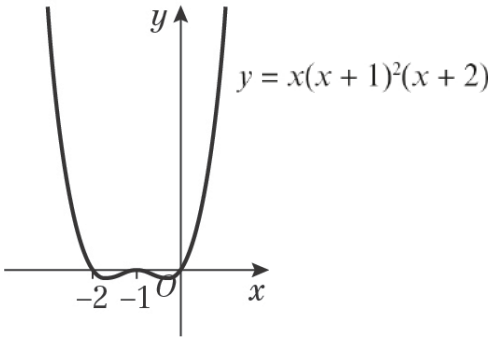


b $y = x(x - 1)(x + 3)(x - 2)$
$0 = x(x - 1)(x + 3)(x - 2)$
So $x = 0$, $x = 1$, $x = -3$ or $x = 2$
The curve crosses the x-axis at $(0, 0)$, $(1, 0)$, $(-3, 0)$ and $(2, 0)$.
$x \to \infty$, $y \to \infty$
$x \to -\infty$, $y \to \infty$

$y = x(x - 1)(x + 3)(x - 2)$

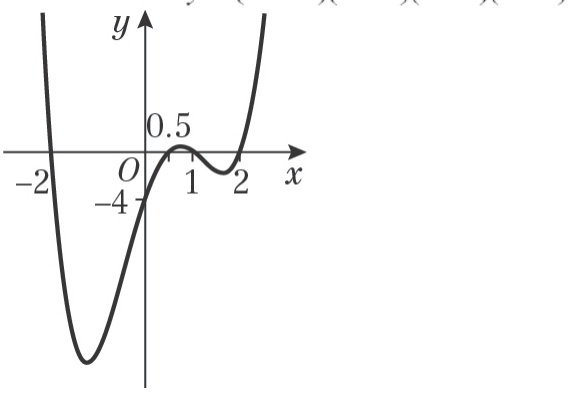


c $y = x(x + 1)^2(x + 2)$
$0 = x(x + 1)^2(x + 2)$
So $x = 0$, $x = -1$ or $x = -2$
The curve crosses the x-axis at $(0, 0)$ and $(-2, 0)$ and touches it at $(-1, 0)$.
$x \to \infty$, $y \to \infty$
$x \to -\infty$, $y \to \infty$

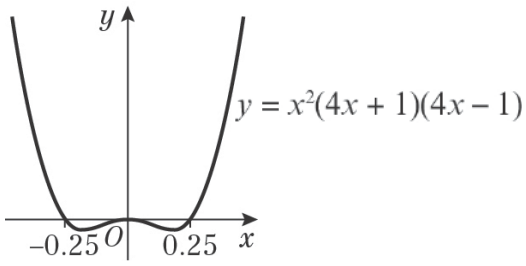


d $y = (2x - 1)(x + 2)(x - 1)(x - 2)$
$0 = (2x - 1)(x + 2)(x - 1)(x - 2)$
So $x = \frac{1}{2}$, $x = -2$, $x = 1$ or $x = 2$
The curve crosses the x-axis at $\left(\frac{1}{2}, 0\right)$, $(-2, 0)$, $(1, 0)$ and $(2, 0)$.
When $x = 0$, $y = (-1) \times 2 \times (-1) \times (-2) = -4$
So the curve crosses the y-axis at $(0, -4)$.
$x \to \infty$, $y \to \infty$
$x \to -\infty$, $y \to \infty$

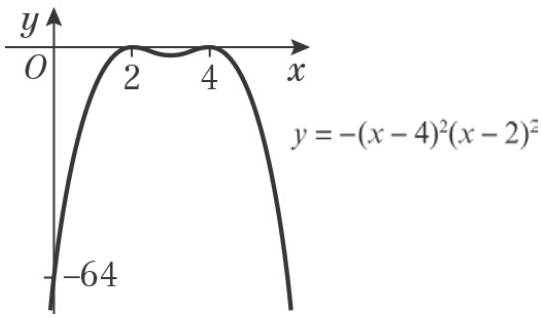
$y = (2x - 1)(x + 2)(x - 1)(x - 2)$



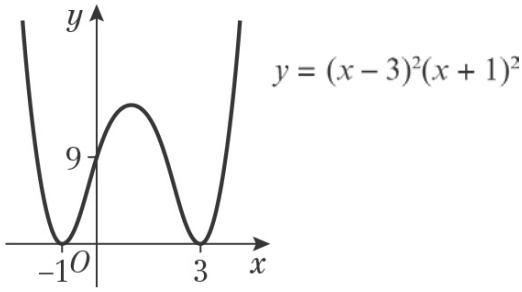
1 e $y = x^2(4x + 1)(4x - 1)$
$0 = x^2(4x + 1)(4x - 1)$
So $x = 0$, $x = -\frac{1}{4}$ or $x = \frac{1}{4}$
The curve crosses the x-axis at $\left(-\frac{1}{4}, 0\right)$ and $\left(\frac{1}{4}, 0\right)$ and touches it at (0, 0).
$x \to \infty$, $y \to \infty$
$x \to -\infty$, $y \to \infty$



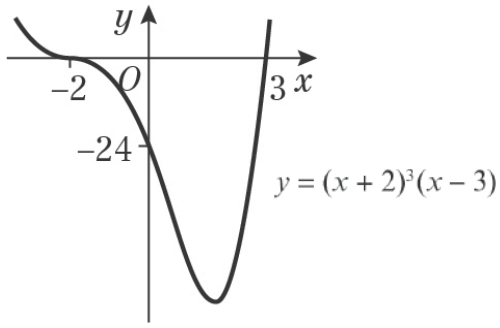
f $y = -(x - 4)^2(x - 2)^2$
$0 = -(x - 4)^2(x - 2)^2$
So $x = 4$ or $x = 2$
The curve touches the x-axis at (4, 0) and (2, 0).
When $x = 0$, $y = -(-4)^2 \times (-2)^2 = -64$
So the curve crosses the y-axis at (0, −64).
$x \to \infty$, $y \to -\infty$
$x \to -\infty$, $y \to -\infty$



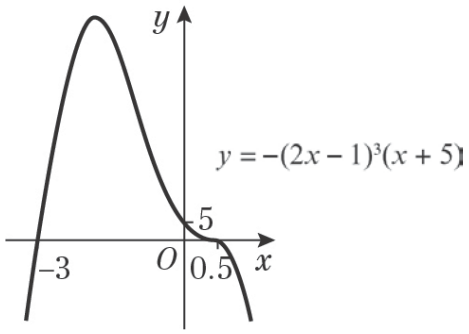
g $y = (x - 3)^2(x + 1)^2$
$0 = (x - 3)^2(x + 1)^2$
So $x = 3$ or $x = -1$
The curve touches the x-axis at (3, 0) and (−1, 0).
When $x = 0$, $y = (-3)^2 \times 1^2 = 9$
So the curve crosses the y-axis at (0, 9).
$x \to \infty$, $y \to \infty$
$x \to -\infty$, $y \to \infty$



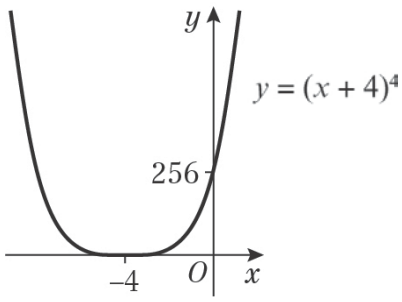
h $y = (x + 2)^3(x - 3)$
$0 = (x + 2)^3(x - 3)$
So $x = -2$ or $x = 3$
The curve crosses the x-axis at (−2, 0) and (3, 0).
When $x = 0$, $y = 2^3 \times (-3) = -24$
So the curve crosses the y-axis at (0, −24).
$x \to \infty$, $y \to \infty$
$x \to -\infty$, $y \to \infty$



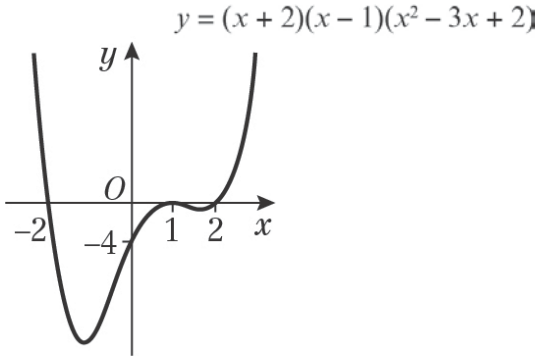
1 i $y = -(2x - 1)^3(x + 5)$
$0 = -(2x - 1)^3(x + 5)$
So $x = \frac{1}{2}$ or $x = -5$
The curve crosses the x-axis at $(\frac{1}{2}, 0)$ and $(-5, 0)$.
When $x = 0$, $y = -(-1)^3 \times 5 = 5$
So the curve crosses the y-axis at $(0, 5)$.
$x \to \infty$, $y \to -\infty$
$x \to -\infty$, $y \to -\infty$



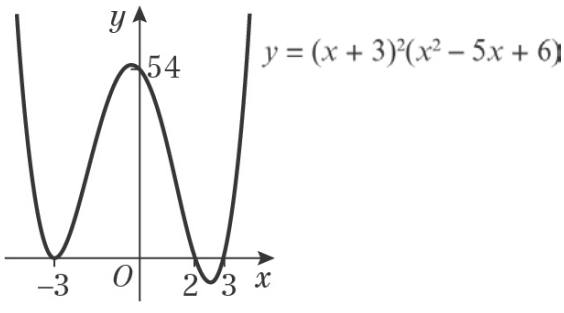
j $y = (x + 4)^4$
$0 = (x + 4)^4$
So $x = -4$
The curve touches the x-axis at $(-4, 0)$.
When $x = 0$, $y = 4^4 = 256$
So the curve crosses the y-axis at $(0, 256)$.
$x \to \infty$, $y \to \infty$
$x \to -\infty$, $y \to \infty$



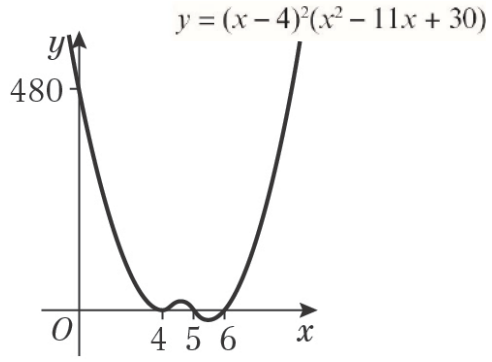
2 a $y = (x + 2)(x - 1)(x^2 - 3x + 2)$
$= (x + 2)(x - 1)(x - 1)(x - 2)$
$= (x + 2)(x - 1)^2(x - 2)$
$0 = (x + 2)(x - 1)^2(x - 2)$
So $x = -2$, $x = 1$ or $x = 2$
The curve crosses the x-axis at $(-2, 0)$ and $(2, 0)$ and touches it at $(1, 0)$.
When $x = 0$, $y = 2 \times (-1)^2 \times (-2) = -4$
So the curve crosses the y-axis at $(0, -4)$.
$x \to \infty$, $y \to \infty$
$x \to -\infty$, $y \to \infty$



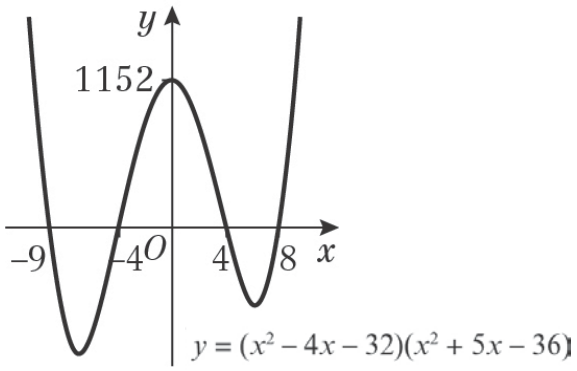
b $y = (x + 3)^2(x^2 - 5x + 6)$
$= (x + 3)^2(x - 2)(x - 3)$
$0 = (x + 3)^2(x - 2)(x - 3)$
So $x = -3$, $x = 2$ or $x = 3$
The curve crosses the x-axis at $(2, 0)$ and $(3, 0)$ and touches it at $(-3, 0)$.
When $x = 0$, $y = 3^2 \times (-2) \times (-3) = 54$
So the curve crosses the y-axis at $(0, 54)$.
$x \to \infty$, $y \to \infty$
$x \to -\infty$, $y \to \infty$



2 c $y = (x - 4)^2(x^2 - 11x + 30)$
$= (x - 4)^2(x - 5)(x - 6)$
$0 = (x - 4)^2(x - 5)(x - 6)$
So $x = 4$, $x = 5$ or $x = 6$
The curve crosses the x-axis at (5, 0) and (6, 0) and touches it at (4, 0).
When $x = 0$, $y = (-4)^2 \times (-5) \times (-6) = 480$
So the curve crosses the y-axis at (0, 480).
$x \to \infty$, $y \to \infty$
$x \to -\infty$, $y \to \infty$



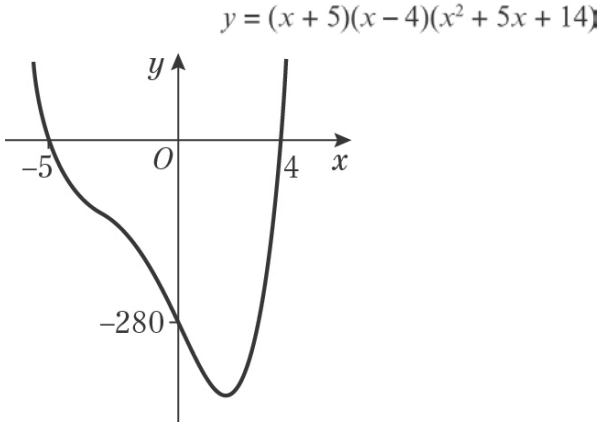
d $y = (x^2 - 4x - 32)(x^2 + 5x - 36)$
$= (x - 8)(x + 4)(x + 9)(x - 4)$
$0 = (x - 8)(x + 4)(x + 9)(x - 4)$
So $x = 8$, $x = -4$, $x = -9$ or $x = 4$
The curve crosses the x-axis at (8, 0), (−4, 0), (−9, 0) and (4, 0).
When $x = 0$, $y = (-8) \times 4 \times 9 \times (-4) = 1152$
So the curve crosses the y-axis at (0, 1152).
$x \to \infty$, $y \to \infty$
$x \to -\infty$, $y \to \infty$



3 a $y = x^4 + bx^3 + cx^2 + dx + e$
$y = (x + 2)(x + 1)(x - 2)(x - 3)$
When $x = 0$, $y = 2 \times 1 \times -2 \times -3 = 12$
So the curve crosses the y-axis at point P, which has coordinates (0, 12).

b $y = (x + 2)(x + 1)(x - 2)(x - 3)$
$= (x + 2)(x + 1)(x^2 - 5x + 6)$
$= (x + 2)(x^3 - 4x^2 + x + 6)$
$= x^4 - 2x^3 - 7x^2 + 8x + 12$
$b = -2$, $c = -7$, $d = 8$ and $e = 12$

4 $y = (x + 5)(x - 4)(x^2 + 5x + 14)$
The discriminant of the quadratic factor
$= b^2 - 4ac$
$= 5^2 - 4 \times 1 \times 14$
$= -31$, so there are no real roots.
$0 = (x + 5)(x - 4)(x^2 + 5x + 14)$
$x = -5$, $x = 4$ or $x^2 + 5x + 14 = 0$
The curve crosses the x-axis at (−5, 0) and (4, 0).
When $x = 0$, $y = 5 \times (-4) \times 14 = -280$
So the curve crosses the y-axis at (0, −280).
$x \to \infty$, $y \to \infty$
$x \to -\infty$, $y \to \infty$



Challenge

$y = ax^4 + bx^3 + cx^2 + dx + e$
$y = a(x + 1)^2(x - 3)^2$
When $x = 0$, $y = 3$:
$3 = a \times 1^2 \times (-3)^2$
$a = \frac{1}{3}$
$y = \frac{1}{3}(x + 1)^2(x - 3)^2$
$= \frac{1}{3}(x + 1)^2(x^2 - 6x + 9)$
$= \frac{1}{3}(x + 1)(x^3 - 5x^2 + 3x + 9)$
$= \frac{1}{3}(x^4 - 4x^3 - 2x^2 + 12x + 9)$
$= \frac{1}{3}x^4 - \frac{4}{3}x^3 - \frac{2}{3}x^2 + 4x + 3$
$a = \frac{1}{3}$, $b = -\frac{4}{3}$, $c = -\frac{2}{3}$, $d = 4$ and $e = 3$