Straight line graphs, Mixed Exercise 5

1 a Gradient $m = -\frac{5}{12}$, $(x_1, y_1) = (2, 1)$

The equation of the line is:

$y - y_1 = m(x - x_1)$

$y - 1 = -\frac{5}{12}(x - 2)$

$y - 1 = -\frac{5}{12}x + \frac{5}{6}$

$y = -\frac{5}{12}x + \frac{11}{6}$

b Substitute $(k, 11)$ into $y = -\frac{5}{12}x + \frac{11}{6}$

$11 = -\frac{5}{12}k + \frac{11}{6}$

$11 - \frac{11}{6} = -\frac{5}{12}k$

$\frac{55}{6} = -\frac{5}{12}k$

Multiply each side by 12:

$110 = 5k$

$k = -22$

2 a The gradient of AB is:

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{1}{3}$

So:

$\frac{(2k - 1) - 1}{8 - k} = \frac{1}{3}$

$\frac{2k - 1 - 1}{8 - k} = \frac{1}{3}$

$\frac{2k - 2}{8 - k} = \frac{1}{3}$

Multiply each side by $(8 - k)$:

$2k - 2 = \frac{1}{3}(8 - k)$

Multiply each term by 3:

$6k - 6 = 8 - k$

$7k - 6 = 8$

$7k = 14$

$k = 2$

b $k = 2$.

So A and B have coordinates (2, 1) and (8, 3).

2 b The equation of the line is:

$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$

$\frac{y - 1}{3 - 1} = \frac{x - 2}{8 - 2}$

$\frac{y - 1}{2} = \frac{x - 2}{6}$

Multiply each side by 2:

$y - 1 = \frac{1}{3}(x - 2)$

$y - 1 = \frac{1}{3}x - \frac{2}{3}$

$y = \frac{1}{3}x + \frac{1}{3}$

3 a The equation of L_1 is:

$y - y_1 = m(x - x_1)$

$y - 2 = \frac{1}{7}(x - 2)$

$y - 2 = \frac{1}{7}x - \frac{2}{7}$

$y = \frac{1}{7}x + \frac{12}{7}$

The equation of L_2 is:

$y - y_1 = (x - x_1)$

$y - 8 = -1(x - 4)$

$y - 8 = -x + 4$

$y = -x + 12$

b Solve $y = \frac{1}{7}x + \frac{12}{7}$ and $y = -x + 12$ simultaneously.

$-x + 12 = \frac{1}{7}x + \frac{12}{7}$

$12 = \frac{8}{7}x + \frac{12}{7}$

$10\frac{2}{7} = \frac{8}{7}x$

$x = \frac{10\frac{2}{7}}{\frac{8}{7}}$

$= 9$

Substitute $x = 9$ into $y = -x + 12$:

$y = -9 + 12$

$= 3$

The lines intersect at $C(9, 3)$.

4 a The equation of l is:

$$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$$

$$\frac{y-0}{6-0}=\frac{x-1}{5-1}$$

$$\frac{y}{6}=\frac{x-1}{4}$$

Multiply each side by 6:

$$y=6\frac{(x-1)}{4}$$

$$y=\frac{3}{2}(x-1)$$

$$y=\frac{3}{2}x-\frac{3}{2}$$

b Solve $2x + 3y = 15$ and $y=\frac{3}{2}x-\frac{3}{2}$ simultaneously.

Substitute:

$$2x+3\left(\tfrac{3}{2}x-\tfrac{3}{2}\right)=15$$

$$2x+\tfrac{9}{2}x-\tfrac{9}{2}=15$$

$$\tfrac{13}{2}x-\tfrac{9}{2}=15$$

$$\tfrac{13}{2}x=\tfrac{39}{2}$$

$$13x=39$$

$$x=3$$

Substitute $x = 3$ into $y=\frac{3}{2}x-\frac{3}{2}$:

$$y=\tfrac{3}{2}(3)-\tfrac{3}{2}$$

$$=\tfrac{9}{2}-\tfrac{3}{2}$$

$$=\tfrac{6}{2}$$

$$=3$$

The coordinates of C are (3, 3).

5 $(x_1, y_1) = (1, 3)$, $(x_2, y_2) = (-19, -19)$

The equation of L is:

$$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$$

$$\frac{y-3}{-19-3}=\frac{x-1}{-19-1}$$

$$\frac{y-3}{-22}=\frac{x-1}{-20}$$

5 Multiply each side by -22:

$$y-3=\tfrac{-22}{-20}(x-1)$$

$$y-3=\tfrac{11}{10}(x-1)$$

Multiply each term by 10:

$$10y-30=11(x-1)$$

$$10y-30=11x-11$$

$$10y=11x+19$$

$$0=11x-10y+19$$

The equation of L is $11x - 10y + 19 = 0$.

6 a $(x_1, y_1) = (2, 2)$, $(x_2, y_2) = (6, 0)$

The equation of l_1 is:

$$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$$

$$\frac{y-2}{0-2}=\frac{x-2}{6-2}$$

$$\frac{y-2}{-2}=\frac{x-2}{4}$$

Multiply each side by -2:

$$y-2=-\tfrac{1}{2}(x-2)\ \left(\text{Note:} -\tfrac{2}{4}=-\tfrac{1}{2}\right)$$

$$y-2=-\tfrac{1}{2}x+1$$

$$y=-\tfrac{1}{2}x+3$$

b The equation of l_2 is:

$$y-y_1=m(x-x_1)$$

$$y-0=\tfrac{1}{4}(x-(-9))$$

$$y=\tfrac{1}{4}(x+9)$$

$$y=\tfrac{1}{4}x+\tfrac{9}{4}$$

7 $A\left(1,3\sqrt{3}\right), B\left(2+\sqrt{3},3+4\sqrt{3}\right)$

The gradient of the line through A and B is:

$$\frac{y_2-y_1}{x_2-x_1}=\frac{3+4\sqrt{3}-3\sqrt{3}}{2+\sqrt{3}-1}$$

$$=\frac{3+\sqrt{3}}{1+\sqrt{3}}$$

7 Rationalising the denominator:

$$\frac{(3+\sqrt{3})\times(1-\sqrt{3})}{(1+\sqrt{3})\times(1-\sqrt{3})}=\frac{3-3\sqrt{3}+\sqrt{3}-3}{1-3}$$

$$=\frac{-2\sqrt{3}}{-2}$$

$$=\sqrt{3}$$

The equation of the line is:

$y=\sqrt{3}x+c$

Substituting $x = 1$ and $y = 3\sqrt{3}$ into $y=\sqrt{3}x+c$:

$3\sqrt{3}=\sqrt{3}+c$

$c=2\sqrt{3}$

The equation of line l is:

$y=\sqrt{3}x+2\sqrt{3}$

Line l meets the x-axis when $y = 0$.
When $y = 0$, $x = −2$.
C is the point (−2, 0).

8 a $A(−4, 6)$, $B(2, 8)$
The gradient of AB is:

$$\frac{y_2-y_1}{x_2-x_1}=\frac{8-6}{2-(-4)}$$

$$=\frac{2}{6}$$

$$=\frac{1}{3}$$

The gradient of a line perpendicular to AB is:

$$-\frac{1}{\frac{1}{3}}=-3$$

The equation of p is:

$y-y_1=m(x-x_1)$

$y-8=-3(x-2)$

$y-8=-3x+6$

$y=-3x+14$

b Substitute $x = 0$ in the equation for AB:
$y = −3(0) + 14 = 14$
The coordinates of C are (0, 14).

9 a The line passes through $A(0, 4)$ and is perpendicular to l: $2x − y − 1 = 0$.

$2x-y-1=0$

$2x-1=y$

$y=2x-1$

The gradient of $2x − y −1 = 0$ is 2.
The gradient of a line perpendicular to $2x − y −1 = 0$ is $-\frac{1}{2}$.
The equation of the line m is:

$y-y_1=m(x-x_1)$

$y-4=-\frac{1}{2}(x-0)$

$y-4=-\frac{1}{2}x$

$y=-\frac{1}{2}x+4$

Or, since A is a y-intercept, the equation can be written once the gradient is known i.e $y=-\left(\frac{1}{2}\right)x+4$.

b To find P, solve $y=-\frac{1}{2}x+4$ and $2x − y −1 = 0$ simultaneously.
Substitute:

$2x-\left(-\frac{1}{2}x+4\right)-1=0$

$2x+\frac{1}{2}x-4-1=0$

$\frac{5}{2}x-5=0$

$5x=10$

$x=2$

Substitute $x = 2$ into $y=-\frac{1}{2}x+4$:

$y=-\frac{1}{2}(2)+4$

$=-1+4$

$=3$

The lines intersect at $P(2, 3)$, as required.

c A line parallel to the line m has gradient $-\frac{1}{2}$.
The equation of the line n is:

$y-y_1=m(x-x_1)$

$y-0=-\frac{1}{2}(x-3)$

$y=-\frac{1}{2}x+\frac{3}{2}$

9 c To find Q solve $2x - y - 1 = 0$ and $y = -\frac{1}{2}x + \frac{3}{2}$ simultaneously.

Substitute:

$$2x - \left(-\frac{1}{2}x + \frac{3}{2}\right) - 1 = 0$$
$$2x + \frac{1}{2}x - \frac{3}{2} - 1 = 0$$
$$\frac{5}{2}x - \frac{5}{2} = 0$$
$$\frac{5}{2}x = \frac{5}{2}$$
$$x = 1$$

Substitute $x = 1$ into $y = -\frac{1}{2}x + \frac{3}{2}$:

$$y = -\frac{1}{2}(1) + \frac{3}{2}$$
$$= -\frac{1}{2} + \frac{3}{2}$$
$$= 1$$

The lines intersect at $Q(1, 1)$.

10 $A(0, -2)$ and $B(6, 7)$

The gradient of the line through A and B is:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - (-2)}{6 - 0}$$
$$= \frac{9}{6}$$
$$= \frac{3}{2}$$

The equation of the line through A and B is:

$y = \frac{3}{2}x + c$

Substituting $x = 0$ and $y = -2$ into $y = \frac{3}{2}x + c$:

$-2 = \frac{3}{2}(0) + c$ so $c = -2$

As in Q9, the point A is the y-intercept so the equation can be written once the gradient has been calculated.

l_1: $y = \frac{3}{2}x - 2$

l_2: $x + y = 8$

To find point D, solve simultaneously by substituting l_1 into l_2.

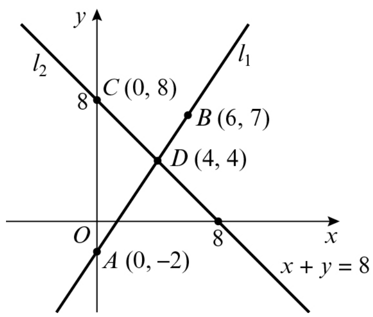
$$x + \frac{3}{2}x - 2 = 8$$
$$\frac{5}{2}x = 10$$
$$x = 4$$

10 when $x = 4$,

$4 + y = 8$,

$y = 4$

$\therefore$ D is the point $(4, 4)$.



The base of the triangle AC is 10 units.

The height of the triangle is 4 units.

Area ΔACD is $\frac{1}{2} \times 10 \times 4 = 20$ units^2

11 a $A(2, 16)$ and $B(12, -4)$

The equation of l_1 through A and B is:

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$
$$\frac{y - 16}{-4 - 16} = \frac{x - 2}{12 - 2}$$
$$\frac{y - 16}{-20} = \frac{x - 2}{10}$$

Multiply each side by -20:

$$y - 16 = -2(x - 2) \quad \left(\text{Note: } -\frac{20}{10} = -2\right)$$
$$y - 16 = -2x + 4$$
$$y = -2x + 20$$
$$2x + y = 20$$

b The equation of l_2 through $C(-1, 1)$ with gradient $\frac{1}{3}$ is:

$$y - y_1 = m(x - x_1)$$
$$y - 1 = \frac{1}{3}(x - (-1))$$
$$y - 1 = \frac{1}{3}(x + 1)$$
$$y - 1 = \frac{1}{3}x + \frac{1}{3}$$
$$y = \frac{1}{3}x + \frac{4}{3}$$

12 a $A(-1, -2)$, $B(7, 2)$ and $C(k, 4)$

The gradient of AB is:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{2-(-2)}{7-(-1)} = \frac{4}{8} = \frac{1}{2}$$

b Since ABC is a right angle the gradient of BC is:

$$\frac{-1}{\frac{1}{2}} = -2$$

$$So\ \frac{y_2 - y_1}{x_2 - x_1} = -2$$

$$\Rightarrow \frac{4-2}{k-7} = -2$$

$$\Rightarrow \frac{2}{k-7} = -2$$

Multiply each side by $(k - 7)$:

$$2 = -2(k-7)$$

$$2 = -2k + 14$$

$$2k = 12$$

$$k = 6$$

c The equation of the line passing through B and C is:

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y-2}{4-2} = \frac{x-7}{6-7}$$

$$\frac{y-2}{2} = \frac{x-7}{-1}$$

Multiply each side by 2:

$$y - 2 = -2(x-7)$$

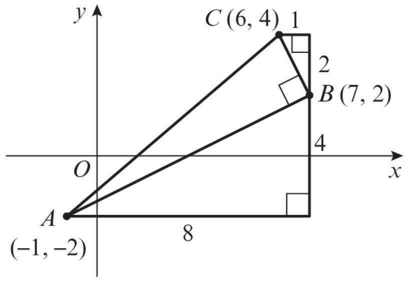
$$y - 2 = -2x + 14$$

$$y = -2x + 16$$

$$2x + y = 16$$

$$2x + y - 16 = 0$$

d Remember angle B is a right angle.



Use the diagram or the distance formula to find the lengths AB and BC.

$$AB = \sqrt{8^2 + 4^2} = \sqrt{80}$$

$$BC = \sqrt{1^2 + 2^2} = \sqrt{5}$$

$$\text{Area of } \Delta ABC = \tfrac{1}{2} \times \sqrt{80} \times \sqrt{5} = \tfrac{1}{2} \times \sqrt{400} = \tfrac{1}{2} \times 20 = 10 \text{ units}^2$$

13 a The equation of the line through $(-1, 5)$ and $(4, -2)$ is:

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y-5}{-2-5} = \frac{x-(-1)}{4-(-1)}$$

$$\frac{y-5}{-7} = \frac{x+1}{5}$$

Multiply each side by −35:

$$5(y-5) = -7(x+1)$$

$$5y - 25 = -7x - 7$$

$$\left(\text{Note: } \tfrac{-35}{-7} = 5 \text{ and } \tfrac{-35}{5} = -7\right)$$

$$7x + 5y - 25 = -7$$

$$7x + 5y - 18 = 0$$

13 b For the coordinates of A, substitute $y = 0$:

$$7x + 5(0) - 18 = 0$$
$$7x - 18 = 0$$
$$7x = 18$$
$$x = \tfrac{18}{7}$$

The coordinates of A are $\left(\tfrac{18}{7}, 0\right)$.

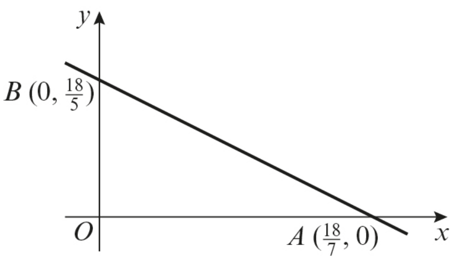
For the coordinates of B, substitute $x = 0$:

$$7(0) + 5y - 18 = 0$$
$$5y - 18 = 0$$
$$5y = 18$$
$$y = \tfrac{18}{5}$$

The coordinates of B are $\left(0, \tfrac{18}{5}\right)$.

The area of ΔOAB is:

$$\tfrac{1}{2} \times \tfrac{18}{7} \times \tfrac{18}{5} = \tfrac{162}{35}$$



14 a Rearrange l_1: $4y + x = 0$ into the form $y = mx + c$:

$$4y = -x$$
$$y = -\tfrac{1}{4}x$$

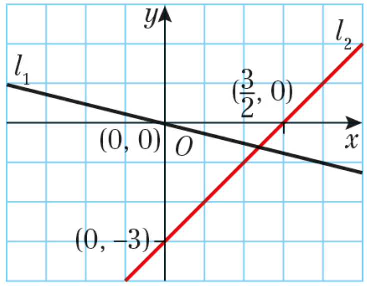
l_1 has gradient $-\tfrac{1}{4}$ and it meets the coordinate axes at (0, 0).
l_2 has gradient 2 and it meets the y-axis at (0, −3).
l_2 meets the x-axis when $y = 0$.

Substitute $y = 0$ into the equation:

$$0 = 2x - 3$$
$$2x = 3$$
$$x = \tfrac{3}{2}$$

l_2 meets the x-axis at $\left(\tfrac{3}{2}, 0\right)$.

14 a



b Solve $4y + x = 0$ and $y = 2x - 3$ simultaneously.
Substitute:

$$4(2x - 3) + x = 0$$
$$8x - 12 + x = 0$$
$$9x - 12 = 0$$
$$9x = 12$$
$$x = \tfrac{12}{9}$$
$$x = \tfrac{4}{3}$$

Now substitute $x = \tfrac{4}{3}$ into $y = 2x - 3$:

$$y = 2\left(\tfrac{4}{3}\right) - 3$$
$$= \tfrac{8}{3} - 3$$
$$= -\tfrac{1}{3}$$

The coordinates of A are $\left(\tfrac{4}{3}, -\tfrac{1}{3}\right)$.

c The gradient of l_1 is $-\tfrac{1}{4}$.
The gradient of a line perpendicular to l_1 is $-\dfrac{1}{-\frac{1}{4}} = 4$.

The equation of this line is:

$$y - y_1 = m(x - x_1)$$
$$y - \left(-\tfrac{1}{3}\right) = 4\left(x - \tfrac{4}{3}\right)$$
$$y + \tfrac{1}{3} = 4x - \tfrac{16}{3}$$
$$y = 4x - \tfrac{17}{3}$$

Multiply each term by 3:

$$3y = 12x - 17$$
$$0 = 12x - 3y - 17$$

The equation of the line is $12x - 3y - 17 = 0$.

15 a $A(4, 6)$ and $B(12, 2)$
The gradient of the line l_1 through A and B is:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{2-6}{12-4} = \frac{-4}{8} = -\frac{1}{2}$$

The equation of l_1 is:
$y = -\frac{1}{2}x + c$
Substituting $x = 4$ and $y = 6$ into $y = -\frac{1}{2}x + c$:
$6 = -\frac{1}{2}(4) + c$
$c = 8$
$y = -\frac{1}{2}x + c$
$x + 2y - 16 = 0$

b The gradient of the line l_2 is $-\frac{2}{3}$, the y-intercept is 0.
$y = -\frac{2}{3}x$

c Solve $x + 2y - 16 = 0$ and $y = -\frac{2}{3}x$ simultaneously.

$$x + 2(-\tfrac{2}{3}x) - 16 = 0$$
$$x - \tfrac{4}{3}x - 16 = 0$$
$$-\tfrac{1}{3}x = 16$$
$$x = -48$$

When $x = -48$:
$y = -\frac{2}{3}(-48)$
$y = 32$
C is the point $(-48, 32)$.

d The gradient of OA is:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{6-0}{4-0} = \frac{3}{2}$$

d The gradient of OC is:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{32-0}{(-48)-0} = -\frac{2}{3}$$

$\frac{3}{2} \times -\frac{2}{3} = -1$.

Therefore the lines OA and OC are perpendicular.

e
$$OA = \sqrt{(4-0)^2 + (6-0)^2} = \sqrt{52} = \sqrt{4 \times 13} = 2\sqrt{13}$$
$$OC = \sqrt{((-48)-0)^2 + (32-0)^2} = \sqrt{3328} = \sqrt{256 \times 13} = 16\sqrt{13}$$

f Area of $\Delta OAB = \frac{1}{2} \times 16\sqrt{13} \times 2\sqrt{13}$
$= 208$ units^2

16 a $(4a, a)$ and $(-3a, 2a)$
The distance d between the points is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{((-3a) - 4a)^2 + (2a - a)^2}$$
$$= \sqrt{49a^2 + a^2}$$
$$= \sqrt{50a^2}$$
$$= \sqrt{25 \times 2a^2}$$
$$= 5a\sqrt{2}$$

b For points $(4, 1)$ and $(-3, 2)$, $a = 1$.
Substitute $a = 1$ into $5a\sqrt{2}$.
Distance $= 5\sqrt{2}$

c For points $(12, 3)$ and $(-9, 6)$, $a = 3$.
Substitute $a = 3$ into $5a\sqrt{2}$.
Distance $= 15\sqrt{2}$

16 d For points (−20, −5) and (15, −10), $a = -5$.

Substitute $a = -5$ into $5a\sqrt{2}$.

Distance = $25\sqrt{2}$

17 a (x, y) is a point on $y = 3x$, so its coordinates are $(x, 3x)$.

The distance between $A(-1, 5)$ and $(x, 3x)$ is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(x-(-1))^2 + (3x-5)^2}$$
$$= \sqrt{x^2 + 2x + 1 + 9x^2 - 30x + 25}$$
$$= \sqrt{10x^2 - 28x + 26}$$

b $\sqrt{10x^2 - 28x + 26} = \sqrt{74}$

$$10x^2 - 28x + 26 = 74$$
$$10x^2 - 28x - 48 = 0$$
$$5x^2 - 14x - 24 = 0$$
$$(5x+6)(x-4) = 0$$

$x = -\frac{6}{5}$ or $x = 4$

When $x = -\frac{6}{5}$, $y = 3(-\frac{6}{5}) = -\frac{18}{5}$

When $x = 4$, $y = 3(4) = 12$

The points are $B(-\frac{6}{5}, -\frac{18}{5})$ and $C(4, 12)$.

c The gradient of the line $y = 3x$ is 3, so the perpendicular line has gradient $-\frac{1}{3}$.

Its equation is:

$y = -\frac{1}{3}x + c$

When $x = -1$ and $y = 5$:

$5 = -\frac{1}{3}(-1) + c$

$c = \frac{14}{3}$

$y = -\frac{1}{3}x + \frac{14}{3}$

d Solving $y = -\frac{1}{3}x + \frac{14}{3}$ and $y = 3x$ simultaneously:

$3x = -\frac{1}{3}x + \frac{14}{3}$

17 d $9x = -x + 14$

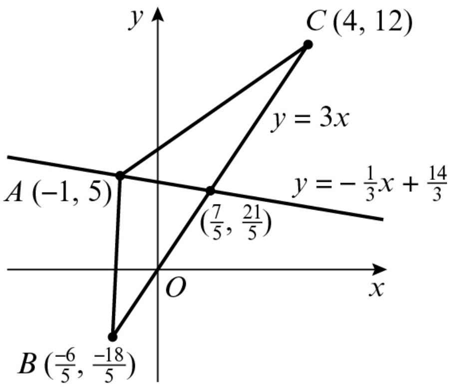
$10x = 14$

$x = \frac{7}{5}$

When $x = \frac{7}{5}$: $y = 3(\frac{7}{5}) = \frac{21}{5}$

The point is $(\frac{7}{5}, \frac{21}{5})$

e



$$BC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(4-(-\tfrac{6}{5}))^2 + (12-(-\tfrac{18}{5}))^2}$$
$$= \sqrt{(\tfrac{26}{5})^2 + (\tfrac{78}{5})^2}$$
$$= \sqrt{\tfrac{6760}{25}}$$

Distance from $A(-1, 5)$ to $(\frac{7}{5}, \frac{21}{5})$ is:

$$\sqrt{(\tfrac{7}{5}-(-1))^2 + (\tfrac{21}{5}-5)^2}$$
$$= \sqrt{(\tfrac{12}{5})^2 + (-\tfrac{4}{5})^2}$$
$$= \sqrt{\tfrac{160}{25}}$$

Area of triangle is:

$\frac{1}{2} \times \sqrt{\frac{6760}{25}} \times \sqrt{\frac{160}{25}} = \frac{520}{25}$

= 20.8 units^2

18 a Gradient of the line of best fit is:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{30\,000 - 23\,000}{3900 - 3200}$$
$$= \frac{7000}{700}$$
$$= 10$$

18 b $C = aP + b$
$a = 10$
Using the point (3200, 23 000):
$23\,000 = 10(3200) + b$
$b = -9000$
$C = 10P - 9000$

c a is the gradient, which is the increase in carbon dioxide emissions in millions of tonnes for every 1 million tonnes of oil pollution.

d The model is not valid for small values of P as a negative amount of carbon dioxide emissions is not possible.

Challenge

1 (−2, −2), B(13, 8) and C(−4, 14)

The equation of AB is:

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\frac{y-(-2)}{8-(-2)} = \frac{x-(-2)}{13-(-2)}$$

$$\frac{y+2}{10} = \frac{x+2}{15}$$

$$3y + 6 = 2x + 4$$

$$3y = 2x - 2$$

$$y = \tfrac{2}{3}x - \tfrac{2}{3}$$

The gradient of AB = $\frac{2}{3}$.

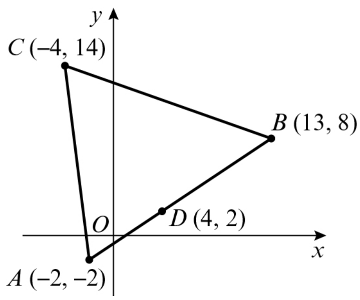
The gradient of a line perpendicular to AB = $-\frac{3}{2}$.

The equation of the perpendicular to AB through C(−4, 14) is:

$$y - 14 = -\tfrac{3}{2}(x - (-4))$$

$$y - 14 = -\tfrac{3}{2}x - 6$$

$$y = -\tfrac{3}{2}x + 8$$



Point D is where the line and the perpendicular intersect.
Solve the equations $y = \frac{2}{3}x - \frac{2}{3}$ and $y = -\frac{3}{2}x + 8$ simultaneously.

$$\tfrac{2}{3}x - \tfrac{2}{3} = -\tfrac{3}{2}x + 8$$

Multiply each term by 6.

$$4x - 4 = -9x + 48$$

$$13x = 52$$

$$x = 4$$

Now substitute $x = 4$ into $y = -\frac{3}{2}x + 8$:

$$y = -\tfrac{3}{2}(4) + 8$$

$$y = 2$$

D is the point (4, 2).

Challenge

1 $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(13-(-2))^2 + (8-(-2))^2}$

$= \sqrt{15^2 + 10^2}$

$= \sqrt{325}$

$CD = \sqrt{(4-(-4))^2 + (2-14)^2}$

$= \sqrt{8^2 + (-12)^2}$

$= \sqrt{208}$

Area of $\Delta ABC = \frac{1}{2} \times \sqrt{325} \times \sqrt{208}$

$= 130$ units^2

2 $A(3, 8)$, $B(9, 9)$ and $C(5, 2)$

The gradient of AB is:

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{9-8}{9-3}$

$= \frac{1}{6}$

l_1 is perpendicular to AB, so its gradient is -6. It passes through C, so its equation is:

$y = -6x + c$

$2 = -6(5) + c$

$c = 32$

The equation of l_1 is $y = -6x + 32$.

The gradient of BC is:

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{2-9}{5-9}$

$= \frac{7}{4}$

l_2 is perpendicular to BC, so its gradient is $-\frac{4}{7}$. It passes through A, so its equation is:

$y = -\frac{4}{7}x + c$

$8 = -\frac{4}{7}(3) + c$

$c = \frac{68}{7}$

The equation of l_2 is $y = -\frac{4}{7}x + \frac{68}{7}$.

2 The gradient of AC is:

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{2-8}{5-3}$

$= -\frac{6}{2}$

$= -3$

l_3 is perpendicular to BC, so its gradient is $\frac{1}{3}$. It passes through B, so its equation is:

$y = \frac{1}{3}x + c$

$9 = \frac{1}{3}(9) + c$

$c = 6$

The equation of l_3 is $y = \frac{1}{3}x + 6$.

Solve l_1 and l_2 simultaneously.

$-6x + 32 = -\frac{4}{7}x + \frac{68}{7}$

$-42x + 224 = -4x + 68$

$38x = 156$

$x = \frac{78}{19}$

$y = -6(\frac{78}{19}) + 32 = \frac{140}{19}$

Their point of intersection is $(\frac{78}{19}, \frac{140}{19})$.

Now solve l_2 and l_3 simultaneously.

$-\frac{4}{7}x + \frac{68}{7} = \frac{1}{3}x + 6$

$-12x + 204 = 7x + 126$

$19x = 78$

$x = \frac{78}{19}$

$y = \frac{1}{3}(\frac{78}{19}) + 6 = \frac{140}{19}$

Their point of intersection is $(\frac{78}{19}, \frac{140}{19})$.

Therefore, l_1, l_2 and l_3 all intersect at $(\frac{78}{19}, \frac{140}{19})$.

3 $A(0, 0)$, $B(a, b)$ and $C(c, 0)$

The gradient of AB is:

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{b-0}{a-0}$

$= \frac{b}{a}$

l_1 is perpendicular to AB so its gradient is $-\frac{a}{b}$.

3 It passes through C so its equation is:

$$y = -\frac{a}{b}x + k \text{ where } k \text{ is the } y\text{-intercept.}$$

At C, $x = c$ and $y = 0$.

$$0 = -\frac{ac}{b} + k$$

$$k = \frac{ac}{b}$$

The equation of line l_1 is:

$$y = -\frac{a}{b}x + \frac{ac}{b}$$

The gradient of BC is:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - b}{c - a}$$

$$= \frac{-b}{c - a}$$

l_2 is perpendicular to BC so its gradient is $\frac{c-a}{b}$.

It passes through A, so its equation is:

$$y = \frac{c-a}{b}x + K \text{ where } K \text{ is the } y\text{-intercept.}$$

At A, $x = 0$, $y = 0$.

$$0 = \frac{c-a}{b}(0) + K$$

$$K = 0$$

The equation of line l_2 is $y = \frac{c-a}{b}x$.

l_3 is the vertical line through (a, b), so its equation is $x = a$.

Solve l_1 and l_3 simultaneously.

$$y = -\frac{a^2}{b} + \frac{ac}{b}$$

$$= \frac{a(c-a)}{b}$$

The intersection of l_1 and l_3 is the point $\left(a, \frac{a(c-a)}{b}\right)$.

Now solve l_2 and l_3 simultaneously.

$$y = \frac{a(c-a)}{b}$$

The intersection of l_2 and l_3 is the point $\left(a, \frac{a(c-a)}{b}\right)$.

Therefore, l_1, l_2 and l_3 all intersect at $\left(a, \frac{a(c-a)}{b}\right)$.